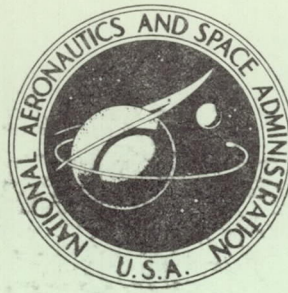


NASA CONTRACTOR
REPORT



NASA CR-2880

NASA CR-2880

F-8C ADAPTIVE FLIGHT CONTROL LAWS

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • SEPTEMBER 1977

1. Report No. NASA CR-2880		2. Government Accession No.		3. Recipient's Catalog No.	
4. Title and Subtitle F-8C ADAPTIVE FLIGHT CONTROL LAWS				5. Report Date September 1977	
				6. Performing Organization Code	
7. Author(s) G. L. Hartmann, C. A. Harvey, G. Stein, D. N. Carlson, and R. C. Hendrick				8. Performing Organization Report No. 76SRC16	
9. Performing Organization Name and Address Honeywell, Inc., Systems and Research Center 2600 Ridgway Parkway, N. E. Minneapolis, Minnesota 55413				10. Work Unit No.	
				11. Contract or Grant No. NAS 1-13383	
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Hampton, Virginia 23665				13. Type of Report and Period Covered	
				14. Sponsoring Agency Code	
15. Supplementary Notes Langley technical monitors: Jarrell R. Elliott and Joseph Gera Final report.					
16. Abstract Three candidate digital adaptive control laws were designed for NASA's F-8C DFBW aircraft. Each design used the same control laws but adjusted the gains with a different adaptative algorithm. The three adaptive concepts were: high-gain limit cycle, Liapunov-stable model tracking, and maximum likelihood estimation. Sensors were restricted to conventional inertial instruments (rate gyros and accelerometers) without use of air-data measurements. Performance, growth potential, and computer requirements were used as criteria for selecting the most promising of these candidates for further refinement. The maximum likelihood concept was selected primarily because it offers the greatest potential for identifying several aircraft parameters and hence for improved control performance in future aircraft application. In terms of identification and gain adjustment accuracy, the MLE design is slightly superior to the other two, but this has no significant effects on the control performance achievable with the F-8C aircraft. The maximum likelihood design is recommended for flight test, and several refinements to that design are proposed.					
17. Key Words (Suggested by Author(s)) Flight control Maximum likelihood estimation Adaptive Liapunov model tracking Identification High-gain limit cycle Digital F-8C				18. Distribution Statement Unclassified - unlimited. Subject Category 08	
19. Security Classif. (of this report) Unclassified		20. Security Classif. (of this page) Unclassified		21. No. of Pages 332	
				22. Price* \$10.00	

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SECTION 1

PROGRAM OVERVIEW AND SUMMARY

INTRODUCTION

NASA is conducting a flight control research program in digital fly-by-wire technology using a modified F-8C aircraft. The first phase of this program used Apollo hardware to demonstrate the practicality of digital fly-by-wire in an actual test vehicle. For the second phase, conventional aircraft sensors and a large floating point digital computer are being utilized to test advanced control laws and redundancy concepts. For control law studies, the new computer's speed, memory, and floating point capability removes most of the constraints imposed by more typical flight control machines.

As part of this research activity, Honeywell initiated work in 1974 under Contract NAS1-13383 to provide a system of digital adaptive flight control laws for flight test in Phase II. These control laws were to be adaptive in the sense of altering the control law on the basis of normal sensed information. One adaptive design was to be refined for flight test from a comparison of several candidate concepts. As part of the ground rules, measurements were restricted to rate gyros, accelerometers, and servo position. Air-data was excluded because aircraft like the F-8C, whose performance requirements are readily satisfied with air-data-scheduled control laws, benefit most directly from adaptive control through the elimination of air-data sensors. This will be particularly valuable for future aircraft requiring flight critical redundant control systems and sensors. The control laws were further constrained to be compatible with the existing airframe without structural modification. Hence, they use only existing elevator, rudder, and ailerons as control effectors--each powered by existing actuators.

Three candidate control algorithms were successfully designed from distinct adaptive concepts:

- Gain adjustment based on self-excited limit cycles,
- Gain adjustment based on explicit identification with Liapunov-stable model trackers, and
- Gain adjustment based on explicit identification with maximum likelihood estimation (MLE).

channel implementation. Several Kalman filter channels operate at fixed locations in parameter space. Likelihood functions are computed for each. Sensitivity equations are then solved only for the maximum likelihood channel and used to interpolate from there to the correct parameter value.

Five parallel channels suffice to handle the F-8C aircraft over its entire operational flight envelope. They estimate three parameters: surface effectiveness (M_{δ_e}), pitching moment due to angle-of-attack (M_q), and airspeed (V). Estimation accuracy depends strongly on the signal levels in the control loop. For the small test signals tolerable in operational situations, errors of 10 to 20 percent in M_{δ_e} and 20 to 30 percent in M_q and V are typical in six-degree-of-freedom simulation runs. Theoretical accuracy analyses confirm these error levels. The gain adjustment in the pitch and lateral control laws is a function of estimated M_{δ_e} only.

CONCEPT COMPARISON

The preceding three designs were compared on the basis of performance, growth potential, and computer requirements. An overview of the comparison is given in Table 1. The MLE design was selected primarily on the basis of growth potential.

TABLE 1. OVERALL COMPARISON OF CONCEPTS

Characteristic	Model tracker	High-gain limit cycle	Maximum likelihood estimation
Performance	Acceptable to good	Good	Good to excellent
Growth potential	Low--limited to single variable explicit gain schedule	Low--limited to single variable implicit adaptation	High--multiple parameter gain adjustment possible
Computer requirements ^a	Minimal	Low	Medium

^a These requirements are relative to AP-101 capacity.

The entire control system is expected to fit comfortably in the F-8C's AP-101 flight computer. The control law plus adaptive algorithm (excluding redundancy management) are estimated to consume less than one-half of the frame time available on that machine (20 msec for a 50 Hz update rate) and to require approximately 2500 memory locations.

SECTION 2

INTRODUCTION

BACKGROUND

NASA is presently conducting research in digital fly-by-wire technology using a modified F-8C aircraft as the test vehicle. In 1972-73 during the first phase of this program, the F-8C was successfully flown using an Apollo guidance computer, an inertial measurement unit, an analog (electrical) backup system, and an electrohydraulic actuation system. The purpose of these flights was to demonstrate the practicality of a digital fly-by-wire flight control system in an actual test vehicle.¹

The second phase of the research program supports technology development of advanced, reliable control systems for future aircraft. Primary objectives of the second phase are to demonstrate benefits of advanced control laws and to study redundant flight control mechanizations. The test aircraft will use a triplex AP-101 digital computer and triply-redundant aircraft gyros and accelerometers. Dual air-data measurements of angle-of-attack, Mach number, and pressure altitude will be available as well as a single sideslip sensor. Of special importance is the extensive computing power provided by the large, floating point computer in the test aircraft. This capability makes it possible to flight test advanced control laws without the constraints imposed by less capable flight control computers.

To date, two sets of control laws have been developed for flight experimentation in Phase II: a "CCV Package" and an "Adaptive Package." The first package is described in Reference 2. It provides basic command augmentation functions, outer loops, and control modes for various Control Configured Vehicle concepts applicable to fighter aircraft. The second control law package is described in this report. It consists of the same command augmentation functions found in the CCV package but adds explicit on-line identification capability for adaptive control. In addition to these packages, research has also proceeded on several other candidate adaptive algorithms for the F-8C^{3,4,5} and on sensor fault detection and isolation algorithms.^{6,7} Some of these may reach flight experimentation later in the program.

MOTIVATION FOR ADAPTIVE CONTROL

NASA's primary reason for studying adaptive control laws for the F-8 aircraft is to identify potential performance and system effectiveness benefits offered by modern adaptation/identification concepts and to demonstrate these in an actual digital flight control environment. Relative to these objectives, adaptive control is viewed in this report as one of several competing ways to achieve successful control actions in the face of aircraft plant uncertainties. Whether it offers benefits for a particular design application then depends on two primary factors: 1) the nature and magnitude of plant uncertainties, and 2) alternate methods available to achieve successful control. The first factor determines feasibility. Most adaptive techniques work well for some types of uncertainties but not for others. The second determines viability. Adaptive controls may or may not be competitive with alternate approaches on economic or operational grounds.

For inner-loop stability and command augmentation of fighters, the feasibility factor tends to be positive. Plant uncertainties generally take the form of unknown parameters in an otherwise known model structure (i.e., coefficients of linearized equations of motion). Their range of uncertainty is largely due to widely varying flight environments (dynamic pressure, Mach, angle-of-attack) and large configuration variations (c.g. locations, fuel and payload, geometry). However, individual coefficients are strongly interrelated, and only a few must be known accurately in order to design good control laws. As a net result, the important uncertainties reduce to a small number of slowly varying parameters. They usually include M_0 , sometimes also M_α , and occasionally other surface effectiveness coefficients. Adaptive controls based either on explicit identification or on implicit control gain adjustment stand a good chance of success with these kinds of uncertainties. This is demonstrated by three separate F-8C adaptive designs and simulator evaluations presented in this report and will be verified, hopefully, in upcoming F-8C adaptive flight tests.

Two alternate approaches for dealing with uncertain aircraft dynamics are to use low-gain flight control laws which can tolerate the full range of uncertainties directly or to provide external air-data for gain adjustment. The first approach is generally ruled out on the basis of performance. Low-gain systems capable of flying the entire flight envelope are unable to satisfy performance requirements for most of today's fighters and will be even less acceptable for future conditionally stable relaxed-static-stability aircraft. The second approach is more successful. It exploits the strong correlations

which exist between externally measurable air-data variables ($\bar{q}$, Mach, α) and unknown aircraft parameters. Thanks to these correlations, control law gains which are originally computed as functions of M_0 , M_α , etc. can be approximated in terms of $\bar{q}$, Mach, and/or α . Although not as good as the original functions, such approximated "air-data schedules" are adequate to meet performance requirements in most current flight control problems.

Given these alternatives, the competition among flight control design techniques reduces to a tradeoff between the economics, reliability, and operational features of air-data-scheduled systems versus adaptive systems. Historically, this tradeoff has favored adaptive techniques because sufficiently accurate and reliable air-data sensors were difficult to build and maintain. In fact, the desire to avoid such sensors was the primary motivation for most adaptive control development efforts of the 1960's, including NASA's X-15 research program,^{6, 8, 9, 10, 11, 12, 13, 14} the F-101 flight tests and X-20 simulations,^{15, 16} the production F-111,¹⁷ and several more recent flight tests with the F-4.^{18, 19, 20*} Of course, the measurement issue also motivated sensor development efforts. These have produced improved air-data systems which have now reversed the tradeoff, making air-data scheduling the preferred approach in most current single-channel analog autopilots.

The present status of the adaptive versus air-data trade-off is not expected to be permanent, however. There are three important trends which will affect future choices. The first of these is digital control. Flight control systems are now being mechanized in on-board digital computers²¹ whose steadily decreasing price tags per function promise to reduce the costs of adaptation substantially.

The second trend is redundancy. Future control configured fly-by-wire aircraft may need dual, triple, or even quad redundant flight control systems to insure safety of flight. Air-data-scheduled approaches will then suffer the added cost and complexity of multiple air-data sensing systems, and they will also face the difficult problem of finding suitable probe locations for adequate data quality and channel tracking (to permit effective redundancy management) but with enough dispersion to avoid common hazards.

* All of these aircraft can be flown quite well with approximated air-data schedules, provided that accurate measurements are available.

The final trend is increased control performance. Several anticipated flight control applications appear to require control gain functions which cannot be adequately approximated by air-data schedules. Prominent examples are active control of flexure^{22, 23} and coupled control modes.²⁴ Successful flexure control laws, for instance, have been found to depend critically on accurate vehicle data: mode frequencies, mode shapes, rigid body/flexure/control interactions. Such data at structural frequencies are difficult to acquire. The same is true for coupled control modes which actuate several surfaces simultaneously in order to achieve deliberate coupling or decoupling of responses (e.g., flight path changes without attitude changes). Here the relative surface effectiveness values and several other stability derivatives must be known with high accuracy.

These flight control technology trends provide strong motivation for continued research and development in adaptive control. Studies with the Phase II F-8C are particularly appropriate because its redundant, digital, fly-by-wire configuration is a forerunner of the evolving technology. Of course, all concepts and design processes studied should be general enough to apply to other aircraft as well. This is important because the F-8C itself is not a state-of-the-art fighter nor is it difficult to control with standard air-data scheduling methods.

CANDIDATE ADAPTIVE CONTROL LAWS FOR THE F-8C

This report presents design details and simulator evaluation results for three candidate adaptive control laws for the Phase II aircraft. Each design provides basic command augmentation functions using only inner-loop inertial sensors (gyros and accelerometers) and no air-data measurements. Hence, each is a potential competitor with air-data-scheduled methods. The purpose of these designs is to establish feasibility of adaptive controls in a digital fly-by-wire setting and to examine the relative merits of different adaptive techniques. They are not intended to demonstrate control performance superiority over air-data scheduling (the F-8C offers little hope here) or to show explicit economic, reliability, or operational advantages. The latter must wait for detailed hardware designs and trade-offs. The three designs achieve adaptation by combining existing control laws from the "CCV Package"² with different on-line adaptive gain adjustment processes. The gain adjustment processes are:

1. Explicit parameter identification via Liapunov-designed model tracking with gains stored as functions of parameters,

2. Explicit parameter identification via maximum likelihood estimation (MLE) with gains stored as functions of parameters, and
3. Direct gain adjustment to maintain self-excited limit cycles.

These specific adaptive techniques were selected from the brief survey of applicable adaptive concepts summarized in Table 2. This survey categorizes various proposed adaptive methods into four groups:

1. Optimal Adaptive

These are concepts based on the "dual control" problem originally posed by Feldbaum.²⁵ They involve optimal identification and control done simultaneously according to an overall criterion of goodness. This problem is so difficult to solve that to date only approximate solutions are available even for trivial cases.

2. Performance Adaptive (implicit)

These are concepts based on on-line measurement and adjustment of performance variables. Included are high-gain, model-following schemes and model reference systems.

3. Separate Identification and Control (explicit)

These concepts are based on assumed separability of the identification function and the control function in adaptive control. While this has been shown to be suboptimal in the "dual control" sense, the assumption seems reasonable in many cases. Table 2 lists several proposed approaches for the separated functions of identifying and recomputing the control law.

4. Learning/Self-Organizing

This group consists of concepts with "memory." Once a control law has been found to work well for a recognizable environment, the law is stored in memory and called for again when the same environment reappears. The "control law" in this case includes not only gain parameters, but the entire structure of the relationship between measurements and control commands.

TABLE 2. ADAPTIVE CONCEPTS

Optimal adaptive			
Dual control approximations		[26-29]	
Performance adaptive (implicit)			
High-gain model following			
→	Limit cycle control	[8-14]	
→	Energy balance control	[15, 16]	
→	Damping ratio control	[17]	
→	Test signal detection	[18]	
Model reference			
	Performance gradient design	[30]	
	Liapunov design	[31-34]	
Separate identification and control (explicit)			
Identification		Control	
Batch processors		Algebraic methods	
	Equation error least squares	Deadbeat control [35, 44]	
	Output error least squares	Pole-zero placement [52]	
	Maximum likelihood	Equation coefficient placement [32]	
	Instrumental variables	Optimization methods	
Recursive processors		Ricatti equation solutions [46, 50]	
	Least squares	Single stage optimization [53, 54]	
	Extended Kalman filtering		
→	Stochastic approximation	→	Stored solutions
→	Model tracking	→	Constant bandwidth [19, 20, 55]
Learning/Self-organizing			
Trainable controllers		[56, 57]	
Learning systems		[58]	

Several references are provided under each category to describe particular techniques more fully. These were selected to treat flight control applications where possible. However, they should be viewed as representative references only. Given the vast adaptive literature, no attempt was made to be all-inclusive.

The surveyed concepts range from ideas which exist only on paper to systems already tested in flight. The latter are highlighted by solid arrows in the table. They include four high-gain, model-following designs:

1. Limit-cycle control (operational on the X-15⁸),
2. Energy balance control (flight tested on the F-101¹⁵ and demonstrated for the X-20 on iron-bird simulations with triply-redundant control hardware¹⁶),
3. Damping ratio control (operational on the F-111¹⁷),
4. Test signal detection (flight tested on an F-4¹⁸),

and two explicit model-tracking identification schemes with stored gain functions:

1. The "Navy Adaptable System" (flight tested on F-4¹⁹), and
2. A simplified "SIDAC" system (flight tested on an F-4²⁰).

Selections of candidate approaches from Table 2 were made to span a range of performance and complexity, to maximize flight-test value, and yet to bound the risk of achieving at least one flight-test-quality design. No concepts were selected from the Optimal Adaptive and Learning/Self-Organizing categories, for example, because these have only limited development status with respect to practical applications. A high-gain limit-cycle system was selected because it offers very low risk (versions have flown) yet still has flight-test value if the design can demonstrate that improved limit-cycle detection and tracking logic can overcome weaknesses of earlier analog systems. * The maximum likelihood procedure was chosen as representative of recent modern estimation theoretic approaches which promise very good identification performance at the expense of on-board computer complexity, while the model-tracking approach was chosen to examine identification capability with more modest computer requirements.

* This is discussed in more detail in Section 10.

Successful simulator-quality designs were developed for these three selected adaptive concepts, and each was evaluated on NASA Langley's F-8C simulator.⁵⁹ As expected, control performance for all three was roughly equivalent. Identification capability turned out to be best for the MLE concept. Hence, this design was refined further and streamlined to fit into the AP-101 flight computer. It is now recommended for flight-test implementation.

DOCUMENT ORGANIZATION

This report is subdivided into 12 sections. The next section, Section 3, presents a list of symbols used throughout. Operational requirements and ground rules for the adaptive designs are set forth in Section 4; aircraft models and simulations are summarized in Section 5; and Section 6 presents basic theoretical identifiability limitations for the F-8C which are imposed by the ground rules of Section 4. The common CCV control law structure for all three concepts is then described in Section 7. Sections 8, 9 and 10 are devoted to the individual design details for the maximum likelihood, model-tracking, and high-gain limit-cycle concepts, respectively. Concept comparisons and selection of the most promising concept for flight test are described in Section 11, and Section 12 presents the conclusions and recommendations.

SECTION 3

SYMBOLS

Operators

$\text{Arg} \{ \min_x f(x) \}$	- minimizing argument of function $f(\cdot)$
$E(\cdot)$	- mathematical expectation
$\exp(\cdot)$	- exponential
$\ln(\cdot)$	- natural logarithm
$\ln \det(\cdot)$	- natural log of determinant
$p(x/y)$	- conditional probability distribution of x given y
∇^2	- Laplace operator
z	- delay operator
$(\dot{\cdot}) = \frac{d}{dt}$	- time derivative
$\Delta(\cdot)$	- increment
$\nabla(\cdot)$	- gradient vector with respect to parameter vector ζ
$\nabla^2(\cdot)$	- second partial derivative matrix with respect to parameters ζ
$\nabla_p(\cdot)$	- p -th component of $\nabla(\cdot)$
Σ	- summation
$ x $	- absolute value
$\ x\ _M^2$	- quadratic form $x^T M x$

SYMBOLS (Continued)

Superscripts

$\hat{(\)}$	- estimated value
$\bar{(\)}$	- one-step predicted value
$(\)^{(i)}$	- value for parallel channel i
$(\)^\circ$	- nominal value

Subscripts

$(\)_{\text{trm}}$	- value at trimmed flight
$(\)_m$	- (1) measured value (2) model value
$(\)_{\text{WL}}$	- water line value
$(\)_k$	- value at time t_k

Upper Case Symbols

A	- discrete system dynamics matrix
B	- (1) discrete system input matrix (2) residual covariance matrix
B _y	- sensor bias
C	- discrete system constant input matrix
C*	- response variable $N_z + V_{co} q$

SYMBOLS (Continued)

D	- (1) measurement matrix, y due to u (2) design parameter matrix in model tracker equations
F	- continuous system dynamics matrix
$G_{ug}(S), G_{vg}(S), G_{wg}(S)$	- gust filter transfer functions
$G_{c*}, G_{LAT}, G_{ROLL}, G_{RSS}$	- control gains
G_1, G_2	- continuous system input matrices for u, η
H, H_x, H_u, H_w	- measurement matrices
I	- identity matrix
J	- partial likelihood function $L - 1/2 \sum \ln \det B$
K, K_k	- continuous and discrete Kalman filter gains
K_p, K_I, k_1, k_2	- control gains
K_c	- critical gain
L	- likelihood function
$L_p, L_r, L_\beta, L_{\delta a}, L_{\delta r}$	- rolling moment coefficients due to indicated variables
L_u, L_v, L_w	- gust field scale lengths
M	- number of parallel channels
$M_q, M_\alpha, M_\delta(M_{\delta e}), M_\alpha$	- pitching moment coefficients due to indicated variables
M_o	- trim pitching moment
$M_{\delta t}$	- true value of M_δ

SYMBOLS (Continued)

N	- (1) number of data samples (2) discrete sensor noise intensity matrix
$N_p, N_r, N_\beta, N_{\delta a}, N_{\delta r}$	- Yawing moment coefficients due to indicated variables
N_z, N_y	- normal and lateral acceleration
P	- weighting matrix in Liapunov function
P_o	- a priori parameter covariance matrix
P_{ij}	- partitions of P_o
Pr	- probability
R	- continuous sensor noise intensity matrix
T	- transformation matrix
T_1, T_2	- thresholds in model tracker error nonlinearity
U_N	- sequence of N control inputs
V	- (1) velocity (2) Liapunov function
V_a	- true airspeed
V_{co}	- crossover velocity in C^* response
$Y_\beta, Y_{\delta a}, Y_{\delta r}$	- lateral force coefficients due to indicated variables
Y_N	- sequence of N measurements
Z_α, Z_δ	- normal force coefficients due to indicated variables

SYMBOLS (Continued)

Lower Case Symbols

a	- hysteresis parameter
$\underline{c}, c_i$	- parameter vector with components c_i
$\underline{c}_0$	- a priori estimate of $\underline{c}$
$\underline{c}_t$	- true value of $\underline{c}$
$\underline{c}_1, \underline{c}_2$	- partitioned parameter vector
$\underline{c}_{t1}, \underline{c}_{t2}$	- true value of $\underline{c}_1, \underline{c}_2$
d	- sensor displacement from c.g. (4.62m)
e	- model tracker error vector
e_0	- nonlinear model tracker error
f_{ij}	- ij-th element of matrix F
g, g_0	- gravity
$\tilde{g}$	- disturbance process to represent gravity variations in maneuvers
g_{ij}	- ij-th element of matrix G
h	- altitude
i^*	- index of the minimum-L channel
p	- roll rate
$\bar{q}$	- dynamic pressure
r	- (1) yaw rate (2) number of measurements

SYMBOLS (Continued)

t	- time
u	- (1) control input vector (2) forward velocity perturbation
u_e	- elevator servo command
u_t	- test input
u_g	- forward gust component
v	- (1) lateral velocity perturbation (2) error parameter in model tracker equations
v_g	- lateral gust component
w	- vertical velocity perturbation
w_g	- vertical gust component
x	- (1) forward position perturbation (2) state vector
y	- (1) lateral position perturbation (2) measurement vector
z	- parameter errors in model tracker equations

Greek Symbols

Upper Case

Γ	- discrete system noise input matrix
Φ	- tracker error dynamics matrix
Ω	- subset of parameter space

SYMBOLS (Continued)

$\nabla L, \nabla L_2$	- partitioned gradient vector
$\nabla^2 L_{ij}$	- partitioned second partial derivative matrix
 <u>Greek Lower Case</u>	
α	- angle-of-attack
α_g	- gust angle-of-attack
β	- angle of sideslip
β_g	- gust angle of sideslip
$\delta_e(\delta), \delta_\alpha, \delta_r, \delta_f, \delta_{rt}$	- aerodynamic surface positions
$\epsilon, \epsilon_1, \epsilon_2$	- error quantities
$\epsilon_k, \overline{\Delta \epsilon_k}$	- generic likelihood filter states
ζ	- (1) damping ratio (2) dummy argument for values of parameter vector $\underline{c}$
η	- white noise process
θ	- pitch attitude
$\lambda, \lambda_i, \lambda_{ij}$	- parameter adjustment gains in model tracker
μ	- (1) $\exp(-\Delta t/\tau)$ (2) design parameter in model tracker
v	- Kalman filter residuals
ξ	- white noise process
ρ_{xy}	- correlation coefficient

SYMBOLS (Concluded)

ρ_i	-	parameter in Liapunov function
σ, σ_x	-	standard deviation of variable x
τ	-	(1) time constant (2) data length
ϕ	-	roll attitude
ψ	-	yaw attitude
ω	-	natural frequency
ω_i, ω_{ij}	-	design parameters in model tracker

SECTION 4

DESIGN GROUND RULES AND REQUIREMENTS

GROUND RULES

The aim of the design effort reported here was not to develop new theoretical procedures or algorithms but to turn existing concepts into flight-worthy control laws for the specific test aircraft. The concepts and design processes, however, should be general enough to apply to other aircraft as well. Other more specific ground rules imposed on the design are described in the following paragraphs.

Inputs

The adaptive system must operate in the presence of normal pilot inputs and also when such inputs are absent. Any test signals required for the latter case must be small enough not to interfere with the aircraft's mission. This generally means that test input normal accelerations, as sensed at the pilot station, should be below ($0.2 - 0.3 \text{ m/sec}^2$), and lateral acceleration should be even lower in the range of (0.1 m/sec^2). This ground rule establishes a crucial distinction between identifiers designed for operational adaptive controls and those designed for post flight-test data processing applications.^{42, 43} In the latter case, test inputs are deliberately large and often optimized for identification accuracy.^{60, 61} For operational adaptive controls, the emphasis must be to make these inputs small and, hopefully, unnoticeable.

Sensors

The system was constrained to operate with the aircraft's existing surface (servo) position transducers and inertial sensors (rate gyros and accelerometers), but without attitude measurements and without air-data measurements (dynamic pressure, velocity, angle-of-attack). These limitations are imposed by the sensor complement available on the test aircraft and, in the case of air data, by the motivation in Section 2.

Computer Capacity and Sample Rate

Since control law calculations typically consume only a small fraction of the total computational load in a flight computer (the majority is I/O, self test, mode, and redundancy control), the adaptive law's goals were restricted to 25 percent of the available frame time of the AP-101 and to a "reasonable" allocation of memory. The sample rate of the computer was pre-specified to be 50 Hz. This gives five msec real-time per sample in which to complete all control and identification calculations. The sample rate is also high enough (for the F-8C) to produce no substantial differences between direct digital design (discrete-time control laws designed for discrete-time models) and continuous time design with after-the-fact discretization.

Control Surfaces and Actuators

The standard elevator, aileron, and rudder surfaces were available for adaptive control. Leading edge and trailing edge flaps were assumed to remain undeflected or to follow their open-loop scheduled minimum drag positions. Actuation systems for these surfaces were represented by models incorporating projected characteristics of the improved systems being installed on the test aircraft.

OPERATIONAL REQUIREMENTS

While no detailed specifications were written for the adaptive control laws, all algorithms were required to operate satisfactorily under the following conditions:

1. Periods of quiescence
2. Periods of high turbulence
3. Large (all-attitude) maneuvers
4. Landings and takeoffs *

Satisfactory operation was demonstrated by simulator evaluations under these conditions.

* Satisfactory operation for these mission phases was assured by inhibiting the adaptive process and using fixed gains.

PERFORMANCE CRITERIA

The performance criteria deliberately concentrated on identifier performance. This was done since overall closed-loop performance for the F-8C provides a less sensitive measure of adaptive capability. Four "measures of goodness" were used: 1) Identification accuracy in steady flight, 2) convergence characteristics, 3) tracking characteristics for standard flight transition, and 4) responses to major maneuvers and configuration changes.

Accuracy at Fixed Flight Condition

Identification accuracy (and thus control gain accuracy) were verified by subjecting the closed-loop adaptive system to a standard sequence of test conditions while in trimmed flight at fixed flight conditions which span the F-8C envelope. The test sequence consisted of a period of quiet, followed by a 6 m/sec^2 (20 ft/sec^2) pilot doublet C*-command, followed by a period of atmospheric turbulence, and then again by a 6 m/sec^2 doublet. This entire sequence was repeated with and without sensor noise. The standard sequence was used to simplify data taking and comparison between concepts.

Convergence Characteristics

Convergence with identified parameters initially in error by $\pm 12 \text{ db}$ was checked during both gust disturbances and pilot commands. Acceptable performance is required within three seconds after initialization, with the disturbance and system engagement occurring simultaneously.

Varying Flight Conditions

Time-varying flight profiles were used to exercise adaptation capabilities. These consisted of acceleration and deceleration maneuvers without pilot stick activity. Since all concepts emphasized adaptation to M_{δ_e} , the profiles were chosen to encounter large variation in M_{δ_e} .

Major Maneuvers

The closed-loop adaptive systems were also tested with large maneuvers, including rolling maneuvers, high-g turns, and step disturbances in angle-of-attack and slideslip.

SECTION 5

MODELING AND SIMULATIONS

AIRCRAFT MODELS

The aircraft models used for the design work were derived from nonlinear aerodynamic data for the F-8C supplied by NASA. Linear airplane models were obtained by numerical differentiation techniques as described in the CCV control law final report.² The resulting pitch and lateral linear models have the form

$$\dot{\mathbf{x}} = \mathbf{F}\mathbf{x} + \mathbf{G}_1\mathbf{u} + \mathbf{G}_2\mathbf{w} \quad (1)$$

$$\dot{\mathbf{y}} = \mathbf{H}_x\mathbf{x} + \mathbf{H}_u\mathbf{u} + \mathbf{H}_w\mathbf{w}$$

The structure and numerical values of these matrices are given in Reference 2. The state, $\mathbf{x}$, is defined in Table 3. The control input, $\mathbf{u}$, is given in Table 4, and the components of the disturbance, $\mathbf{w}$, and output vector, $\mathbf{y}$, are listed in Tables 5 and 6, respectively. Twenty-five flight conditions used for design are listed in Table 7. The aircraft modeling was completed by adding actuator dynamics for the control surfaces, including the gust model, and by defining sensor noise.

TABLE 3. STATE DEFINITION

Symbol	Definition	
p	Roll rate (rad/sec)	Lateral axis
r	Yaw rate (rad/sec)	
v	Lateral velocity (m/sec)	
ϕ	Bank angle (rad)	
ψ	Yaw angle (rad)	
y	Lateral displacement (m)	
q	Pitch rate (rad/sec)	Longitudinal axis
w	Vertical velocity (m/sec)	
u	Forward velocity (m/sec)	
θ	Pitch angle (rad)	
h	Vertical displacement (m)	
x	Forward displacement (m)	

TABLE 4. CONTROL DEFINITION

Symbol	Definition	
δ_a	Differential aileron deflection (rad)	Lateral axis
δ_r	Rudder deflection (rad)	
δ_{rt}	Differential elevator deflection (rad)	
δ_e	Symmetric elevator deflection (rad)	Longitudinal axis
δ_{sb}	Speed brake (rad)	
δ_g	Gear (%)	
δ_t	Throttle (%)	

TABLE 5. DISTURBANCE VECTOR

Symbol	Definition
u_g	Forward gust (m/sec)
v_g	Side gust (m/sec)
w_g	Vertical gust (m/sec)

TABLE 6. OUTPUT VECTOR

Symbol	Definition
p	Roll rate (rad/sec)
q	Pitch rate (rad/sec)
r	Yaw rate (rad/sec)
N_z	Normal acceleration (m/sec ²)
N_y	Lateral acceleration (m/sec ²)

TABLE 7. TWENTY-FIVE FLIGHT CONDITIONS

No.	h		Mach	$\bar{q}$ (psf)	$\bar{q}$ (n/m ²)	V (m/sec)	α_{trm} (deg)	Condition
	(Kft)	(Km)						
1	20	6.1	0.670	305	14,603	212	3.45	Cruise
2	20	6.1	0.670	305	14,603	212	6.10	$\Delta N_z = 1g$ (climb)
3	20	6.1	0.670	305	14,603	212	12.12	$\Delta N_z = 3g$ (climb)
4	20	6.1	0.670	305	14,603	212	4.32	Cruise
5	20	6.1	0.400	109	5,219	126	8.86	Cruise
6	20	6.1	0.900	551	26,382	285	2.18	Cruise
7	40	12.2	0.700	134	6,416	207	6.73	Cruise
8	40	12.2	1.200	395	18,913	354	2.72	Cruise
9	10	3	0.800	652	31,218	263	1.96	Cruise
10	0	0	0.700	725	34,713	238	1.86	Cruise
11	0	0	0.300	133	6,368	102	7.64	Cruise
12	0	0	0.530	416	19,918	180	2.88	Cruise
13	20	6.1	0.600	245	11,731	190	4.25	Cruise
14	20	6.1	0.800	435	20,828	253	2.54	Cruise
15	40	12.2	0.800	175	8,379	236	5.15	Cruise
16	40	12.2	0.900	222	10,629	266	4.08	Cruise
17	0	0	0.189	53	2,538	64	7.48	Power approach
18	0	0	0.219	71	3,400	75	2.76	Power approach
19	20	6.1	0.670	305	14,603	212	2.12	$\Delta N_z = 0.5g$ (dive)
20	20	6.1	0.600	245	11,731	190	15.45	$\Delta N_z = 3g$ (climb)
21	40	12.2	1.400	537	25,712	414	2.64	Cruise
22	40	12.2	1.300	463	22,168	384	2.65	Cruise
23	30	9.1	1.200	633	30,308	364	1.92	Cruise
24	30	9.1	1.100	532	25,472	334	1.89	Cruise
25	30	9.1	0.600	158	7,565	182	6.09	Cruise

ACTUATOR MODELS

Actuator models consisting of a first-order lag for the primary actuator and a second-order lag for the secondary actuator were included for the elevator aileron and rudder. The parameters were again taken from Reference 2 and are listed in Table 8. The secondary actuator dynamics were approximated as a unity transfer function for the design of the model tracker and the maximum likelihood estimation concept. The hysteresis values shown were estimates of the values expected in the Phase II hardware following modifications. The complete model is shown in Figure 2.

For the majority of the simulation runs, the hysteresis was set to zero. This was considered since the adaptive algorithms used measured secondary servo position, and in flight the hysteresis of the primary actuator is expected to be small. This results from the aerodynamic loading of the surface.

TABLE 8. ACTUATOR MODEL PARAMETERS

Actuator	Primary (first-order lag)	Secondary (second-order lag)
Pitch	$\tau_1 = 0.0800$ $2a = 0.00436 \text{ rad}$ Rate limit = 0.44 rad/sec	$\omega = 125.6 \text{ rad/sec}$ $\zeta = 0.7$ $2a = 0.00175 \text{ rad}$
Roll	$\tau_2 = 0.0333$ $2a = 0.006$ Rate limit = 2.44 rad/sec	Same as above
Yaw	$\tau_3 = 0.0400$ $2a = 0.00855 \text{ rad}$ Rate limit = 122 rad/sec	Same as above

GUST MODEL

The gust model used has the form attributed to Dryden and contained in MIL-STD-8785A.⁶⁴ Random gusts with the appropriate spectra are obtained by passing gaussian white noise through a linear filter. The filter transfer functions for the three gust components are given in Table 9. The scale length (L_1) and rms velocities (σ_1) have the following altitude dependence:

$$L_u = L_v = 145.0 h^{1/3}$$

$$h > 533.4 \text{ m}$$

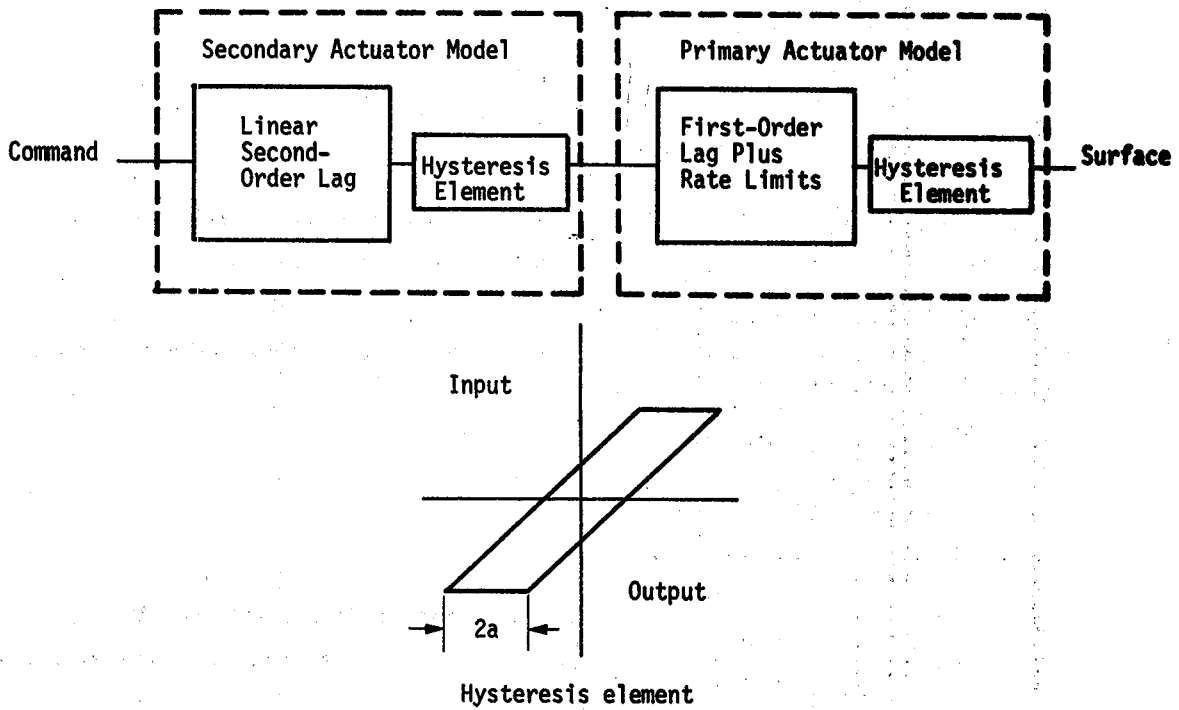


Figure 2. Simulation model of actuator.

TABLE 9. GUST MODEL

Gust Spectrum	Transfer Function
$G_{ug}(s)$	$\sigma_u \frac{2L_u}{V_a} \frac{1}{1 + \frac{L_u}{V_a} s}$
$G_{vg}(s)$	$\sigma_v \frac{L_v}{V_a} \frac{1 + \frac{3L_v}{V_a} s}{1 + \frac{L_v}{V_a} s^2}$
$G_{wg}(s)$	$\sigma_w \frac{L_w}{V_a} \frac{1 + \frac{3L_w}{V_a} s}{1 + \frac{L_w}{V_a} s^2}$

$$L_w = L_u = L_v = 1750 \text{ ft} = 533.4 \text{ m}$$

$$\sigma_w = 5.25 \left(-\log_{10} \frac{h}{3046} \right)^{1.25} \quad h \text{ in meters}$$

This expression approximates the MIL-ST-8785A values from 30.48 to 18,288 m. For $0 < h < 30.48$, the value of $h = 30.48$ is used in the above equations.

For design work, the transfer function for vertical gusts was approximated by a first-order filter:

$$\frac{\sigma_w \sqrt{\frac{2L}{V}}}{1 + \frac{L}{V} s} \quad (2)$$

For the MLE concept, the vertical gust model incorporated in the Kalman filter was further approximated. Using the first-order differential equation model corresponding to (2),

$$\dot{\alpha} = -\frac{V}{L} \alpha + \frac{\sigma_w}{V} \sqrt{\frac{2V}{L}} \eta \quad (3)$$

The low frequency content of the derivative was increased to yield

$$\dot{\alpha} = \frac{\sigma_w}{V} \sqrt{\frac{2V}{L}} \eta \quad (4)$$

SENSOR MODELS

Sensors were modeled as unity transfer functions plus additive noise. Sensor noise effects were modeled for the rate gyros, accelerometers, and servo position transducers. The noises were modeled as white noise shaped by a first-order low-pass filter. The filter time constants and rms values are given in Table 10. This spectrum approximates the bandwidth of most flight control sensors, as well as the nominal pre-filter used on sampled CAS signals. There is some difficulty in defining "sensor noise" because sensors in flight will measure unmodeled dynamics (body bending effects) and unmodeled disturbances (engine noise, etc.). The values presented here are considered representative of internal noise only.

TABLE 10. SENSOR NOISE PARAMETERS

Sensor	Correlation time ^a	rms value
Pitch rate gyro	0.01 sec	0.0026 rad/sec
Yaw rate gyro	0.01 sec	0.0026 rad/sec
Roll rate gyro	0.01 sec	0.0131 rad/sec
Normal accelerometer	0.01 sec	2 m/sec ²
Lateral accelerometer	0.01 sec	0.03 m/sec ²
Servo position	0.01 sec	0.0007 rad

^a Implemented as white noise shaped by a first-order lag with this time constant.

DESIGN MODELS AND SIMULATIONS

The limit cycle, model tracker, and maximum likelihood estimation designs were developed using several versions of the above models. Pitch and lateral-directional designs were done separately using uncoupled models. The pitch axis designs used only the short-period dynamics, elevator dynamics, and gusts. The lateral designs used a fourth-order aircraft model, aileron and rudder control effectors, and gusts. Other modeling details peculiar to a specific design will be covered in their specific sections of this report.

A different simulation was used for the design of each adaptive concept. Each simulation used linear airplane equations of motion and constant parameters representing a specific flight condition. The simulations were tailored to facilitate design trade-offs for a specific concept. Additional details and rationale are provided in the sections devoted to the design details of each concept.

The simulation used for design verification was the complete nonlinear six-degree-of-freedom simulation at NASA Langley Research Center. It operated in real-time on a CDC6600 and permitted man-in-the-loop simulation. Provision for driving actual F-8C servos with computer signals was included. Details of this simulation are contained in Reference 59.

SECTION 6

FUNDAMENTAL LIMITATIONS OF ADAPTIVE CONTROLS FOR THE F-8C

Like most physical control processes, adaptive controllers for the F-8C aircraft cannot work arbitrarily well. There are fundamental limitations imposed by aircraft physics, measurement constraints, disturbances, and operational requirements which bound achievable performance. It is important to recognize and to evaluate these limitations before detailed adaptive designs are seriously undertaken. This establishes realistic "levels of expectation" for the performance of final designs and helps to define promising design configurations and initial design parameters for particular adaptive concepts.

In the design program reported here, such fundamental limitations of adaptive controls for the F-8C were evaluated with separate identifiability studies. The purpose of these studies was to determine how accurately parameters in the aircraft's linearized equations of motion can be explicitly identified in an operational setting--that is, with realistic sensor imperfections, realistic atmospheric disturbances, and, most importantly, with operationally tolerable test signal levels. Identification accuracy was selected as the key index of adaptive capability because it translates readily into control-loop performance for explicit adaptive concepts. It also bears a strong, though indirect, relationship to the performance of implicit concepts and can be computed theoretically without elaborate simulation evaluations.

Separate identifiability studies were carried out for the pitch and lateral-directional axes. In each case, an identification model was first developed which included dominant short-period dynamics, disturbances, sensor characteristics, and trim conditions. This model was parameterized by expressing individual equation coefficients as functions of certain dominant aircraft parameters ($\bar{q}$, M_0 , Mach) plus a set of small perturbation parameters.

A linear system identification problem was then formulated to estimate these dominant and small perturbation parameters from on-line, closed-loop, input/output records. As shown in the literature,⁶³ the theoretical accuracies attainable in such identification problems are bounded from below by the inverse "Fisher information matrix." Hence, it remained only to compute this matrix for various operational situations and to

use its diagonal elements as fundamental identification accuracy limitations for the F-8C aircraft.

Results show very limited identification potential. For example, out of six parameters describing pitch dynamics, only two can be readily identified. These are elevator effectiveness (M_{δ}) and a small perturbation parameter corresponding to pitching moment due to angle-of-attack (M_{α}). The remaining parameters are known more accurately from a priori data than from on-line estimation. The estimation process does not significantly reduce their a priori uncertainties. Also, theoretical identification accuracies associated with M_{δ} and M_{α} are no better than 10 to 20 percent depending on the length of data samples used.

Similar results hold for the lateral-directional axes. In this case, only two out of 16 parameters can be identified. These are dynamic pressure ($\bar{q}$) and the perturbation parameter corresponding to yawing moment due to sideslip (N_{β}). On-line identification does not significantly improve a priori knowledge for any of the rest. Theoretical accuracies for the identifiable parameters are again in the 10 to 20 percent range.

The primary reason for these weak identifiability properties is the operational ground rule governing test signal magnitudes (Section 4). This ground rule makes explicit identification for adaptive control qualitatively different from, say, explicit identification for post-flight data reduction.^{42, 43} In one case we are free to choose the magnitudes and shapes of control inputs so as to optimize identifiability. In the other case we must try to make the inputs as small as possible, hopefully to the point that they blend in and get lost among natural noise and gust induced signals of the control loop. This difference is fundamental to the design work reported here. It affects the algorithms used for identification, their structure, the number of parameters identified, and anticipated identification accuracies.

The remainder of this section provides support for the identification properties summarized above. Identification models and their parameterizations are presented for the pitch and lateral-directional axes, together with identification accuracy results as functions of data length, test signal magnitudes, and gust and sensor noise conditions.

PITCH AXIS IDENTIFIABILITY

The Identification Model

The following model was used for the pitch axis identifiability study:

$$\frac{d}{dt} \begin{bmatrix} q \\ \alpha + \alpha_g \\ \alpha_g \end{bmatrix} = \begin{bmatrix} M_q & M_\alpha & 0 \\ 1 & Z_\alpha & -\frac{V}{L_w} \\ 0 & 0 & -\frac{V}{L_w} \end{bmatrix} \begin{bmatrix} q \\ \alpha + \alpha_g \\ \alpha_g \end{bmatrix} + \begin{bmatrix} M_\delta \\ Z_\delta \\ 0 \end{bmatrix} \delta + \begin{bmatrix} M_o \\ g/V \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \left(\sigma \sqrt{\frac{2}{V L_w}} \right) \eta \quad (5)$$

$$y_k = \begin{bmatrix} q_m \\ N_{z_m} \end{bmatrix}_k = \begin{bmatrix} 1 & 0 & 0 \\ dM_q & dM_\alpha - (Z_\alpha V) & 0 \end{bmatrix} \begin{bmatrix} q \\ \alpha + \alpha_g \\ \alpha_g \end{bmatrix}_k + \begin{bmatrix} 0 \\ dM_\delta - (Z_\delta V) \end{bmatrix} \delta_k + \begin{bmatrix} 0 \\ dM_o \end{bmatrix} + \begin{bmatrix} \epsilon_q \\ \epsilon_{nz} \end{bmatrix}_k \quad (6)$$

$$k = 0, 1, 2, \dots \quad (t_{k+1} - t_k) = \Delta$$

This is a simplified version of the aircraft models described in Section 5. It was derived by partitioning the 12-state models obtained from numerical linearizations into longitudinal and lateral-directional dynamics. The longitudinal states (q, w, u, θ, h, x) were then transformed to ($q, \alpha = w/V, u, \gamma = \theta - \alpha, h, x$) and partitioned into the fast short-period dynamics (q, α) and the slower states (u, γ, h, x). Coupling between these two groups was ignored. To complete the model, a first-order approximation of the Dryden vertical gust spectrum was appended; constant disturbances ($M_o, g/V$) were added to represent trim; and output equations were written for the gyro, q_m , and the accelerometer, N_{zm} .

the theoretical identification error levels reported in this section, provided that test signal levels are not changed. Conversely, they would permit the same error levels to be achieved with proportionately smaller test signals.

Model Parameterization

The above identification model contains 11 parameters--

six coefficients describing pitch dynamics:

$$M_q, M_{\alpha}, V, (Z_{\alpha} V), M_{\delta}, (Z_{\delta} V)$$

two gust level and trim terms:

$$M_o, \sigma_{wg}$$

three initial conditions:

$$q(t_o), [\alpha(t_o) + \alpha_g(t_o)], \alpha_g(t_o)$$

In the identifiability study, all 11 were treated as unknowns. However, because there is a good deal of a priori knowledge about their values and correlations, the parameters were not treated as independent, unrelated variables. Instead, they were modelled as dependent functions of two dominant aircraft parameters (M_o and Mach) plus a set of small perturbation parameters. The latter parameters (denoted by vector $\underline{c}$) were then identified.

The functional relationships between new variables and old were obtained by approximating scatter plots for the original parameters evaluated by 25 flight conditions given in Section 5. This gives very simple functions--linear (or quadratic at worst) in M_o with two sets of coefficients, one for subsonic and the other for supersonic flight. Scatter about these functions is small, providing tight a priori bounds for most of the new parameter values. The functions are summarized in Table 11, and their associated scatter plots are given in Appendix A.

It should be noted that this parameterization is airplane-specific. The functions in Table 12 apply to the F-8C only. However, it is known that other aircraft also exhibit strong correlations between model coefficients, and, hence, similar tables can be readily constructed for other design applications.

TABLE 11. PITCH MODEL PARAMETERIZATION

Original parameter			New parameter	New parameter a priori one sigma uncertainty ^a
M_q	$= -0.23 + 0.028 M_\delta + c_1$	Mach < 1	c_1	0.065
	$= -0.23 + 0.010 M_\delta + c_1$	Mach > 1		
M_α	$= (0.61 + 0.92 c_2) M_\delta$	Mach < 1	c_2	0.135
	$= (1.53 + c_2) M_\delta$	Mach > 1		
$\dot{V}$	$= (200 + c_3) \sqrt{-M_\delta}$	Mach < 1	c_3	31.5
	$= (260 + c_3) \sqrt{-M_\delta}$	Mach > 1		
$(Z_\alpha \dot{V})$	$= (53 + c_4) M_\delta$	All Mach	c_4	5.0
M_δ	$= c_5$	All Mach	c_5	15.0
$(Z_\delta \dot{V})$	$= (7.7 + c_6) M_\delta$	All Mach	c_6	0.750
σ_{wg}	$= 3.0 + c_7$	All Mach	c_7	1.50
M_o	$= 0.017 M_\delta + c_8$	Mach < 1	c_8	0.125
	$= c_8$	Mach > 1		
$q(t_o), \alpha(t_o) + \alpha_g(t_o), \alpha_g(t_o)$			Same as original	0.02 rad/sec, 0.02 rad, 0.01 rad

^a Taken to be one-half of the $\pm$ max uncertainty range in the figures in Appendix A.

Accuracy Analysis Procedure

As reported in much recent literature,^{42,43} one way to identify the vector $\underline{c}$ is to maximize the conditional probability density function of the observed outputs given the unknowns and the inputs, i.e.,

$$\hat{\underline{c}} = \text{Arg} \left\{ \max_{\underline{c}} p(q_{mk}, N_{zmk}; k = 1, 2, \dots, N \mid \underline{c} = \underline{c}; \delta_k, k = 0, 1, \dots, N-1) \right\} \quad (15)$$

Without repeating too much in the references, this maximization reduces to the following minimization problem:

$$\hat{\underline{c}} = \text{Arg} \left\{ \min_{\underline{c}} \frac{1}{2} \left[\left\| \underline{c} - \underline{c}_0 \right\|_{P_0}^2 + \sum_{k=0}^N \left(\left\| v_k \right\|_{B_k}^2 + \ln \det B_k \right) \right] \right\} \quad (16)$$

where

$\underline{c}_0$ = initial parameter estimate vector,

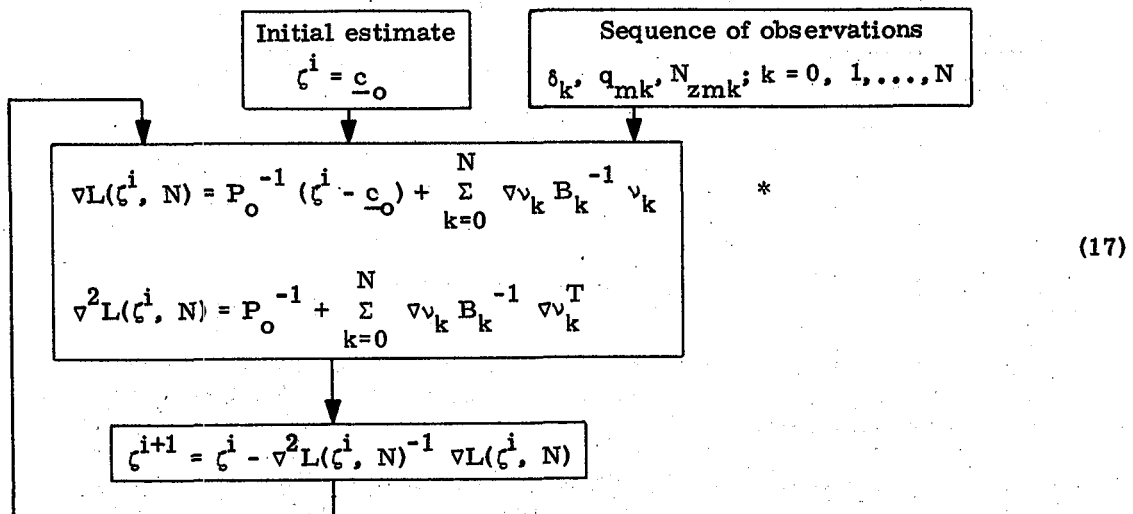
P_0 = initial error covariance matrix (Table 11),

v_k = residual (innovations) sequence of Kalman filter computed for parameters $\underline{c} = \underline{c}$, and

B_k = computed covariance matrix for the residual sequence.

Let the term in brackets above be denoted by $L(\underline{c}, N)$, the log likelihood function.

Then the required minimization operation can be accomplished iteratively using the following modified Newton-Raphson procedure:



* These expressions assume high sample rates, so that $B_k \approx R/\Delta t$ and $\nabla B_k = 0$.

Under suitable identifiability assumptions, this procedure converges to the true parameter vector, $\underline{c}^i \rightarrow \underline{c}_t$. The final iteration produces the second-partials matrix $\nabla^2 L(\underline{c}_t, N)$ which is the so-called "Fisher Information Matrix." Its inverse is a theoretical lower bound for the parameter error covariance matrix. Diagonal elements of the inverse are lower bounds on the variances of the individual parameter identification errors.[†]

The identifiability study basically consisted of carrying out the last iteration of the above algorithm for various noise, test signal, and data length conditions. The procedure is illustrated in Figure 3. For each case, the identification model (Equations (5) and (6) with $\underline{c} = \underline{c}_t$) was used to generate a sequence of observations $\{q_{mk}, N_{zmk}; k = 1, 2, \dots, N\}$. This was done in closed-loop fashion, using a simple proportional-plus-integral pitch rate controller. The loop was excited by a small test signal (u_t) and, for some cases, by a pseudorandom gust input (η). The sequence of observations was then used to generate the likelihood function, $L(\underline{c}_t, N)$, its gradient, $\nabla L(\underline{c}_t, N)$, and its second-partials matrix, $\nabla^2 L(\underline{c}_t, N)$.^{*}

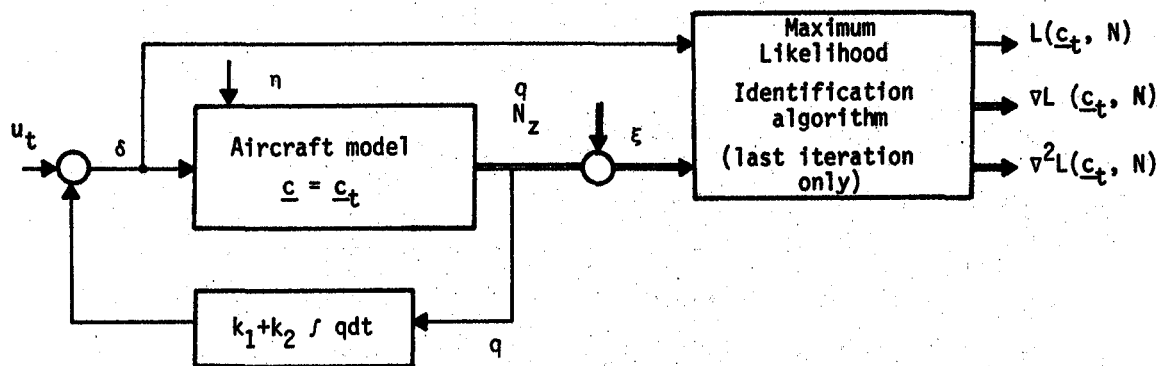


Figure 3. Accuracy analysis procedure.

[†] Strictly speaking, the actual identification error approaches the inverse from above asymptotically (as N gets large). Hence, the inverse represents a lower bound for the identification errors at all finite N .

^{*} In noise and/or gust-driven cases, L , ∇L , $\nabla^2 L$ are stochastic quantities. For such cases, these quantities were computed as sample averages of several (usually five) separate samples.

Three different types of test inputs were used: square waves, sine waves, and random signals. The first two were motivated by References 63 and 69 which indicate that cyclic test signals near the natural frequencies of the plant are good for identification accuracy. The last was chosen to provide a test input which is difficult to distinguish from atmospheric turbulence. Magnitudes for all signals were scaled to produce rms nominal acceleration levels in the range of 1 to 1.5 m/sec² at the pilot station. These levels fall into the "comfortable" range of recent ride quality studies (for example, Reference 64) but are well above the so-called "threshold of detectability" which falls around 0.2 - 0.3 m/sec².

The random test signal consisted of white noise shaped with the following linear filter:

$$G(s) = \frac{k s}{s^2 + 2\zeta\omega s + \omega^2} \quad (18)$$

The frequency (ω) was selected near the short period of the aircraft; the damping ratio ζ was set to 1.5; and k was adjusted to provide the desired 1.0 - 1.5 m/sec² acceleration at the pilot station.

The study also addressed reduced-parameter identification, i. e., identifying fewer than 11 error parameters. This is an important practical question since all 11 parameters comprise a fairly high order identification problem whose solution is probably not feasible on board. For reduced-parameter identification, the error covariance matrix, $(\nabla^2 L)^{-1}$, must be modified slightly to account for the errors produced by the presence of incorrect unidentified parameters. The modified matrix is derived as described in the following paragraph.

Let the parameter vector $\underline{c}$ be partitioned into two groups, $[\underline{c}_1, \underline{c}_2]^T$. The first group is to be identified, while the second is to be neglected. Then the likelihood function near the true parameters, $\underline{c}_t = [\underline{c}_{t1}, \underline{c}_{t2}]^T$, can be written as

$$L(\zeta, N) = L(\underline{c}_t, N) + \frac{1}{2} \left\| \zeta - \underline{c}_t \right\|_{\nabla^2 L}^2 + \delta \quad (19)^*$$

$$\nabla L = \nabla^2 L (\zeta - \underline{c}_t) + \epsilon$$

* The first-order term vanishes because $L(\zeta, N)$ is minimized at $\zeta = \underline{c}_t$.

$$\begin{bmatrix} \nabla L_1 \\ \nabla L_2 \end{bmatrix} = \begin{bmatrix} \nabla^2 L_{11} & \nabla^2 L_{12} \\ \nabla^2 L_{21} & \nabla^2 L_{22} \end{bmatrix} \begin{bmatrix} \zeta_1^i - \underline{c}_{t1} \\ \zeta_2 - \underline{c}_{t2} \end{bmatrix} + \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \end{bmatrix} \quad (20)$$

where δ and ϵ represent higher order term and random errors.

A reduced parameter Newton-Raphson minimization procedure will now iterate on ζ_1^i according to Algorithm (17) until the gradient components ∇L_1 (not the whole vector ∇L) is zero. This condition is satisfied when

$$\zeta_1^i - \underline{c}_{t1} = -(\nabla^2 L_{11})^{-1} [\nabla^2 L_{12}(\zeta_2 - \underline{c}_{t2}) + \epsilon_1] \quad (21)$$

Hence, the error covariance of the converged estimate will be

$$\begin{aligned} \text{Cov}(\hat{\underline{c}}_1) &\triangleq E[(\zeta_1^i - \underline{c}_{t1})(\zeta_1^i - \underline{c}_{t1})^T] \\ &= (\nabla^2 L)_{11}^{-1} [(\nabla^2 L)_{11} + (\nabla^2 L)_{12} P_{22} (\nabla^2 L)_{21}] (\nabla^2 L)_{11}^{-1} \end{aligned} \quad (22)$$

where $P_{22} = E[(\zeta_2 - \underline{c}_{t2})(\zeta_2 - \underline{c}_{t2})^T]$ is the a priori covariance matrix of the unidentified parameter components and $(\nabla^2 L)_{11}$ can be shown to be the covariance matrix of ϵ_1 . Equation (22) agrees with the full parameter identification theory if $P_{22} = 0$.

Analysis Results

Results of the pitch axis accuracy study are summarized in Table 12 and Figures 4 through 7. The principal result is Table 12 which shows before and after accuracy comparisons for a 0.13 g square wave test signal and true parameters corresponding to Flight Condition 1. The table compares a priori parameter uncertainties from Table 11 (the "Before" column) with parameter uncertainties obtained after identifying with five seconds and ten seconds of observed on-line data (the "After" columns). Scanning these numbers shows that only c_2 , c_5 , c_8 , c_9 , and c_{10} can be improved by more than a factor of two. Among the first six dynamic parameters, only $c_5(M_\delta)$ is improved dramatically, c_2 (small perturbation for M_α) improves marginally, and the rest are barely touched at all. The corresponding percentage accuracy at ten seconds for M_δ is

TABLE 12. BASIC IDENTIFICATION ACCURACY

0.13 g square wave test signal

 c_t for Flight Condition 1

40 samples/second

Parameter	One sigma uncertainty (τ = data length)		
	Before	After	
	$\tau = 0$ sec	$\tau = 5$ sec	$\tau = 10$ sec
c_1 (M_q)	0.065	0.0630	0.0620
c_2 (M_α)	0.135	0.0650	0.0560
c_3 (V)	31.5	30.5	29.6
c_4 ($Z_\alpha V$)	5.0	4.45	4.20
c_5 (M_δ)	15.0	1.88	1.46
c_6 ($Z_\delta V$)	0.750	0.750	0.740
c_7 (σ_{wg})	6.0	6.0	6.0
c_8 (M_o)	0.125	0.0063	0.0050
c_9 (q_o)	0.015	0.0066	0.0067
c_{10} ($\alpha + \alpha_{go}$)	0.017	0.0058	0.0053
c_{11} (α_{go})	0.008	0.0063	0.0061

$$\frac{\sigma_{c_5}}{|M_{\delta t}|} = \frac{1.46}{14.8} \approx 10 \text{ percent} \quad (M_{\delta t} = \text{true value of } M_{\delta})$$

and for M_{α} it is

$$\frac{\sigma_{c_5}}{|M_{\delta t}|} + \frac{\sigma_{c_2}}{0.61} + \rho_{c_5 c_2} \left(\frac{\sigma_{c_5}}{|M_{\delta t}|} \right) \left(\frac{\sigma_{c_2}}{0.61} \right) \approx 10 \text{ percent}$$

If we look at the M_{δ} parameter alone, its identification accuracy depends on data length (as seen above) and also on the true parameter vector $\underline{c}_t$. This is shown in Figure 4 which presents $\sigma_{M_{\delta}}$ results for three flight conditions plotted as a function of data length. The most significant point to notice is that M_{δ} errors are more or less proportional to the true M_{δ} values. This is consistent with the figures in Appendix A which show that a priori uncertainties are also proportional to the true values. It permits us to express subsequent identification errors as normalized quantities. The data length dependence of M_{δ} errors is also worth noting. Errors are reduced to 20 to 30 percent almost immediately and then improve very slowly with time from there on. As shown later in Section 8, it is not possible to use more than five to ten seconds for identification because the aircraft parameters can change significantly over such time spans. Hence, the accuracies in Table 12 and Figure 4 represent practical lower bounds on explicit on-line identification performance.

The possibility of reduced-parameter identification is examined in Figure 5. This figure was obtained by selectively deleting error parameters from the list of identified unknowns but accounting for their uncertainty in subsequent M_{δ} error calculations. The results show that many parameters are insignificant with respect to accurate M_{δ} identification. In fact, only M_{δ} itself, M_0 , and the initial condition $(\alpha + \alpha_g)_0$ really matter much to the accuracy of M_{δ} . This is an important conclusion because it justifies simplified identification procedures for use on board the F-8C.

The results presented so far include sensor noise and data uncertainties, but not gusts.* The latter are added in Figure 6 and improve identification slightly. Figure 6 also shows

* It should be noted that the maximum likelihood procedure was in all cases set up to expect gusts even when none were included in the true aircraft responses.

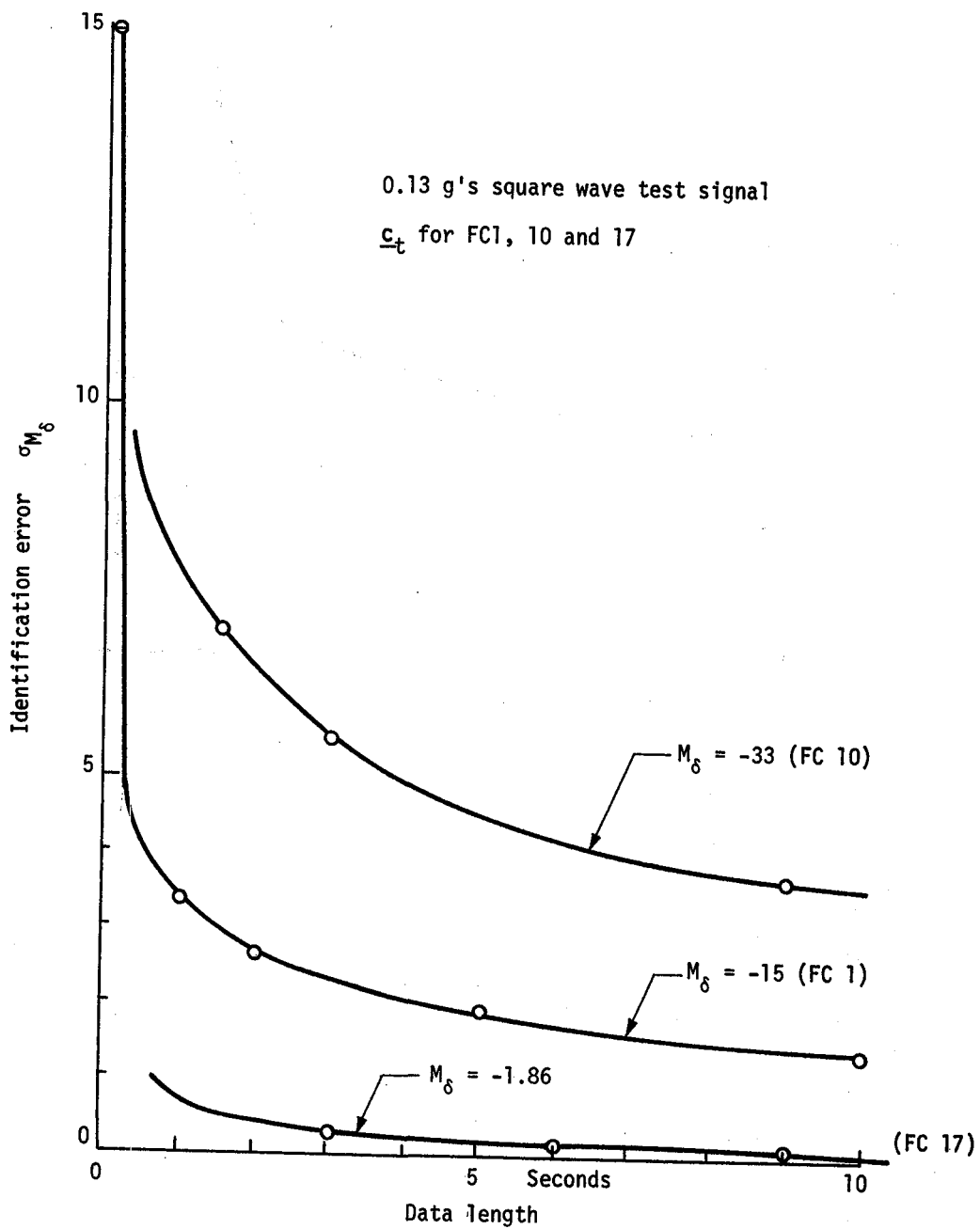


Figure 4. Identification accuracy for M_δ .

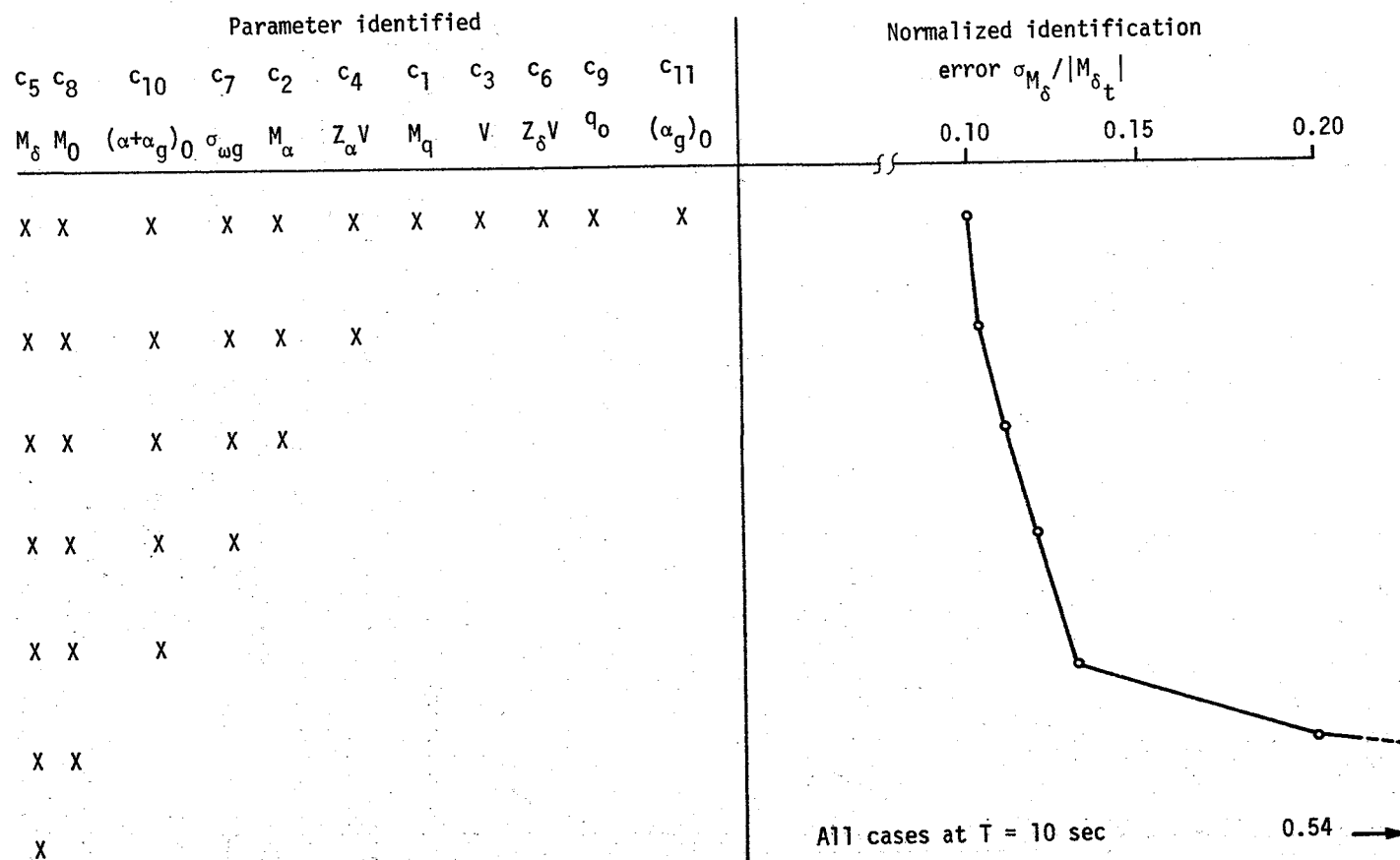


Figure 5. Reduced-parameter identification.

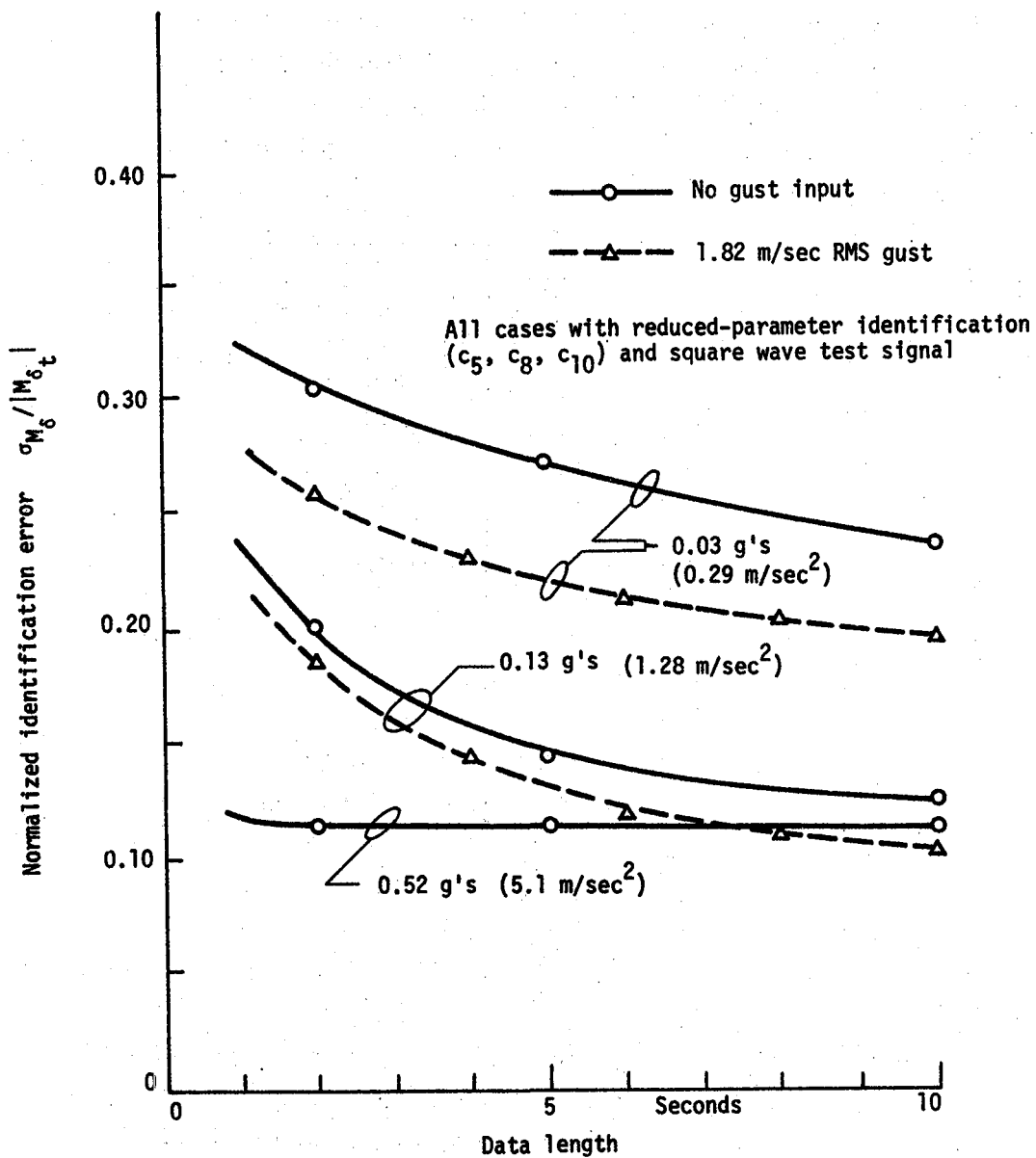


Figure 6. Effects of gusts and test signal magnitude.

effects of test signal magnitude. As expected, larger test signals improve accuracy, at least up to a point. About 10 percent error remains even with very large test inputs. This is caused by errors in the unidentified model parameters. (Note that Figure 6 uses reduced-parameter identification.) For purely cyclic test inputs, these parameters have identical effects on M_{δ} error over each period of the input, and, hence, no improvement occurs with time.

The latter effect can be reduced by gust inputs and by using test signals which are not cyclic. This is shown in Figure 7, where random test signals are compared with square waves and sine waves. Both cyclic signals are seen to be inferior to random commands. Compared against each other, they are roughly equivalent.

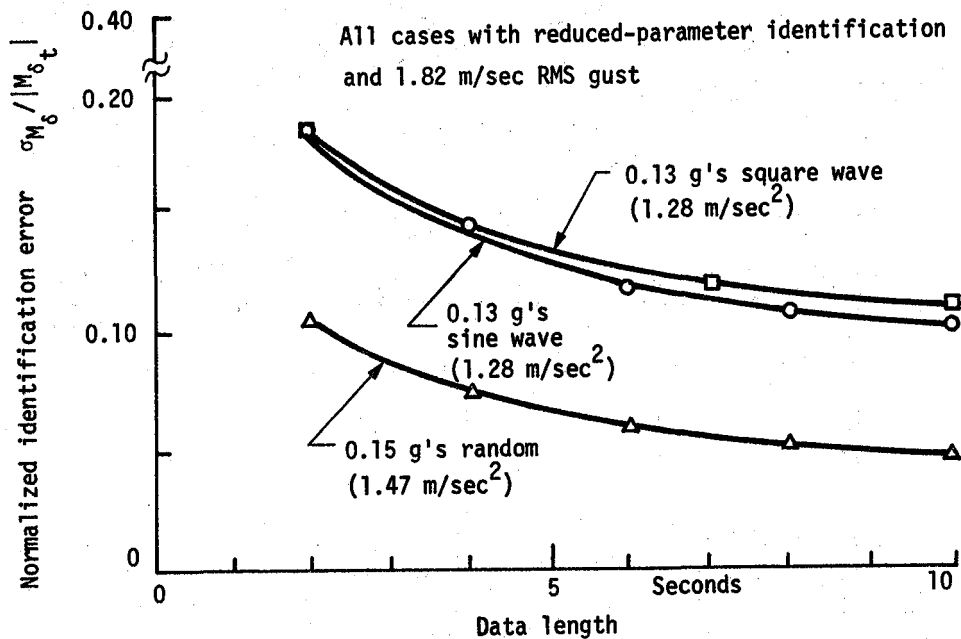


Figure 7. Effects of test signal type.

LATERAL-DIRECTIONAL IDENTIFIABILITY

Similar accuracy analyses were also conducted for the F-8C lateral-directional axes. Results for this case are reported here in abbreviated form. They point to the same basic observations--only a few parameters are readily identifiable on-line and those have only limited accuracy.

The identification model for the lateral-directional axes is shown in Figure 8. Like pitch, this model was developed from the original 12-state aircraft linearizations in Section 5. These were first partitioned to separate lateral-directional dynamics. The lateral states (p, r, v, ϕ, ψ, y) were then transformed to ($p, r, \beta = \frac{v}{V}, \phi, \psi, y$), and the fast modes (p, r, β) were isolated.

Finally, the model was appended with an approximate lateral gust model, a disturbance term ($\tilde{g}$) to represent $g \cos \theta \sin \phi$ gravity inputs, and algebraic sampled output equations for the roll and yaw rate gyros and for the lateral accelerometer.

Like the vertical spectrum, the lateral gust spectrum was approximated from the complete Dryden model in Section 5. The approximation follows Equations (7) through (10) exactly (except, of course, for symbols) but adds the following additional simplification:

$$\begin{aligned} \frac{d}{dt} \beta_g(t) &= -\left(\frac{V}{L_v}\right) \beta_g(t) + \left(\sigma_{vg} \sqrt{\frac{2}{VL_v}}\right) \eta(t) \\ &\approx \sigma_{vg} \sqrt{\frac{2}{VL_v}} \eta(t) \end{aligned} \tag{23}$$

In effect, this added simplification turns the gust derivative into white noise (i.e., a flat spectrum). This is pessimistic in the sense that the actual derivative spectrum is flat at high frequencies only and falls off below (V/L_v) rad/sec.

The gravity disturbance ($\tilde{g}$) was added in lieu of the bank angle state which was not considered to be available for the study (Section 4). It was modelled as a first-order random process with bandwidth $a = 1$ sec and rms level $\sigma_{\tilde{g}} = 9.8 \text{ m/sec}^2$. The sensors were again assumed to have wide-band noise outputs so that low frequency sampling will produce white discrete noise sequences. The rms sensor noise levels were identical to the pitch study.

The lateral-directional model was parameterized in terms of two dominant aircraft parameters-- $\bar{q}$ and Mach--and a new set of small perturbation parameters. The transformations are again simple linear functions (or quadratic, at worst) with different coefficients for subsonic and supersonic flight. The functions are summarized in Table 13.

$$\frac{d}{dt} \begin{bmatrix} p \\ r \\ \beta + \beta_g \\ \dot{g} \end{bmatrix} = \begin{bmatrix} L_p & L_r & L_\beta & 0 \\ N_p & N_r & N_\beta & 0 \\ \alpha & -1 & Y_\beta & 1/V \\ 0 & 0 & 0 & -a \end{bmatrix} \begin{bmatrix} p \\ r \\ \beta + \beta_g \\ \dot{g} \end{bmatrix} + \begin{bmatrix} L_{\delta a} & L_{\delta r} \\ N_{\delta a} & N_{\delta r} \\ Y_{\delta a} & Y_{\delta r} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta a \\ \delta r \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \sigma_{vg} \sqrt{\frac{2}{VLV}} & 0 \\ 0 & \sigma_{ng} \sqrt{2a} \end{bmatrix} \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix}$$

$$\begin{bmatrix} p_m \\ r_m \\ N_{ym} \end{bmatrix}_k = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ dN_p & dN_r & dN_\beta - Y_\beta V & 0 \end{bmatrix} \begin{bmatrix} p \\ r \\ \beta + \beta_g \\ \dot{g} \end{bmatrix}_k + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ dN_{\delta a} - Y_{\delta a} V & dN_{\delta r} - Y_{\delta r} V \end{bmatrix} \begin{bmatrix} \delta a \\ \delta r \end{bmatrix}_k + \begin{bmatrix} \varepsilon_p \\ \varepsilon_r \\ \varepsilon_{ny} \end{bmatrix}_k$$

$$k = 0, 1, 2, \dots$$

Figure 8. Lateral-directional identification model.

TABLE 13. LATERAL-DIRECTIONAL MODEL PARAMETERIZATION

Original parameter			New parameter	New parameter one sigma ^a uncertainty
$\bar{q}$	$= c_1$	All Mach	c_1	0.20 c_1
V	$= (41 + c_2) \sqrt{\bar{q}}$	Mach < 1	c_2	7.0
	$(54 + c_2) \sqrt{\bar{q}}$	Mach > 1		
L_p	$= -1.5 - 0.0071 \bar{q} + c_3$	Mach < 1	c_3	0.60
	$0.5 - 0.0071 \bar{q} + c_3$	Mach > 1		0.30
L_r	$= 0.44 + c_4$	Mach < 1	c_4	0.26
	$-0.03 + c_4$	Mach > 1		0.020
L_β	$= -10 - 0.12 \bar{q} + c_5$	All Mach	c_5	5.0
N_p	$= -0.090 - 0.00012 \bar{q} + c_6$	Mach < 1	c_6	0.025
	$-0.025 - 0.00012 \bar{q} + c_6$	Mach > 1		0.009
N_r	$= -0.10 - 0.00090 \bar{q} + c_7$	Mach < 1	c_7	0.040
	$0.02 - 0.00090 \bar{q} + c_7$	Mach > 1		0.030
N_β	$= -2.3 + 0.019 \bar{q} + c_8$	All Mach	c_8	1.20
$Y_\beta V$	$= (-0.52 + c_9) \bar{q}$	All Mach	c_9	0.060
$L_{\delta a}$	$= (0.075 + c_{10}) \bar{q}$	Mach < 1	c_{10}	0.008
	$(0.032 + c_{10}) \bar{q}$	Mach > 1		0.004
$L_{\delta r}$	$= (0.029 + c_{11}) \bar{q}$	Mach < 1	c_{11}	0.004
	$(0.014 + c_{11}) \bar{q}$	Mach > 1		0.002
$N_{\delta a}$	$= (0.0038 + c_{12}) \bar{q}$	Mach < 1	c_{12}	0.0006
	$(0.0013 + c_{12}) \bar{q}$	Mach > 1		0.0004

^a Taken to be one-half of the $\pm$ max uncertainty range of the scatter plots.

TABLE 13. Concluded.

Original parameter			New parameter	New parameter one sigma ^a uncertainty
$N_{\delta r}$	$= (-0.013 + c_{13}) \bar{q}$	Mach < 1	c_{13}	0.0020
	$= (-0.005 + c_{13}) \bar{q}$	Mach > 1		0.0004
$Y_{\delta a} V$	$= (0.0094 + c_{14}) \bar{q}$	All Mach	c_{14}	0.0028
$Y_{\delta r} V$	$= (0.110 + c_{15}) \bar{q}$	Mach < 1	c_{15}	0.016
	$= (0.042 + c_{15}) \bar{q}$	Mach > 1		0.004
$p(t_o)$	$= c_{16}$	All Mach	c_{16}	0.017
$p(t_o)$	$= c_{17}$	All Mach	c_{17}	0.017
$\beta(t_o) + \beta_g(t_o)$	$= c_{18}$	All Mach	c_{18}	0.017
$\tilde{g}(t_o)$	$= c_{19}$	All Mach	c_{19}	32.2

^a Taken to be one-half of the $\pm$ max uncertainty range of the scatter plots.

The primary accuracy results for the lateral-directional axes are summarized in Table 14. This table provides a before and after comparison of a priori parameter uncertainties ($\tau = 0$) with uncertainties obtained after identification with two, five, and 10 seconds of on-line data. As in the pitch case, only a few parameters are readily identifiable--two out of 16 dynamic parameters ($\bar{q}$ and $c_g(N_\beta)$) and three of four initial conditions. Accuracies are again in the 10 percent range. These results were obtained for closed-loop operation with a simple roll rate control loop. The loop was excited by random test signals generating 0.05 g's rms lateral acceleration. The lower level of lateral acceleration is consistent with ride quality requirements.⁶⁴

F-8C IDENTIFIABILITY SUMMARY

The pitch and lateral-directional identifiability studies led to four major observations:

- Only a few F-8C parameters can be realistically identified in an operational on-board setting. These include M_δ and M_α in pitch and $\bar{q}$ and N_β in lateral.
- Accuracies no better than 10 to 20 percent can be anticipated for the identifiable parameters. This accuracy range applies only for the sensor noise, gust levels, and test signal magnitudes assumed in the study, of course. It can be approximately scaled to other assumptions, however, by noting, in Algorithm (9), that the covariance matrix is directly proportional to B_k (sensor noise variance) and inversely proportional to ∇w^T (signal level squared). Using this scaling, for example, we can expect to get the same accuracy with lower test inputs if we have proportionately cleaner sensors.
- Reduced-parameter identification can successfully find identifiable parameters with little loss of accuracy. The parameter set in pitch can be reduced to as few as three parameters-- M_δ and two initial conditions, M_0 and $(\alpha + \alpha_g)$ --and still provide good M_δ estimates.
- Random test inputs appear to be superior to cyclic inputs for reduced-parameter identification.

These results have significant impact on the adaptive control concepts formulated in later sections.

TABLE 14. LATERAL AXIS BASIC IDENTIFICATION ACCURACY

Parameter	One sigma uncertainty (τ = data length)			
	$\tau = 0$	$\tau = 2$ sec	$\tau = 5$ sec	$\tau = 10$ sec
c_1 ($\bar{q}$)	60.	33.	26.	23.
c_2 (V)	7.0	6.7	6.6	6.3
c_3 (L_p)	0.60	0.58	0.57	0.55
c_4 (L_r)	0.26	0.26	0.26	0.26
c_5 (L_β)	5.0	4.1	4.1	3.7
c_6 (N_p)	0.025	0.025	0.025	0.025
c_7 (N_r)	0.040	0.040	0.038	0.038
c_8 (N_β)	1.20	0.66	0.61	0.58
c_9 ($Y_\beta V$)	0.060	0.060	0.060	0.060
c_{10} ($L_{\delta a}$)	0.008	0.008	0.008	0.008
c_{11} ($L_{\delta r}$)	0.004	0.004	0.004	0.004
c_{12} ($N_{\delta a}$)	0.0006	0.0006	0.0006	0.0006
c_{13} ($N_{\delta r}$)	0.001	0.0009	0.0009	0.0009
c_{14} ($Y_{\delta a} V$)	0.0028	0.0028	0.0028	0.0028
c_{15} ($Y_{\delta r} V$)	0.016	0.016	0.016	0.016
c_{16} (p_o)	0.017	0.011	0.011	0.011
c_{17} (r_o)	0.017	0.003	0.003	0.003
c_{18} ($\beta + \beta_{g_o}$)	0.017	0.004	0.004	0.004
c_{19} ($\tilde{g}_o$)	32.	18.	18.	18.

FC1 - 0.052g rms random test signal
 0.012 rad/sec rms roll rate
 0.005 rad/sec rms yaw rate

SECTION 7

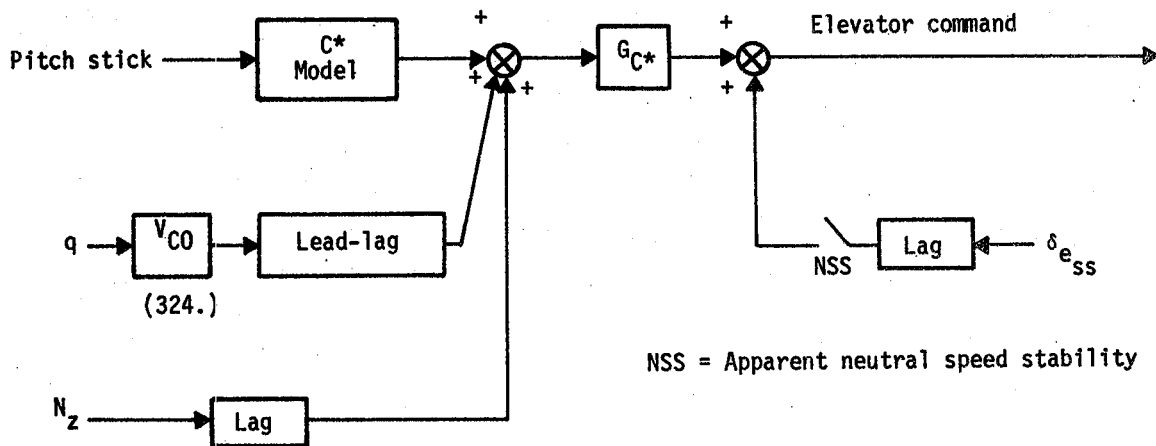
THE CONTROL LAW STRUCTURE

The optimal control laws of the CCV package were used as a basis in designing the adaptive control laws. The major difference being that the gains scheduled with air-data quantities are replaced with gains determined by the adaptive concepts.

The pitch and lateral-directional control laws will be described below. These designs are explained in greater detail in Reference 2.

PITCH CONTROL AUGMENTATION SYSTEM

The structure of the pitch control law is shown in Figure 9. It is a C*-model-following design that was designed with linear quadratic optimal control theory. Signal shaping and an integrator were included in the optimal control problem formulation to satisfy design specifications. The optimal design has been redrawn here to illustrate the sensors used and the signal shaping employed. The C*-quantity is formed by a fixed



NSS = Apparent neutral speed stability

Figure 9. Pitch CAS structure.

sum of normal acceleration and pitch rate multiplied by V_{co} (the "crossover" velocity), a constant of 324 ft/sec. The normal acceleration is lagged and a lead-lag compensation is applied to pitch rate. Values for these filters resulted from the quadratic design. Apparent natural speed stability is provided by generating an integration in the forward path of the elevator control system. This is mechanized by positive feedback of the elevator secondary servo position. In actual operation, the average position is used since the F8-C has a split horizontal tail surface. The loop gain (G_{C*}) was previously scheduled with dynamic pressure as shown in Figure 10. For the two explicit adaptive concepts, this gain is adjusted from estimates of elevator surface effectiveness. For the implicit concept (high-gain limit cycle), the gain is adjusted to remain at or near critical gain. The specific values of the filters are given in Figure 11 for the digital pitch CAS at a 32 sps sample rate, which was the rate on the F-8C simulator. The flight computer will operate at 50 sps. The analog equivalents of the digital filters are indicated in parenthesis.

LATERAL-DIRECTIONAL CONTROL AUGMENTATION SYSTEM

The structure of the lateral-directional CAS used with each of the adaptive concepts is shown in Figure 12 in a standard block diagram form. This design resulted from a reduced measurement solution of an optimal control problem. The full-state optimal controller was designed to follow an explicit roll rate model and to minimize sideslip and lateral acceleration for roll stick commands. The resulting design produced good turn coordination and Dutch roll damping over the entire flight envelope.

For implementation on the F-8C, the full-state design was simplified to use a reduced set of measurements. The reduced measurements consisted of p , r , δ_a , and N_y . For the CCV package, the feedback and feedforward gain schedules were modified to be a simple function of angle-of-attack. This eliminated any need for true airspeed or attitude measurements. Details are contained in Reference 2.

For use with the adaptive control laws, the feedback gains of the reduced measurement CAS were grouped into a single-loop gain that is now a function of surface effectiveness (or dynamic pressure). (Refer to Figure 13.)

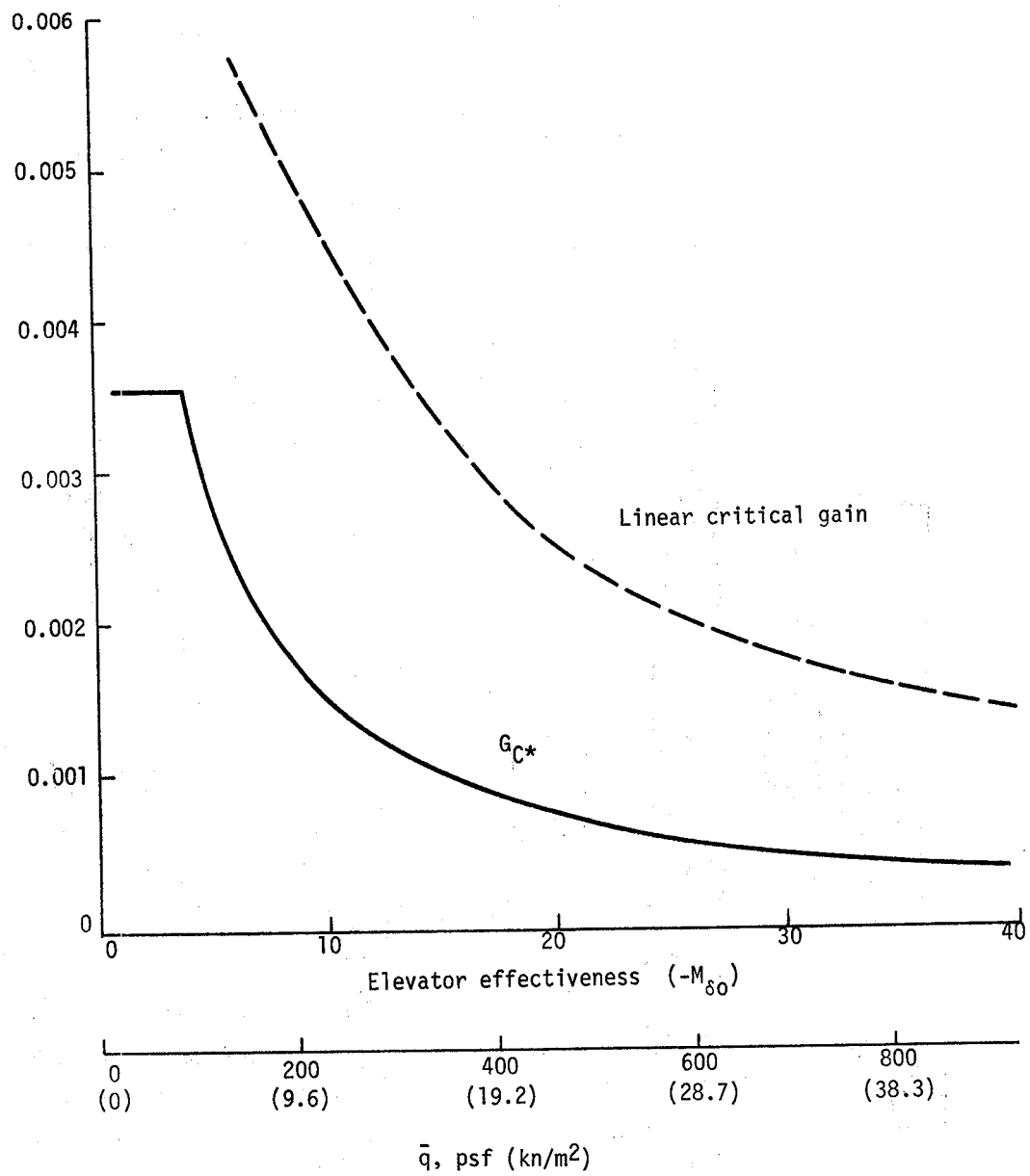


Figure 10. G_{C*} gain schedule.

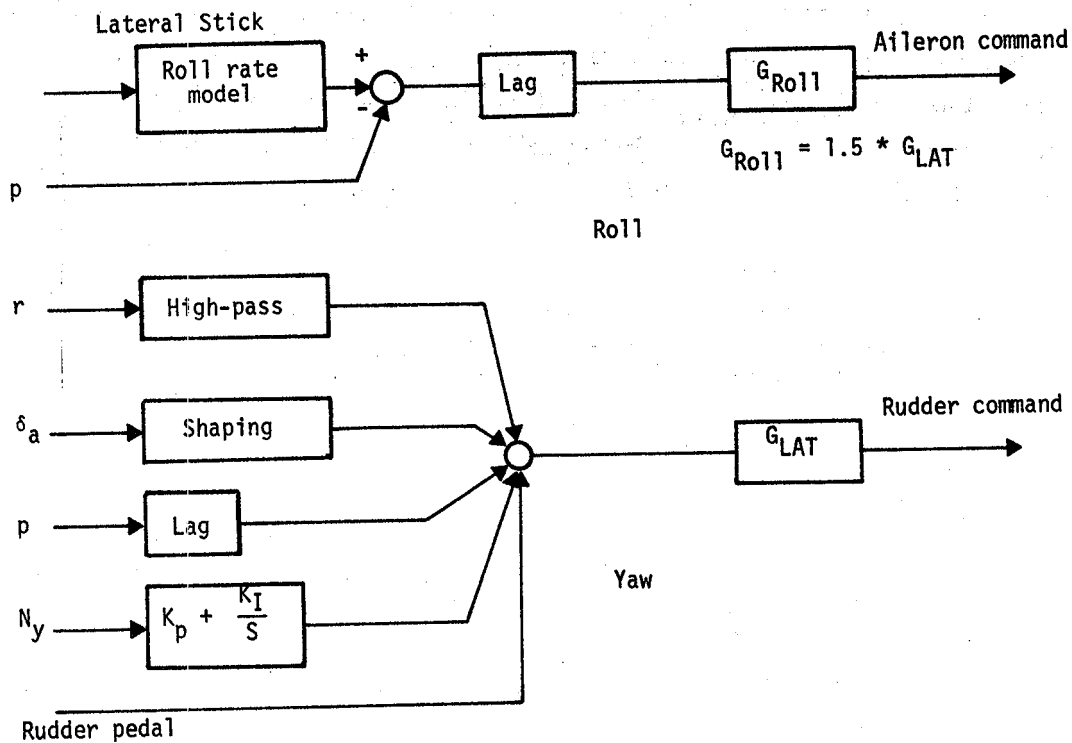


Figure 12. Structure of adaptive lateral-directional CAS.

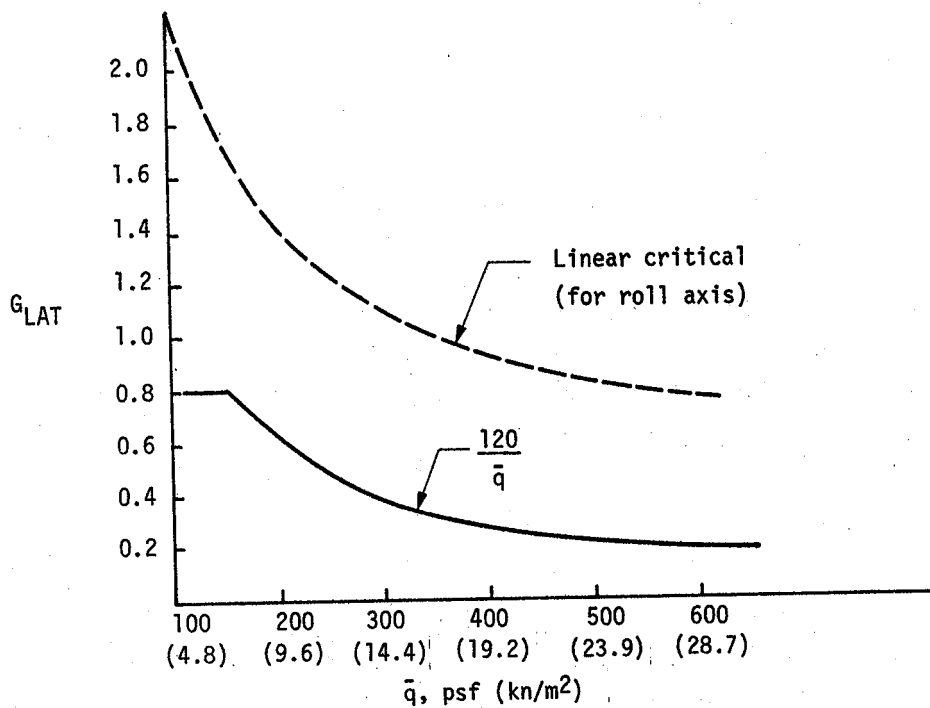


Figure 13. G_{LAT} gain schedule.

The two explicit adaptive concepts determine the loop gain from surface effectiveness estimates in the pitch axis. The limit cycle operates with a gain changer in the roll axis that keeps G_{LAT} near critical gain. The yaw axis loop gain is slaved to the roll axis. The digital lateral-directional design is shown in Figure 14 for a 32 sps sample rate.

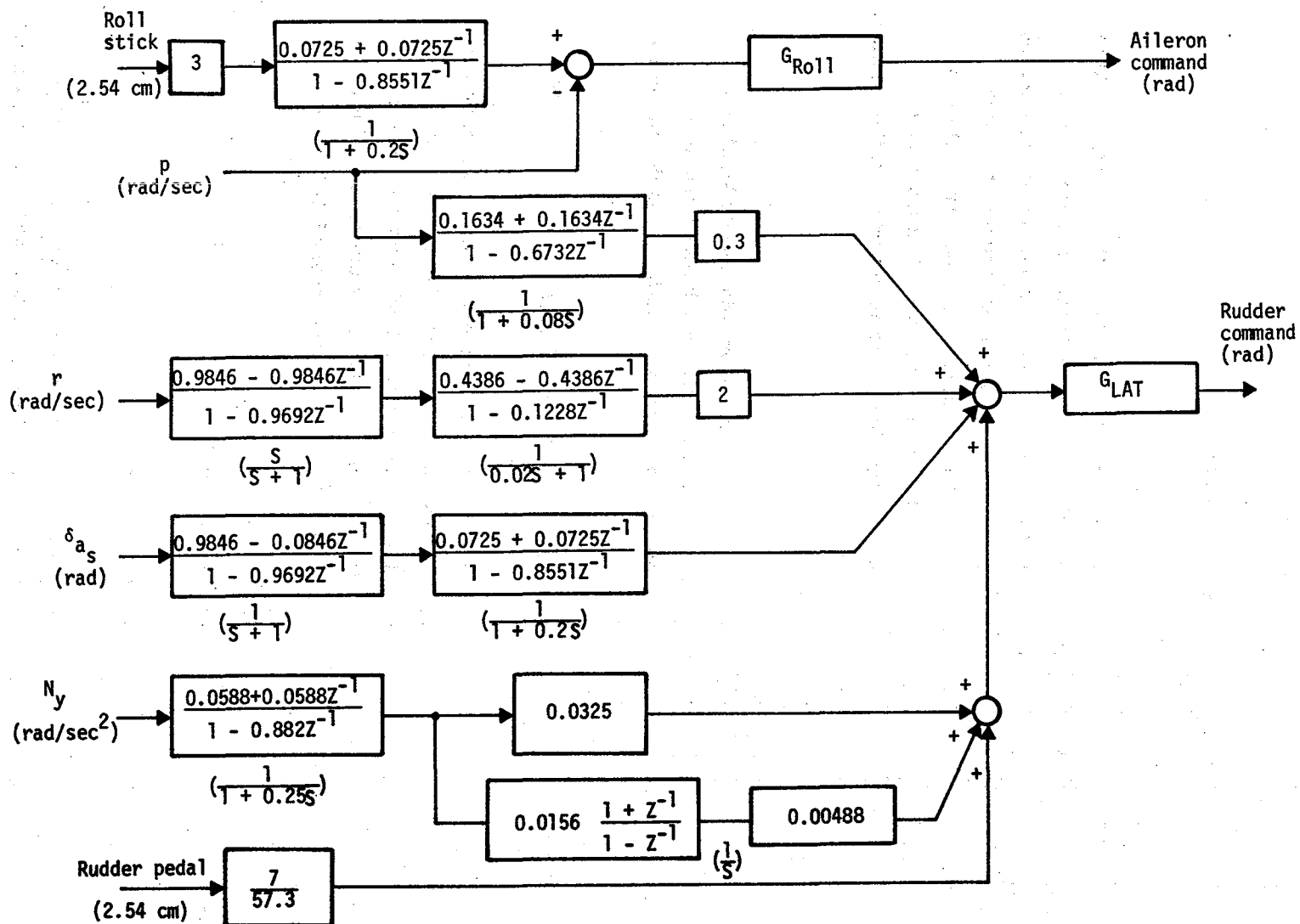


Figure 14. Lateral-directional CAS.

SECTION 8

THE MLE DESIGN

One of the three candidate adaptive concepts selected at the outset of the design program was intended to explore modern estimation-theoretic methods for explicit parameter identification. These were to be combined with simple algebraic gain update calculations to make up the complete adaptive concept. After some initial experimentation with two identification approaches, maximum likelihood estimation and nonlinear filtering (the latter largely unsuccessful), it was decided to base the concept on maximum likelihood methods. The resulting adaptive controller design is described in this section.

MAXIMUM LIKELIHOOD IDENTIFICATION

We have already introduced the general process of maximum likelihood identification in Section 6. The principle is to find parameter estimates which maximize the a posteriori probability distribution for the observed outputs conditioned on the unknowns and the measured inputs, i.e.,

$$\hat{\underline{c}} = \underset{\underline{c}}{\text{Arg}} \{ \max_{\underline{c}} p(y_1, y_2, \dots, y_N | \underline{c} = \underline{c}; u_0, u_1, \dots, u_{N-1}) \} \quad (24)$$

When the unknowns are constant and the plant dynamics are linear, this maximization problem leads to the solution shown in Figure 15.^{42, 43} The solution consists of a Kalman filter designed for the true system structure but with parameters equal to an estimate $\underline{c} = \underline{c}$. The filter tracks the true system outputs and generates a residual sequence $\{v_k = y_k - \hat{y}_k \quad k = 1, 2, \dots\}$. This sequence is accumulated into a likelihood function,

$$L(\underline{c}, N) \triangleq - \ln p(y_1, y_2, \dots, y_N | \underline{c} = \underline{c}; u_0, u_1, \dots, u_{N-1}) \quad (25)$$

which is then minimized over the parameter estimate $\underline{c}$.

At first glance, this solution appears ideal for on-board applications. The Kalman filter is relatively simple and runs recursively, processing data samples as they appear. The same is true for the likelihood accumulation operation. The difficulty, of course, is the last step of the solution: the minimization. This requires repeated or parallel

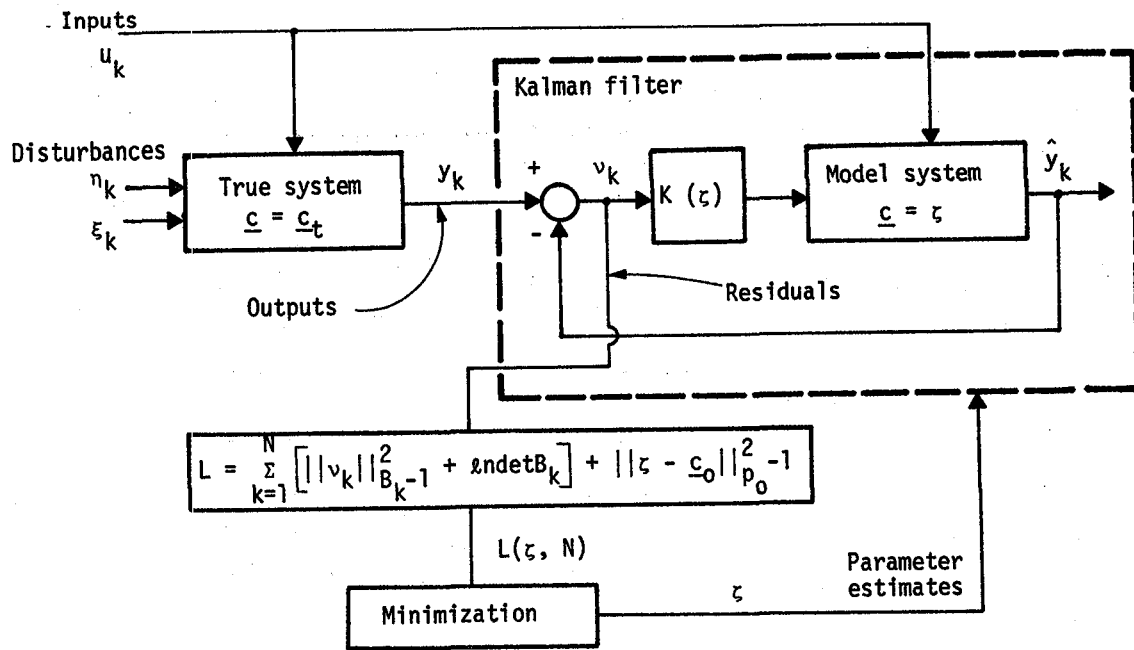


Figure 15. Maximum likelihood estimation.

processing through the data and adds significantly to computational complexity. Approaches to minimization and the equations involved are discussed further below.

Minimization Algorithms

Two algorithms were considered for the likelihood minimization operation: 1) iterative Newton-Raphson calculations, and 2) parallel non-iterative calculations. The first algorithm is illustrated in Figure 16. It begins by collecting a sequence of input/output data,

$$\begin{aligned} Y_N &= \{y_1, y_2, \dots, y_N\} \\ U_N &= \{u_0, u_1, \dots, u_{N-1}\} \end{aligned} \tag{26}$$

The Kalman filter is then run with parameters \$\underline{c}\$ equal to a priori estimates, \$\underline{c} = \zeta^0 = \underline{c}_0\$. This generates the likelihood function, \$L\$. At the same time, a set of sensitivity equations are processed which permit calculation of first- and second-partial derivatives,

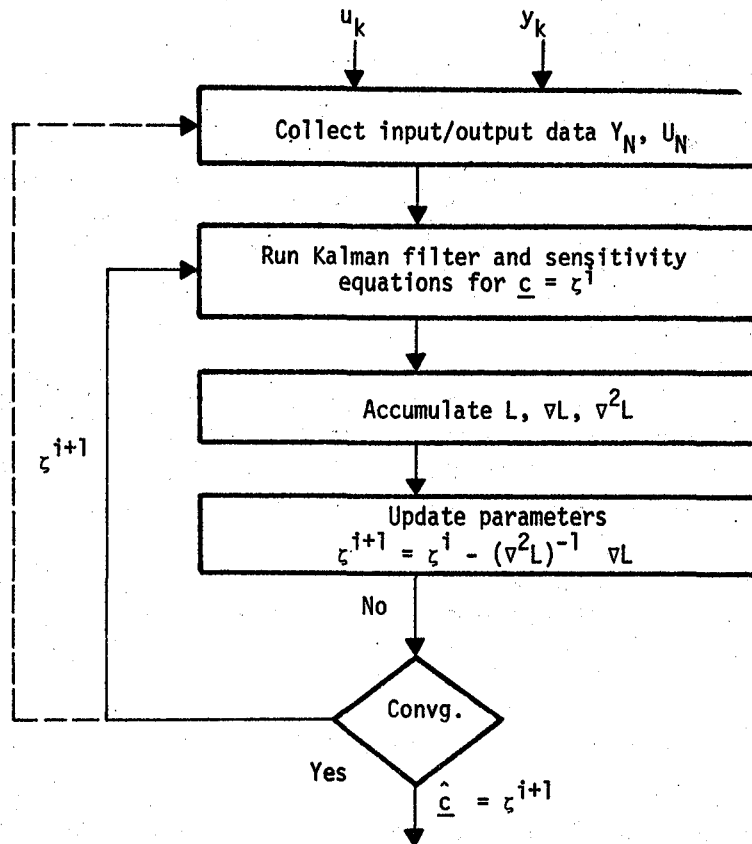


Figure 16. Newton-Raphson minimization (Algorithm 1).

∇L and $\nabla^2 L$ (some form of approximation is usually used to simplify equations for the latter). These derivatives are then used to obtain a new parameter estimate using a standard Newton-Raphson formula. The filtering, accumulation, and Newton-Raphson operations are performed repeatedly for the same data set until convergence is achieved. The data set itself is usually kept current by a "sliding window" process which increments the lower and upper indices of the input/output sequences (Figure 16) while keeping the total number of samples, N , unchanged. The overall algorithm has been implemented successfully for various post-flight data processing applications. ^{42, 43}

The second algorithm is illustrated in Figure 17. It achieves the same end result as Algorithm 1 but replaces iterative calculations with parallel ones. The sequence of input/output observations is sent simultaneously to M Kalman filter channels, each with its own sensitivity calculations and likelihood accumulations. The channels are distinguished by their assumed parameter values. Each one operates with a different parameter estimate, $\underline{c} = \zeta^{(i)}$, and hence computes, $L^{(i)}$, $\nabla L^{(i)}$, and $\nabla^2 L^{(i)}$ at a different

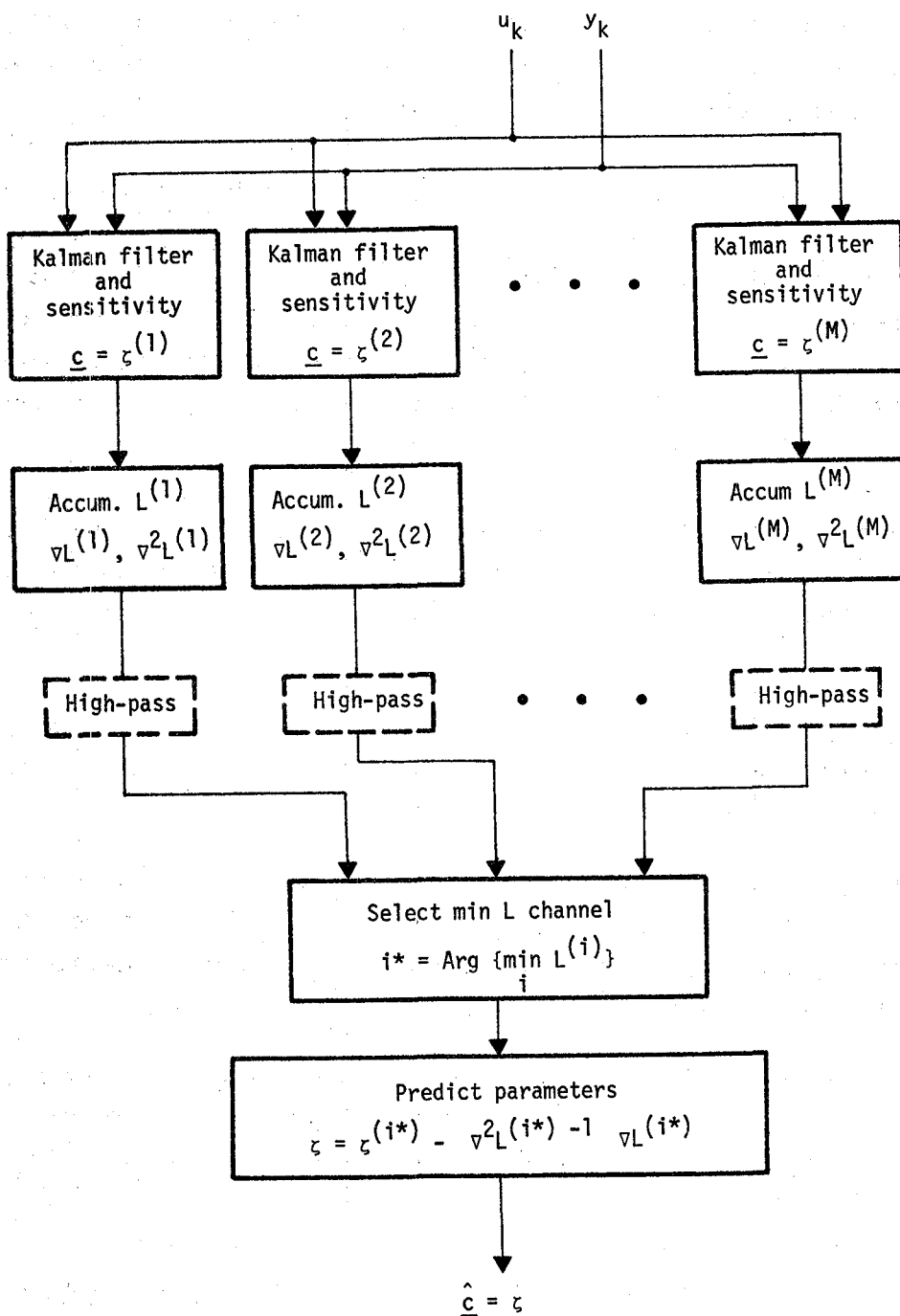


Figure 17. Non-iterative parallel minimization (Algorithm 2).

fixed point in parameter space.* The likelihood functions at these points are then compared to find the approximate minimum point, and a single Newton-Raphson step is taken from there to estimate the true extremum.

As in Algorithm 1, some procedure must generally be added to keep the processed data current. This is done in Figure 17 by high-pass operations which provide exponential de-weighting of past accumulated data samples. This can be contrasted with the above-mentioned sliding window process, which provides uniform weighting for the past N samples and zero weighting for all earlier ones. While not proposed in the exact format of Figure 17, general parallel estimation structures have been suggested in several references.^{3, 26, 65, 66}

Assuming that we have a well-posed identification problem (i.e., $L(\zeta, N)$ has a unique minimum at $\zeta = \underline{c}_t$ as $N \rightarrow \infty$), it is clear that both algorithms will perform the desired estimation function. They have important differences, however, which favor Algorithm 2 for on-board application. The chief differences are:

- Recursiveness--Algorithm 2 is completely recursive. It processes data samples as they appear and needs no data base storage and data base management operations as required in Algorithm 1.
- Iteration--Algorithm 1 is iterative. This leaves both the data processing flow and computing time uncertain, and it raises convergence issues. In contrast, the computation flow and time of Algorithm 2 are completely defined and inviolate from cycle to cycle.
- Accuracy--Both algorithms can find the minimum with arbitrary accuracy if given enough iterations or enough channels, respectively. However, for equal computing time (the number of iterations equal to the number of channels), Algorithm 1 is probably better.

As criteria for on-board application, Items 1 and 2 far outweigh the importance of Item 3. This is especially true because the location of the minimum of a realization of $L(\zeta, N)$ is itself so inaccurate (Section 6) that errors made in finding the minimum contribute little to overall identification error. Hence, it was decided to proceed with the parallel MLE algorithm.

* The enclosed superscript (i) is used to denote variables and parameter values for Channel i.

Filter, Sensitivity, and Likelihood Equations

The basic data processing equations which must be solved by each filter channel are summarized in this subsection. They are stated in terms of general symbols corresponding to the following discretized plant equations:

$$\begin{aligned} \mathbf{x}_{k+1} &= \mathbf{A}\mathbf{x}_k + \mathbf{B}\mathbf{u}_k + \mathbf{\Gamma}\eta_k \\ \mathbf{y}_k &= \mathbf{H}\mathbf{x}_k + \mathbf{D}\mathbf{u}_k + \mathbf{N}\xi_k \end{aligned} \quad (27)$$

The matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{\Gamma}$, $\mathbf{H}$, $\mathbf{D}$, and $\mathbf{N}$ should all be thought of as dependent functions of the parameter vector $\underline{\mathbf{c}}$. Then the channel equations are:

Filter equations,⁶⁷

$$\begin{aligned} \bar{\mathbf{x}}_{k+1} &= \mathbf{A}\hat{\mathbf{x}}_k + \mathbf{B}\mathbf{u}_k \\ \mathbf{v}_{k+1} &= \mathbf{y}_{k+1} - \mathbf{H}\bar{\mathbf{x}}_{k+1} - \mathbf{D}\mathbf{u}_{k+1} \\ \hat{\mathbf{x}}_{k+1} &= \bar{\mathbf{x}}_{k+1} + \mathbf{K}_{k+1} \mathbf{v}_{k+1} \end{aligned} \quad (28)$$

Sensitivity equations for each component c_p of $\underline{\mathbf{c}}$ (derived by differentiating (28) with respect to c_p),

$$\begin{aligned} \nabla_p \bar{\mathbf{x}}_{k+1} &= \mathbf{A} \nabla_p \hat{\mathbf{x}}_k + (\nabla_p \mathbf{A}) \hat{\mathbf{x}}_k + (\nabla_p \mathbf{B}) \mathbf{u}_k \\ \nabla_p \mathbf{v}_{k+1} &= -\mathbf{H} \nabla_p \bar{\mathbf{x}}_{k+1} - (\nabla_p \mathbf{H}) \bar{\mathbf{x}}_{k+1} - (\nabla_p \mathbf{D}) \mathbf{u}_{k+1} \\ \nabla_p \hat{\mathbf{x}}_{k+1} &= \nabla_p \bar{\mathbf{x}}_{k+1} + \mathbf{K}_{k+1} \nabla_p \mathbf{v}_{k+1} + (\nabla_p \mathbf{K}_{k+1}) \mathbf{v}_{k+1} \end{aligned} \quad (29)$$

Filter gains,⁶⁷

$$\begin{aligned} \bar{\mathbf{P}}_{k+1} &= \mathbf{A} \mathbf{P}_k \mathbf{A} + \mathbf{\Gamma} \mathbf{\Gamma}^T \\ \mathbf{B}_{k+1} &= (\mathbf{H} \bar{\mathbf{P}}_{k+1} \mathbf{H}^T + \mathbf{N} \mathbf{N}^T) \\ \mathbf{K}_{k+1} &= \bar{\mathbf{P}}_{k+1} \mathbf{H}^T \mathbf{B}_{k+1}^{-1} \\ \mathbf{P}_{k+1} &= \bar{\mathbf{P}}_{k+1} - \mathbf{K}_{k+1} \mathbf{H} \bar{\mathbf{P}}_{k+1} \end{aligned} \quad (30)$$

Likelihood accumulation,

$$L_{k+1} = \mu L_k + 1/2 [\|v_{k+1}\|_{B_{k+1}^{-1}}^2 + \ell n \det B_{k+1}] \quad (31)$$

$$\nabla L_{k+1} = \mu \nabla L_k + \nabla v_{k+1}^T B_{k+1}^{-1} v_{k+1} + 1/2 \text{Trace} (v_{k+1} v_{k+1}^T - B_{k+1}) \nabla (B_{k+1}^{-1})$$

$$\nabla^2 L_{k+1} = \mu \nabla^2 L_k + \nabla v_{k+1}^T B_{k+1}^{-1} \nabla v_{k+1}$$

with $\mu = \exp(-\Delta t/\tau)$ for exponential de-weighting of past data. The choice of τ is discussed later in this section.

These equations warrant two explanatory comments. First, there are no sensitivity equations for the filter gains. This is because the matrix ∇K was computed by numerical differentiation throughout the design program, i.e.,

$$\nabla_p K = [K(\underline{c} + \lambda_p \underline{e}_p) - K(\underline{c})]/\lambda_p$$

where $\underline{e}_p$ is a unit vector in the i -th coordinate direction and λ_p was chosen small compared to the range of $\underline{c}_p$.

Second, since $\nabla^2 L$ must be a derivative of ∇L , it is evident that the last equation in (31) includes approximations. All second-partial derivatives and products of derivatives have been ignored. This is a common approximation for so-called modified Newton-Raphson procedures³⁷ and has the important advantage of eliminating second-partial derivative sensitivity equations. It is also common practice to eliminate the last term of the ∇L equation and to replace the filter gain equations with their steady state solutions. These additional approximations are considered later.

IDENTIFIER DESIGN FOR THE F-8C

In view of the identifiability results in Section 6 and the known scheduling requirements of the control laws (Section 7), a specific implementation of Algorithm 2 was developed for the pitch axis only and for a reduced-parameter set with three components. Design details and trade-offs for this identifier are presented below. Key design issues include:

- Identification models,
- Channel selection,
- Kalman filter design,
- Adaptation to noise statistics, and
- Likelihood filters.

Identification Models

Following Section 6, a simplified short-period model was chosen to represent the pitch plant dynamics (27).

State equations:

$$\frac{d}{dt} \begin{bmatrix} q \\ \alpha + \alpha_g \\ g/V \\ M_O \end{bmatrix} = \begin{bmatrix} M_q & M_\alpha & 0 & 1 \\ 1 & Z_\alpha & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} q \\ \alpha + \alpha_g \\ g/V \\ M_O \end{bmatrix} + \begin{bmatrix} M_\delta \\ Z_\delta \\ 0 \\ 0 \end{bmatrix} \delta + \begin{bmatrix} 0 & 0 & 0 \\ \gamma_{21} & 0 & 0 \\ 0 & \gamma_{32} & 0 \\ 0 & 0 & \gamma_{43} \end{bmatrix} \eta \quad (32)$$

Measurements:

$$y_k = \begin{bmatrix} q_m \\ n_{zm} \\ \delta_m \end{bmatrix}_k = \begin{bmatrix} 1 & 0 & 0 & 0 \\ dM_q & dM_\alpha - Z_\alpha V & 0 & d \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} q \\ \alpha + \alpha_g \\ g/V \\ M_O \end{bmatrix}_k + \begin{bmatrix} 0 \\ dM_\delta - Z_\delta V \\ 1 \end{bmatrix} \delta_k + \begin{bmatrix} \sigma_q & 0 & 0 \\ 0 & \sigma_{nz} & 0 \\ 0 & 0 & \sigma_\delta \end{bmatrix} \xi_k \quad (33)$$

$$k = 0, 1, 2, \dots \quad (t_{k+1} - t_k) = \Delta t$$

This model includes short-period dynamics (q , α), turbulence (α_g), trim states (g/V , M_0), and noisy sampled algebraic sensors for pitch rate, normal acceleration, and elevator surface position. Turbulence is again modelled as an approximated Dryden spectrum without the low frequency drop-off in its derivative, i.e.,

$$\frac{d}{dt} \alpha_g = -\left(\frac{V}{L_w}\right) \alpha_g + \sigma_w \sqrt{\frac{2}{VL_w}} \eta \simeq \sigma_w \sqrt{\frac{2}{VL_w}} \eta \quad (34)$$

$$\text{Hence } \gamma_{21} = \sigma_w \sqrt{\frac{2}{VL_w}}$$

The trim terms (which according to Section 6 must be included for accurate reduced-parameter identification) are appended as slowly varying states with Brownian motion representations. The term g/V was given a variance growth rate proportional to roll angle variance, and M_0 has a variance growth rate proportional to the variance of the trim pitching moment due to control. Hence,

$$\gamma_{32} = \sigma_\varphi \frac{g_0}{V} \quad (35)$$

$$\gamma_{43} = \sigma_{\delta_{\text{trm}}} M_\delta$$

All sensor noises are modelled as independent and identically distributed random variables from sample to sample. Their magnitudes σ_q , σ_{nz} , σ_δ as well as the disturbance magnitudes σ_w , σ_φ , $\sigma_{\delta_{\text{trm}}}$ were treated as design parameters.

Using Section 6, Table 11, the identification model was parameterized in terms of M_δ and two small-perturbation parameters for M_α and V . The parameter M_δ was chosen because it is most identifiable and also most important for control. M_α was chosen because it falls next in the hierarchy of identifiability. Velocity was included because it is desirable for lateral control. However, since V is not readily identifiable from the pitch axis alone, we cannot expect much in the way of accuracy. The rest of the perturbations in Table 11 were assumed to be zero for reduced-parameter identification. The parameterization was also modified to account for quasi-static flexibility of the airframe (not included in Section 6). This phenomenon reduces all control effectiveness terms at high dynamic pressure (high M_δ) and can be accounted for by expressing the model in terms of an "un-flexed" M_{δ_0} parameter instead of M_δ directly. The resulting parameterization is summarized on the following page.

$$\begin{bmatrix} M_\delta \\ M_q \\ M_\alpha \\ V \\ Z_\alpha V \\ Z_\delta V \end{bmatrix} \approx \begin{bmatrix} M_{\delta 0} (1 + 0.016 M_{\delta 0} + 0.0002 M_{\delta 0}^2) \\ -0.23 + (0.028 - 0.017 c_2) M_{\delta 0} \\ (0.61 + 0.92 c_2) M_{\delta 0} \\ (200 + c_3) M_{\delta 0} \\ 53 M_{\delta 0} \\ 7.7 M_\delta \end{bmatrix} \quad (36)$$

$$\underline{c} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} M_{\delta 0} \\ c_2 \\ c_3 \end{bmatrix} \quad \begin{matrix} c_2 \approx \begin{cases} 0 & \text{subsonic} \\ 1 & \text{supersonic} \end{cases} \\ c_3 \approx \begin{cases} 0 & \text{subsonic} \\ 60 & \text{supersonic} \end{cases} \end{matrix} \quad (37)$$

Channel Selection

The three-dimensional parameterization above generates a parameter space illustrated in Figure 18. The space consists of two regions, one for subsonic and the other for supersonic flight. There is also a loose interconnection through which the aircraft migrates rapidly in the transonic flight regime. The problem of channel selection is to choose both the number and the location of points in this space at which to operate Kalman filter and sensitivity equations. Criteria for this choice are derived from the interpolation function of the channels. We obviously want as few channels as possible, but they must be close enough together to be able to interpolate throughout the space.

For the F-8C problem, a bit of experimentation showed that the c_2 and c_3 directions could be readily interpolated from channels located at their nominal values, i.e., (0, 0) or (1, 60). The $M_{\delta 0}$ direction, however, requires several channels, each positioned so that it interpolates over an $M_{\delta 0}$ region which is a fixed percentage of its nominal value. This follows from results in Section 6 which indicate that accuracy, and hence curvature of the likelihood function, is proportional to true M_δ values

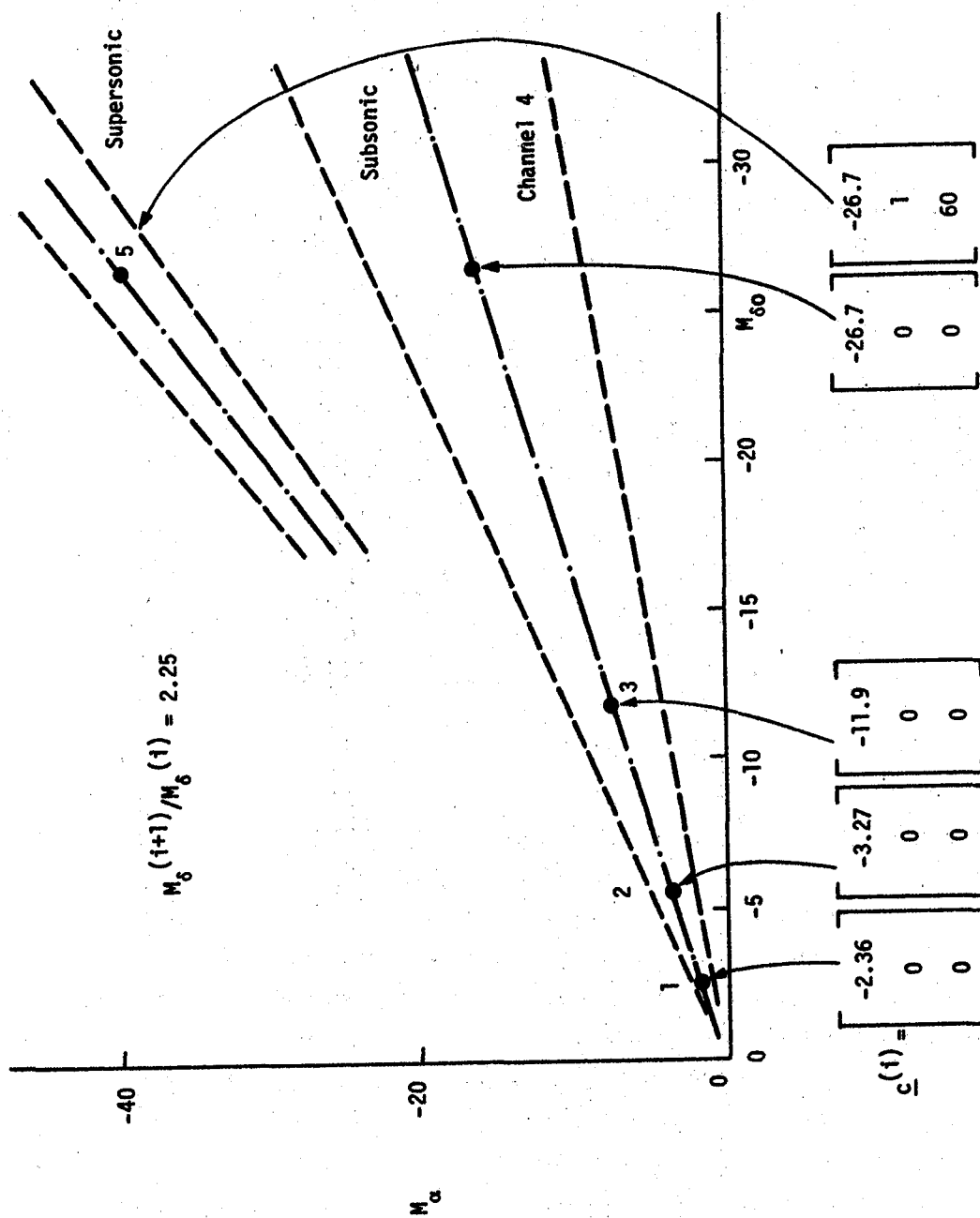


Figure 18. MLE channel selection.

(percentage accuracy is constant). Using these findings and a 50 percent interpolation range for each channel gives the following candidate $M_{\delta o}$ values:

$$M_{\delta o}^{(4)} = -40/1.5 = -26.7$$

$$M_{\delta o}^{(3)} = M_{\delta o}^{(4)}/2.25 = -11.9$$

$$M_{\delta o}^{(2)} = M_{\delta o}^{(3)}/2.25 = -5.27$$

$$M_{\delta o}^{(1)} = M_{\delta o}^{(2)}/2.25 = -2.34$$

These values must be combined with $(c_1, c_2) = (0, 0)$ for subsonic flight and $(c_1, c_2) = (1, 60)$ for supersonic. A total of five channels are required--one for supersonic, which has an $M_{\delta o}$ range entirely covered by interpolation about $M_{\delta o}^{(5)} = M_{\delta o}^{(4)}$, and four for subsonic. The resulting channel locations are shown in Figure 18.

Kalman Filter Design

According to Algorithm 2, a fixed set of Kalman filter and sensitivity equations must be operated at each set of parameter values shown in Figure 18. While the design of these filters is straightforward, the following features of the F-8 design are worth noting.

Time Invariance--A few experiments showed that steady state filter gains achieve almost the same speed of convergence (of the identifier) as time-varying gains. Hence, Equations (30) were replaced by their steady state equivalents (set $k+1 = k$) and solved off-line. Stored values of K and ∇K are used in the on-line identifier.

Self-Initialization--When the adaptive mode is engaged, the filter and sensitivity equations in each channel are initialized on the basis of current sensor outputs (q_m, n_{zm}, δ_m) . This step avoids long channel transients associated with the slow trim states, g/V and M_o . The initialization assumes that the aircraft is in equilibrium at the time of engagement (i.e., $\dot{x} = 0$), which gives the following equations for each channel:

$$q(0) = q_m$$

$$\alpha(0) = -[n_{zm} + (Z_\delta V) \delta_m] / (Z_\alpha V)$$

(38)*

$$\frac{g}{V}(0) = -[A_{21} q_m + (A_{22} - 1) \alpha(0) + B_{21} \delta_m] / A_{23}$$

$$M_o(0) = -[(A_{11} - 1) q_m + A_{12} \alpha(0) + B_{11} \delta_m] / A_{14}$$

$$\nabla q(0) = 0$$

$$\nabla \alpha(0) = -[(Z_\alpha V) \alpha(0) + \nabla(Z_\delta V) \delta_m] / (Z_\alpha V)$$

(39)*

$$\nabla \frac{g}{V}(0) = -[\nabla A_{21} q(0) + \nabla A_{22} \alpha(0) + (A_{22} - 1) \nabla \alpha(0) + \nabla B_{21} \delta_m] / A_{23}$$

$$\nabla M_o(0) = -[\nabla A_{11} q(0) + \nabla A_{12} \alpha(0) + A_{12} \nabla \alpha(0) + \nabla B_{11} \delta_m] / A_{14}$$

Noise Statistics--The filter gains for each channel were designed for the following assumed noise statistics:

<u>Sensor Noise</u>	<u>Disturbances</u>
$\sigma_q = 0.0026 \text{ r/s}$	$\sigma_w = 6.0 \text{ ft/s (1.83 m/s)}$
$\sigma_{nz} = 0.02 \text{ g's (0.2 m/s}^2\text{)}$	$\sigma_\varphi = 0.017 \text{ rad}$
	$\sigma_{\delta_{\text{trm}}} = 0.001 \text{ rad}$
	$\sigma_\delta = 0.0008 \text{ rad}^\dagger$

* The A_{ij} symbols in these equations refer to elements of the discretized dynamics matrix corresponding to Equation (32).

† The Kalman filter Equations (28) treat $u_k = \delta_k$ as perfectly known. The actual measurement, of course, is noisy. This was recognized by adding the control measurement error as an extra random disturbance in the discrete model Equation (27), i.e.,

$$x_{k+1} = Ax_k + Bu_{mk} + (\Gamma \eta_k - B\xi_{\delta k}) \quad (27')$$

Since these numbers are fixed for all operating conditions (i. e., with and without sensor noise, with and without gusts, with slow or fast trim changes, etc.), they represent experimental compromises over many simulation runs. As seen later in the performance summaries, some performance benefit can be gained by adapting these statistics to the environmental situation.

State Estimate Interpolation -- Since each filter operates at a fixed location in parameter space, no one of them provides proper state estimates unless the parameters happen to fall close to one of the channels. This difficulty is alleviated, at least for the α estimate, by interpolating between adjacent channels. Because trim angles-of-attack are inversely related to $M_{\delta o}$, the following interpolating function was chosen:

$$\hat{\alpha}(M_{\delta o}) = a_1 + \frac{1}{M_{\delta o}} a_2 \quad (40)$$

The coefficients a_1 and a_2 for this function are computed from estimates of α at two adjacent channels (i, j) and then inserted back into (40). The resulting formula is:

$$\hat{\alpha}(M_{\delta o}, t) = \frac{(\hat{M}_{\delta o} - M_{\delta o}^{(i)}) \hat{\alpha}(M_{\delta o}^{(j)}, t) + (M_{\delta o}^{(j)} - \hat{M}_{\delta o}) \hat{\alpha}(M_{\delta o}^{(i)}, t)}{(M_{\delta o}^{(j)} - M_{\delta o}^{(i)})} \quad (41)$$

Adaptation to Proportional Noise Statistics

We noted in the filter design section that fixed statistics were used to compute filter gains. This is desirable because it generates an invariant set of gains for each channel which can be computed off-line and stored for on-line use. Invariant gains are actually obtained under slightly less restrictive circumstances--namely, when the disturbance and noise statistics remain in constant relationship to one another. This means that a filter for disturbances statistics ($\sigma_w, \sigma_\varphi, \sigma_{\delta_{trm}}, \sigma_\delta$) and sensor noise statistics (σ_q, σ_{nz}) remains unchanged when those statistics are changed to ($\sigma_w, \sigma_\varphi, \sigma_{\delta_{trm}}, \sigma_\delta$) and (σ_q, σ_{nz}) for arbitrary σ . As a result, the total identification algorithm designed for one set of statistics can still be valid when those statistics are scaled up or down, provided only that the likelihood functions are also scaled. This can be done adaptively as described in the following paragraphs.

Assume that all statistics are known to within a constant scale factor σ . Then the likelihood function, in terms of the usual unknowns ($\underline{c}$) and the new unknown σ , is given by

$$L = 1/2 \sum_{k=1}^N \left(v_k^T \frac{B^{-1}}{\sigma^2} v_k + \ln \det \sigma^2 B \right) \quad (42)$$

We find estimates for $\underline{c}$ and σ in the usual way by letting

$$0 = \nabla L = \sum_{k=1}^N \left[\nabla v_k^T \frac{B^{-1}}{\sigma^2} v_k + 1/2 \text{Trace} (v_k v_k^T - \sigma^2 B) \nabla \left(\frac{B^{-1}}{\sigma^2} \right) \right] \quad (43)$$

$$0 = \frac{\partial L}{\partial \sigma} = \sum_{k=1}^N 1/2 \text{Trace} (v_k v_k^T - \sigma^2 B) \left(-2 \frac{B^{-1}}{\sigma^3} \right)$$

The last equation yields the solution

$$\hat{\sigma}^2 = J/rN$$

with

$$J(\zeta, N) \triangleq \sum_{k=1}^N v_k^T B^{-1} v_k \quad (44)$$

where r is the number of measurements and N is the length of the data sample. The product rN is the degree of freedom in the quadratic form (44). * The $\hat{\sigma}^2$ estimate can now be used to scale the likelihood function; i. e.,

$$2 \hat{\sigma}^2 L = \sum_{k=1}^N \left(v_k^T B^{-1} v_k + \hat{\sigma}^2 \ln \det B + \text{constants} \right) \quad (45)$$

In the F-8 identifier, Equation (44) is evaluated at the minimum likelihood channel in order to estimate $\hat{\sigma}^2$. This estimate is then used to scale all channel likelihood function via Equation (45), before subsequent comparisons of these functions are made.

* Because the actual likelihood functions are being filtered, the degrees of freedom in the identifier are different from rN . They are determined by passing the constant r through the same filters and hence rise exponentially to the steady state level $r/(1-\mu)$.

Likelihood Filters

Figure 17 shows a high-pass filtering operation on the accumulated likelihoods in each channel of Algorithm 2. This operation keeps the accumulations current by exponentially de-weighting the past. The rate at which de-weighting occurs (or the choice of time constants for the high pass) is determined by two conflicting requirements:

1. A well-defined, correct minimum of the likelihood function requires slow de-weighting.
2. Small tracking errors when aircraft parameters change requires fast de-weighting.

The first requirement exists because likelihood functions do not necessarily have minima in the correct place (i. e., at $\underline{c} = \underline{c}_t$) for short data samples. Some examples of this behavior are given in Figure 19. The figure shows plots of experimental likelihood functions taken during our identifiability studies. The functions were accumulated without de-weighting and are shown at several time points. It is clear that times shorter than three to five seconds can give dramatically incorrect answers. It should be noted that this phenomenon is not inconsistent with maximum likelihood theory. The theory guarantees correct answers only asymptotically.

The second requirement is less esoteric. As the aircraft changes flight condition, its parameters change in ramp-like fashion. The accumulated likelihood functions will then be out of data and cause parameter estimates to lag behind the true parameter values. The faster we de-weight accumulated data, the less the lag.

The design compromise for these requirements is a de-weighting time constant of five seconds. This satisfies the experimental curves in Figure 19 and produces 30 percent lag errors in $\hat{M}_{\delta_0}$ when the aircraft is accelerated with full afterburner starting at Flight Condition 5. (Traces for this maneuver are shown later in this section.) Errors in M_{δ_0} of the order of 30 percent are quite tolerable for scheduling the control laws in Section 7. It should be recognized, however, that the F-8C can be maneuvered so as to produce substantially faster flight transitions, and hence we must maintain tolerance in the control laws for errors well above 30 percent.

In addition to low frequency de-weighting of accumulated likelihoods, it was also found desirable to prevent very high frequency data (such as sharp edged gust or command

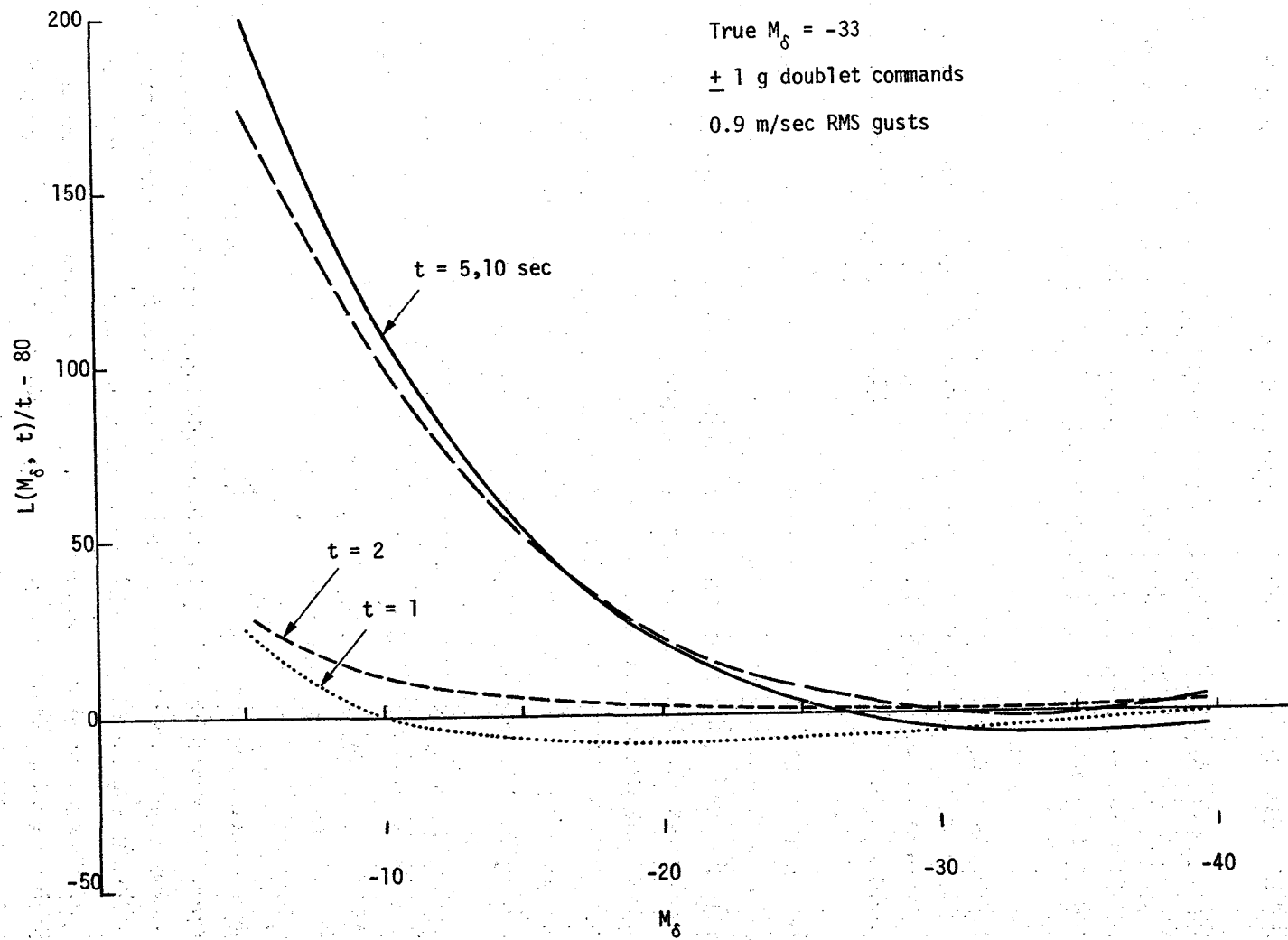


Figure 19. Typical likelihood functions.

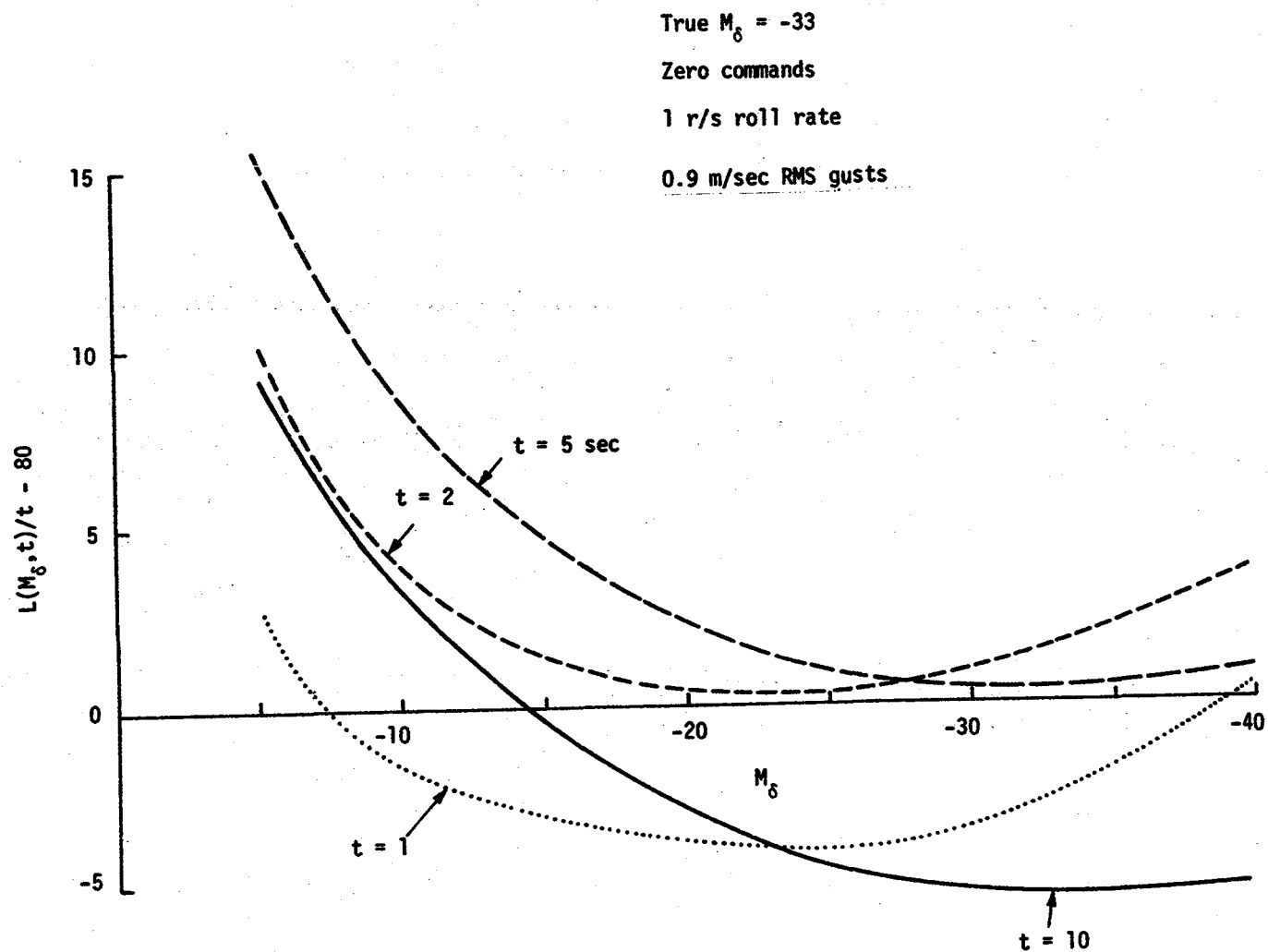


Figure 19. Concluded.

responses) from being accumulated. This was done by adding relatively high frequency, low-pass filter to the accumulation and high-pass network. The total likelihood filter then takes the following (digital) form:

$$\Delta \epsilon_k \rightarrow \boxed{\overline{\Delta \epsilon}_{k+1} = \mu_2 \overline{\Delta \epsilon}_k + (1 - \mu_2) \Delta \epsilon_k} \xrightarrow{\overline{\Delta \epsilon}_k} \boxed{\epsilon_{k+1} = \mu \epsilon_k + \overline{\Delta \epsilon}_k} \rightarrow \epsilon_k \quad (46)$$

Unity gain low pass

$$\tau = 0.6 \text{ sec}$$

$$\mu_2 = \exp(-\Delta/0.6)$$

Accumulation and high pass

$$\tau = 5 \text{ sec}$$

$$\mu = \exp(-\Delta/5)$$

The symbols $\Delta \epsilon_k$, $\overline{\Delta \epsilon}_k$, and ϵ_k denote generic inputs, intermediate states, and outputs, respectively. The filter is actually used to generate all likelihood functions, $L^{(i)}$ $i = 1, 2, \dots, 5$, and all components of ∇L and $\nabla^2 L$.

CONTROL LAW ADJUSTMENT

To complete the adaptive concept, the identifier described above was combined with a simple control gain calculation. Both pitch and lateral gains are adjusted as a function of the primary identified parameter, $M_{\delta 0}$. In the pitch axis, the basic CCV control law gain schedule is utilized, but with a somewhat higher gain ratio. The CCV schedule (Section 7) calls for

$$G_{c*} = \frac{0.158}{\bar{q}}$$

This was replaced with the schedule

$$G_{c*} = \frac{0.35}{\bar{q}} \doteq - \frac{0.015}{M_{\delta 0}} \quad (47)$$

where the conversion from $\bar{q}$ to $M_{\delta 0}$ is based on the F-8C's very good linear $\bar{q} - M_{\delta 0}$ relationship shown in the scatter plots of Appendix A ($\bar{q} \doteq -23 M_{\delta 0}$). The gain schedule (46) is a little more than twice as high as the CCV schedule, yet it still provides well above six db of stability margin. For additional protection, the schedule is restricted to a maximum gain output of

$$G_{c* \max} = \frac{0.35}{100} = 0.0035$$

and to a minimum gain of

$$G_{c* \min} = \frac{0.35}{600} = 0.00058$$

Similarly, the lateral gain was defined by the following schedule:

$$G_{LAT} = \frac{120}{\bar{q}} = 343 G_{c*} \quad (48)$$

This is automatically restricted to the range

$$1.20 = \frac{120}{100} \leq G_{LAT} \leq \frac{120}{600} = 0.20$$

by the limits imposed on G_{c*} .

SIGNIFICANCE TESTS

Although the above schedules provide ample stability margin when the identifier is working properly, they do not exclude system instabilities for extreme identification errors such as might occur with no signals present in the control loop. To protect against this possibility, a self-checking feature was added to the identifier which monitors the statistical significance of M_{δ_0} estimates. This self check calculates approximate theoretical probabilities that the true parameter values fall into the range covered by each channel. Whenever this probability exceeds a specified threshold (10^{-6}) for any channel, the upper G_{c*} limit is reset to a gain value low enough to assure stability for true parameters actually in the range of that channel. The calculations associated with the self-check operation are summarized below.

Probability Derivations

Let $\Pr^{(i)}$ denote the conditional probability that the true parameters are actually in the range $\Omega^{(i)}$ covered by Channel i . This probability is defined by

$$\text{Pr}^{(i)} = \int_{\Omega^{(i)}} p(\zeta | Y_N, U_N) d\zeta$$

(49)

$$= \int_{\Omega^{(i)}} \frac{p(Y_N | \zeta, U_N) p(\zeta)}{p(Y_N | U_N)} d\zeta$$

Here $p(\zeta)$ is the a priori probability distribution for parameter vector $\underline{c}$. This distribution can be assumed to be uniform with equal probability for each channel range; i. e.,

$$p(\zeta) = \lambda^{(i)} \text{ for all } \zeta \in \Omega^{(i)}$$

$$\lambda^{(i)} \int_{\Omega^{(i)}} d\zeta = 1/M$$

Then (48) reduces to

$$\begin{aligned} \text{Pr}^{(i)} &= \frac{\lambda^{(i)}}{p(Y_N | U_N)} \int_{\Omega^{(i)}} p(Y_N | \zeta, U_N) d\zeta \\ &= \frac{\lambda^{(i)}}{p(Y_N | U_N)} p(Y_N | \zeta^{(i)}, U_N) \int_{\Omega^{(i)}} d\zeta \\ &= \frac{p(Y_N | \zeta^{(i)}, U_N)}{M p(Y_N | U_N)} \end{aligned} \quad (51)$$

Next, it is convenient to divide through by $\text{Pr}^{(i^*)} = 1$ (i^* denotes the min-L channel) and to take natural logs:

$$\text{Pr}^{(i)} \sim \frac{\text{Pr}^{(i)}}{\text{Pr}^{(i^*)}} = \frac{p(Y_N | \zeta^{(i)}, U_N)}{p(Y_N | \zeta^{(i^*)}, U_N)} \quad (52)$$

$$\begin{aligned} \ln \text{Pr}^{(i)} &\sim \ln \text{Pr}^{(i)} - \ln \text{Pr}^{(i^*)} \\ &\sim -L^{(i)} + L^{(i^*)} \end{aligned}$$

This final equation forms the basis for the significance test. Since we want to detect cases for which $\Pr^{(i)} > 10^{-6}$, it suffices to test for

$$L^{(i)} - L^{(i*)} \geq -\ln(10^{-6}) = 13.8 \quad (53)$$

and to reset $G_{c* \max}$ whenever this test fails.

Gain Limit Reset

The actual limit imposed on G_{c*} in the event that the significance test fails is given by

$$G_{c* \max} = -0.039/1.5 M_{\delta 0}^{(i)} \quad (54)$$

This is eight db above the standard schedule, Equation (47), evaluated at the maximum $|M_{\delta 0}|$ value covered by Channel i. Of course, if the significance test fails for several channels, then the smallest limit applies.

DESIGN REFINEMENTS

The identifier and control laws described above were tested on linear simulations in Minneapolis and on the nonlinear F-8 simulator at Langley Research Center with a high degree of success. They only posed one serious problem--computing time. On the CDC-6600 computers at both simulation facilities, the identifier consumed approximately four to six msec per control cycle. Roughly scaled to the F-8C's IBM AP-101 machine, this corresponds to 20 to 30 msec and exceeds the total frame time available at the nominal 50 Hz update rate planned for F-8C flight tests. A serious effort was therefore undertaken to develop design refinements which would reduce running time of the identifier. Three such refinements were developed:

1. Single-channel sensitivity calculations,
2. Multi-rate structures, and
3. Index-free Fortran code for primary channel calculations.

These refinements reduce CDC computing times for the complete adaptive concept to approximately one msec/cycle. They are discussed in detail in the following pages.

Single Channel Sensitivities

Since interpolation between channels of Algorithm 2 utilizes sensitivities from the min-L channel only, it is clear that a great deal of redundant computing is being done in the other channels. Redundant operations include sensitivity calculations for $\hat{\nabla} \mathbf{x}$ and likelihood accumulations for $\nabla \mathbf{L}$ and $\nabla^2 \mathbf{L}$. These operations have been eliminated by providing a data hand-off procedure which allows one channel's sensitivity calculations to be started up and another's terminated whenever a channel change occurs. The transition occurs smoothly and without severe estimator transients. It is accomplished with the following transfer sequence from Channel j to Channel i at time t_k :

1. Initialize $\hat{\nabla} \mathbf{x}(t_k)$ in Channel i

Equations (39) with states $\mathbf{q}(0)$ and $\alpha(0)$ replaced by their current estimates from Channel i, $\hat{\mathbf{q}}^{(i)}(t_k)$, $\hat{\alpha}^{(i)}(t_k)$.

2. Transfer Accumulated $\nabla^2 \mathbf{L}$ Matrix from Channel j to Channel i

$$\nabla^2 \mathbf{L}^{(i)} = \mathbf{T} \nabla^2 \mathbf{L}^{(j)} \mathbf{T}^T \quad (55)$$

where

$$\mathbf{T} = \text{diag} \left\{ \rho \left(1 + \frac{M_{\delta o}^{(j)}}{M_{\delta o}^{(i)}} \right), 1, 1 \right\}$$

with

$$\rho = 1.10 - \exp(-t_k/5)$$

Matrix $\mathbf{T}$ adjusts the implied estimation accuracy of the transferred second-partials matrix. Basically, the accuracy is increased for every transfer in order to avoid rapid estimation transients. However, larger increases are used for transfers from high to low $|M_{\delta o}|$ values, and actual decreases are used early in a flight (through the parameter ρ). The larger increases preserve the identifier's constant percentage error property (Section 6), while the decrease speeds initial convergence.

3. Compute New ∇L Vector for Channel i

$$\nabla L^{(i)} = \nabla^2 L^{(i)} (\underline{c}^{(i)} - \hat{\underline{c}}_k) \quad (56)$$

This equation assures that the identifier will interpolate to the current estimate $\hat{\underline{c}}_k$ from the new channel immediately after transfer. Hence the parameter estimate remains continuous.

After transfer, the propagation of $\nabla \hat{x}$ and accumulation of ∇L and $\nabla^2 L$ proceed as normal in Channel i, starting from the initial conditions $\nabla \hat{x}(t_k)$, $\nabla L_k^{(i)}$, and $\nabla^2 L_k^{(i)}$ defined above. The same calculations in Channel j are terminated.

Multi-rate Structure

The second timesaving refinement of the adaptive concept consists of a rearranged sequence of computation. Since many operations do not need to be repeated at every control sample time, the calculations were broken into six separate groups, each containing those calculations which must be performed at every minor cycle plus a subset of calculations which can be performed less frequently. These six groups are then executed sequentially, one per minor cycle. The resulting multi-rate structure is illustrated in simplified flow diagram form in Figure 20.

Fortran Implementation

Other substantial time savings were achieved with modified Fortran code. In particular, a separate subroutine was prepared to implement five Kalman filter equations and one set of sensitivity equations in index-free Fortran. This subroutine accomplishes all minor cycle calculations shown in Figure 20.

Figure 20 also illustrates several other features of the final identifier implementation. These include:

1. Temporary likelihood accumulation for each minor cycle in order to prevent data loss between likelihood filter updates (which occurs once per six minor cycles),
2. Limits on $\hat{\sigma}^2$ to prevent excessively large noise level estimates induced by model mismatch at the min-L channel,

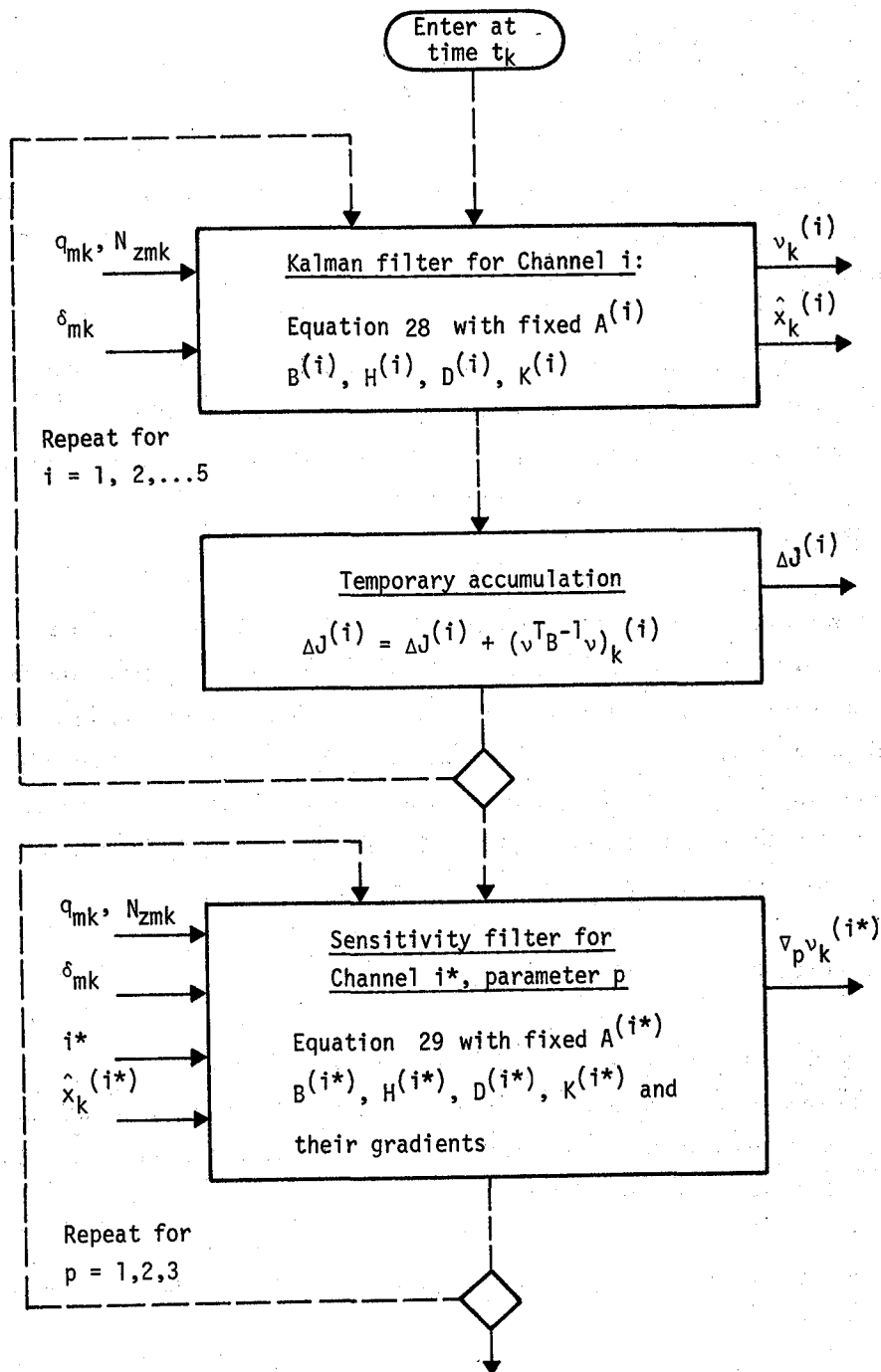
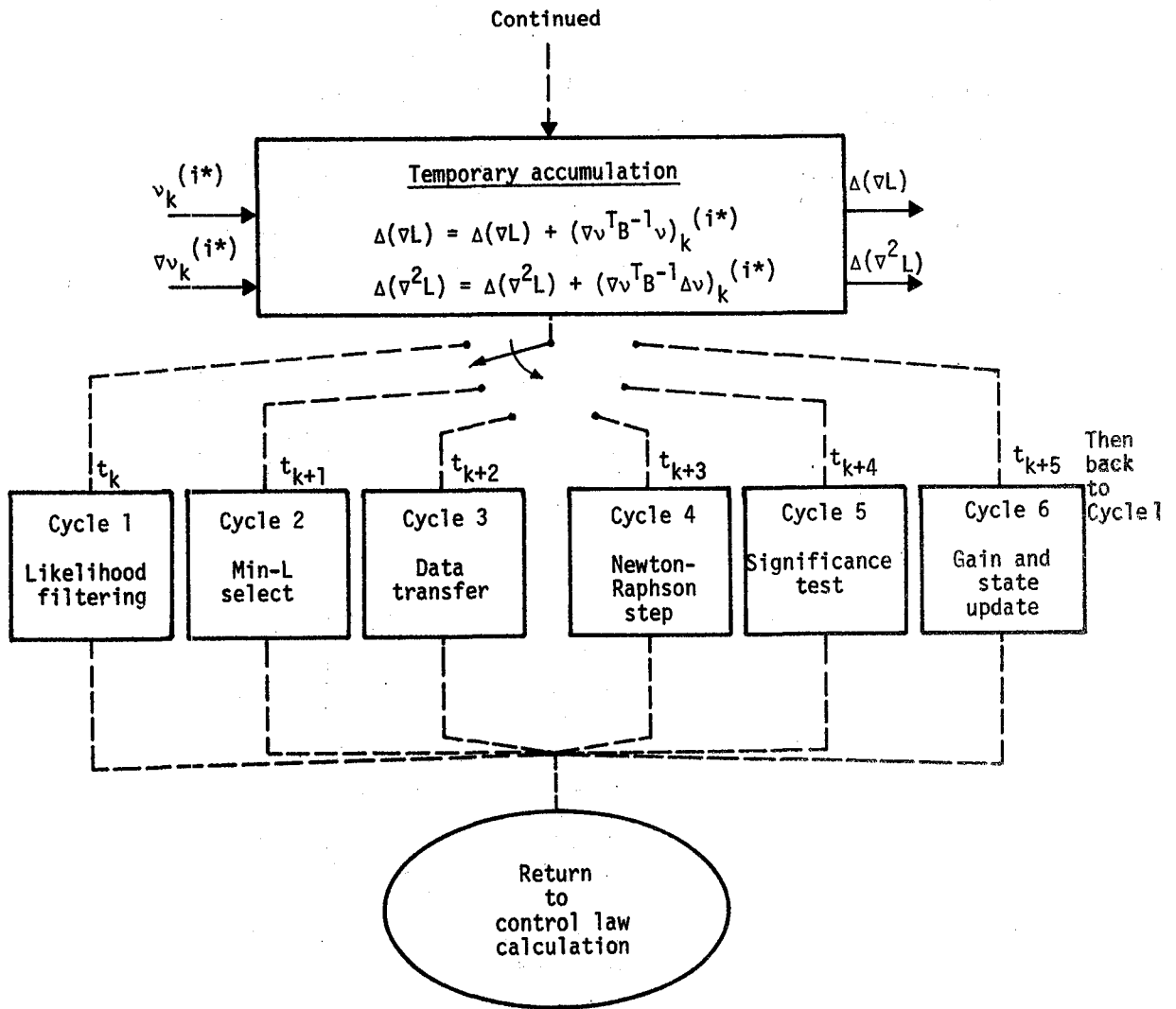
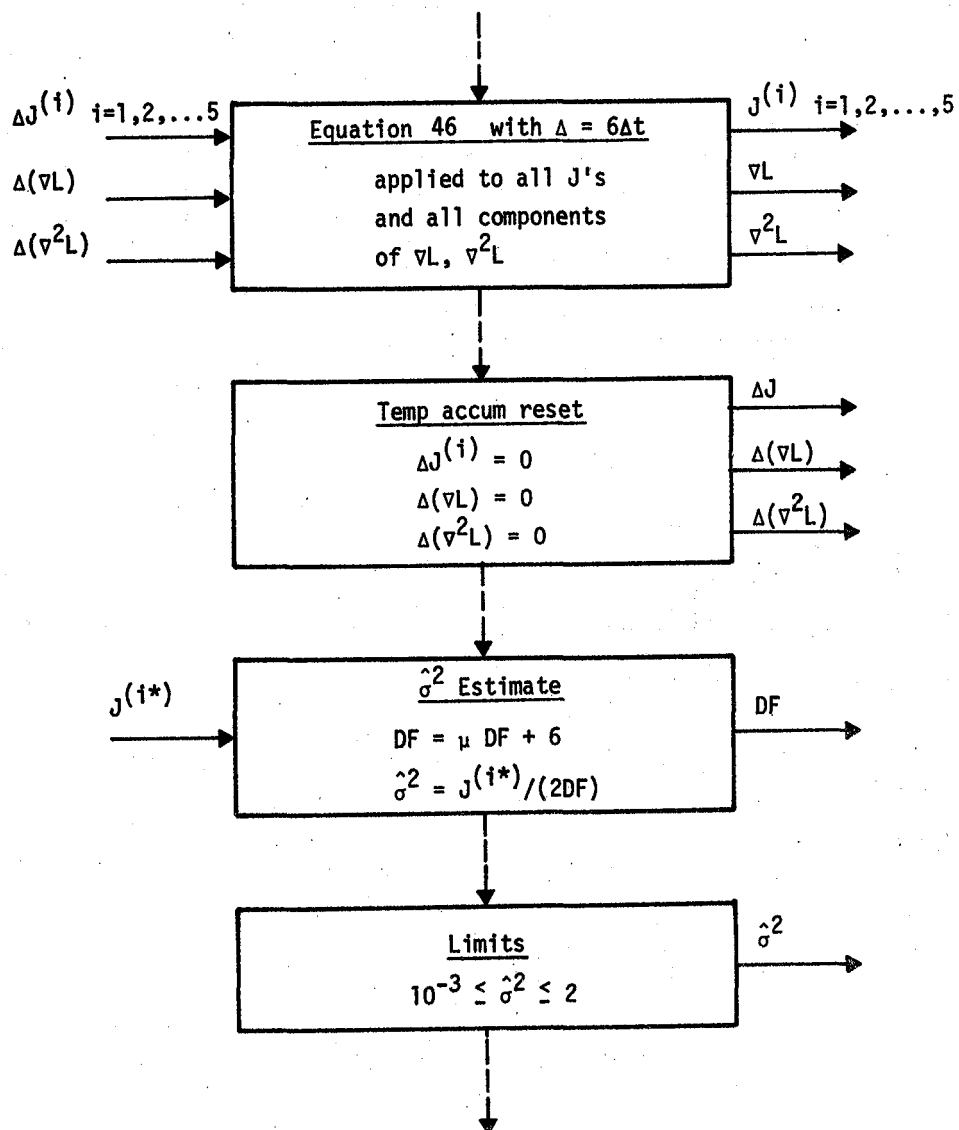


Figure 20. MLE identifier flow diagram.



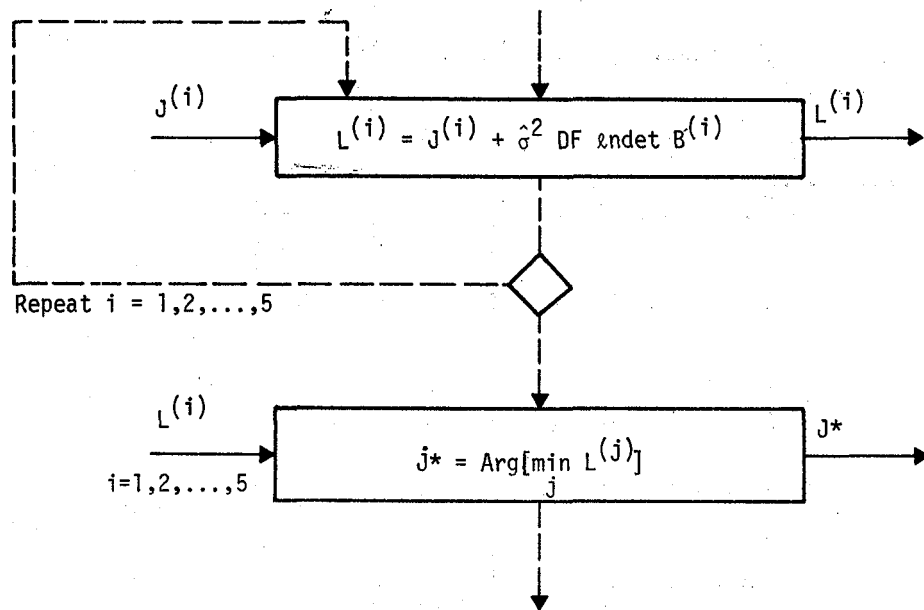
MLE Identifier Flow Diagram

Figure 20. Continued.

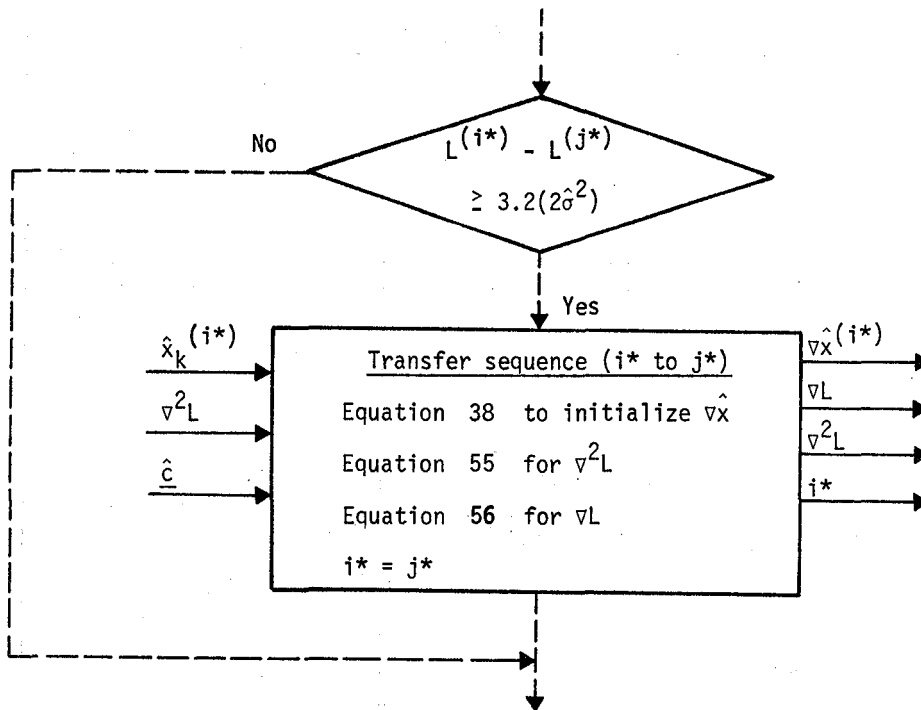


Cycle 1: Likelihood filtering

Figure 20. Continued.

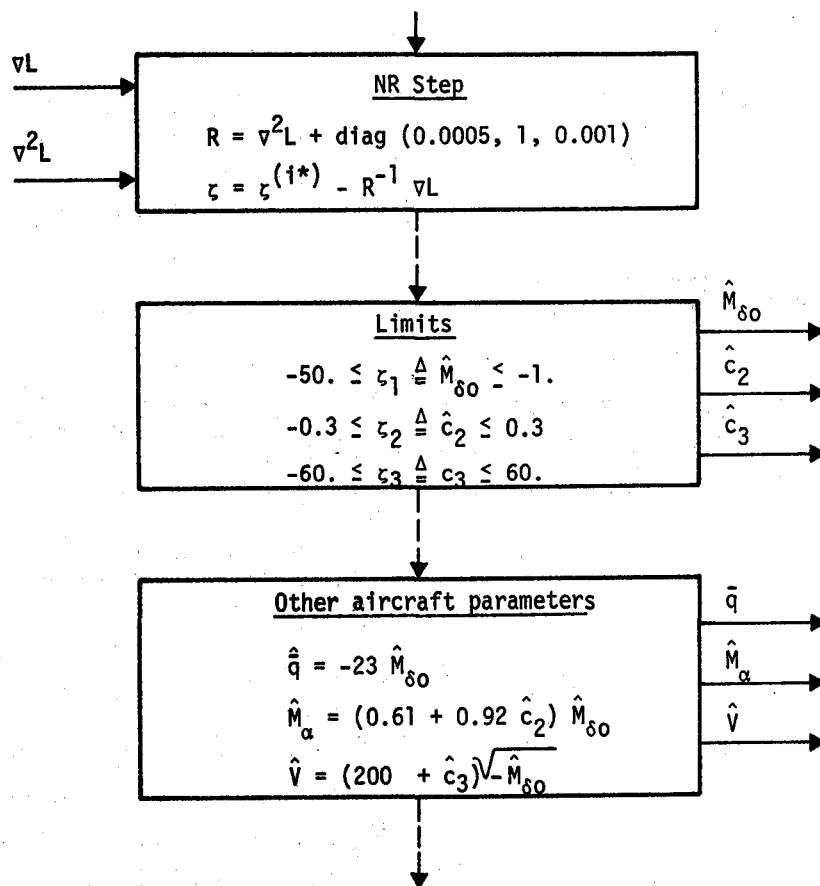


Cycle 2: Min-L select



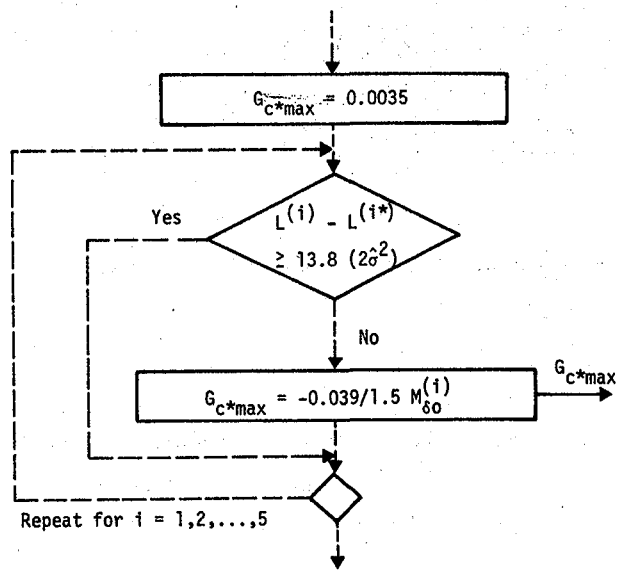
Cycle 3: Channel data transfer

Figure 20. Continued.



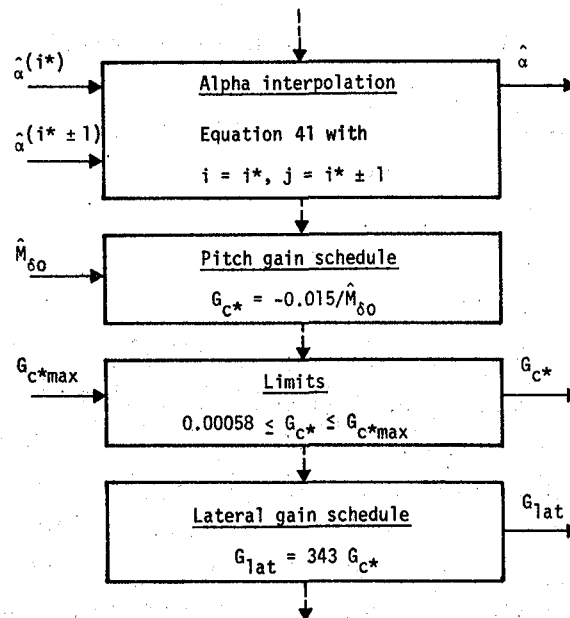
Cycle 4: Newton-Raphson correction

Figure 20. Continued.



Cycle 5: Significance test

Figure 20. Continued.



Cycle 6: Gain and state update

Figure 20. Concluded.

3. Hysteresis on channel transfer decisions to prevent chattering when two channels have nearly equal likelihood functions,
4. Addition of small diagonal elements to $\nabla^2 L$ in order to assure invertability, and
5. Limits on $\hat{c}$ to protect against gross estimation errors.

MLE CONCEPT PERFORMANCE

The complete adaptive control concept was verified with extensive nonlinear simulation trials on NASA Langley's F-8 simulator.⁵⁹ System performance was assessed with respect to four "measures of goodness:"

- Identification accuracy at fixed flight conditions,
- Convergence characteristics,
- Tracking characteristics for standard flight transitions, and
- Response to major maneuvers and configuration changes.

The controller was also "flown" under pilot control from the iron bird and under simulated relaxed-static-stability conditions.

Accuracy at Fixed Flight Conditions

Identification performance (and thus control gain accuracy) were verified by subjecting the closed-loop adaptive control system to a standard sequence of test conditions while in trimmed flight at various fixed flight conditions. The test conditions consisted of a period of quiet, followed by a 20 ft/sec² (6.1 m/sec²) C*-command doublet, followed by a period of six ft/sec (1.83 m/sec) atmospheric turbulence, followed by another command doublet. This entire sequence was run with and without sensor noise, and each run included a small random test signal inserted at the C*-command point.

Accuracy tests were run at five flight conditions: FC1, FC5, FC8, FC10, and power approach, FC17. Strip chart recordings for these flights are shown in Figures 45 through 54 in Appendix B. Each run is described by two pages of recordings. The first page shows major aircraft variables, and the second shows variables from the adaptive control law.

Performance summaries of the accuracy tests are given in Tables 15 through 18 for FC1, FC5, FC8, and FC10, respectively. These tables show estimation error percentages for the various test conditions. The errors were computed by comparing linearized derivatives ($M_{\delta o}$, M_α) or actual quantities from the simulation ($\bar{q}$, V , α) with estimated values. The estimates $\hat{M}_{\delta o}$, $\hat{M}_\alpha$, $\hat{V}$, and $\hat{\alpha}$ were taken directly from the identifier, and $\hat{\alpha}_{wL}$ was synthesized as follows:

$$\hat{\alpha}_{wL} = \hat{\alpha} + 0.5$$

The summary tables show basic accuracies of 10 to 20 percent, which are very consistent with the predictions in Section 6. Poorest performance occurs at max- $\bar{q}$ "on-the-deck" (FC10), where $M_{\delta o}$ errors and $\bar{q}$ errors are as high as 30 percent and velocity errors are as high as 40 percent. These errors are well within the stability margins of the control laws.

TABLE 15. MLE IDENTIFICATION ACCURACY SUMMARY AT
FIXED FLIGHT CONDITIONS

FC1

Conditions	Parameter error (percent)				
	$\bar{q}$	$M_{\delta o}$	M_α	V	α_{wL}
Quiet	2	0	10	4	1
Turbulence only	8	7	2	8	
Sensor noise only	5	4	10	15	
Turbulence plus sensor noise	10	7	2	1	
Pilot doublets	2	4	2	6	
Resolution of percent error calculation	1	4	8	3	5

TABLE 16. MLE IDENTIFICATION ACCURACY SUMMARY AT
FIXED FLIGHT CONDITIONS

FC5

Conditions	Parameter error (percent)				
	$\bar{q}$	$M_{\delta o}$	M_{α}	V	α_{wL}
Quiet	2	5	8	8	8
Turbulence only	5	5	8	2	
Sensor noise only	10	5	7	17	
Turbulence plus sensor noise	10	5	7	3	
Pilot doublets	4	5	7	8	
Resolution of percent error calculation	1	11	15	5	5

TABLE 17. MLE IDENTIFICATION ACCURACY SUMMARY AT
FIXED FLIGHT CONDITIONS

FC8

Conditions	Parameter error (percent)				
	$\bar{q}$	$M_{\delta o}$	M_{α}	V	α_{wL}
Quiet	16	0	2	1	4
Turbulence only	22	6	6	1	
Sensor noise only	15	0	2	4	
Turbulence plus sensor noise	30	12	2	4	
Pilot doublets	15	0	2	4	
Resolution of percent error calculation	1	3	2	2	16

TABLE 18. MLE IDENTIFICATION ACCURACY SUMMARY AT
FIXED FLIGHT CONDITIONS

FC10

Conditions	Parameter error (percent)				
	$\bar{q}$	$M_{\delta o}$	M_{α}	V	α_{wL}
Quiet	2	2	17	40	3
Turbulence only	20	23	9	23	
Sensor noise only	30	29	5	29	
Turbulence plus sensor noise	15	23	9	23	
Pilot doublets	4	6	9	29	
Resolution of percent error calculation	1	2	4	3	9

Convergence Characteristics

To verify convergence characteristics, the initial L , $\sqrt{L}$, and $\sqrt[2]{L}$ functions of the identifier were set to zero and the initial min- L index, i^* , was set to a channel not covering the true flight condition. This was done for several combinations of channels and flight conditions and for various inputs. Strip chart recordings for these tests are given in Figures 55 and 56 in Appendix B, and the performance results are summarized in Table 19. Convergence times at or below one second were realized in all cases. It is particularly interesting to observe convergence characteristics obtained for C*-doublets without test signals (Figure 56, Runs 104 and 77). For these cases, the identifier drifts until the first command occurs and then responds almost immediately with proper estimates (and control gains). Comparing the initial transient with subsequent ones shows no major differences. This rapid response to commands may make it possible to operate the control loop without any test excitation. More experimentation will be required, however, to substantiate this potential.

TABLE 19. MLE CONVERGENCE SUMMARY

Run	Condition	Convergence		
		From channel	To FC	Time to 80% (sec)
71	C*-doublets	4	5	≤ 1
68	99% turbulence	4	5	≤ 1
103	C*-doublets	4	1	≤ 1
104	C*-doublets no test signal	4	1	^a < 1
77	C*-doublets no test signal	2	1	^a < 1
74	99% turbulence	2	1	< 1

^a Measured from occurrence of first command.

Tracking

To verify tracking capabilities, the aircraft was flown through two standard flight transitions:

1. Max accelerations with full afterburner starting in trimmed flight at FC5, and
2. Half-power deceleration starting in trimmed flight at FC8.

Both transitions were flown with the CAS engaged (with zero C*-command). For the first transition, this produces rapid acceleration and a gentle climb to about 27,000 to 28,000 feet (8.5K m) altitude and barely supersonic speed. The second transition produces slow decelerations to subsonic speeds with little change in altitude.

Each transition was flown in quiet air, in 99 percent turbulence, and with 99 percent turbulence and sensor noise. All cases included a small C*-test signal. Strip charts for these cases are again given in Appendix B. Figures 57 through 59 cover the first

transition, and Figures 60 through 62 cover the second. Performance is summarized in Table 20 in terms of $\bar{q}$ tracking errors only (these were the easiest to calculate since $\bar{q}$ is available continuously from the simulation).

TABLE 20. $\bar{q}$ TRACKING ERROR SUMMARY

Condition	Transition 1	Transition 2
Quiet	- 31% peak	+ 27% peak
99% turbulence	- 28% peak	+ 38% peak
99% turbulence plus sensor noise	- 28%, +32% peaks	+ 38% peak (+50% with speed brake applied)

Maneuvers and Configuration Changes

The adaptive controller was also tested under various maneuver conditions and configuration changes. These runs are primarily intended to explore qualitative properties of the concept in large signal situations and to assure that no drastic upsets occur in normal operation. The runs are summarized below with figures given in Appendix B.

Figure 62. Speed brake extended

Figure 63. Roll maneuvers at FC1, FC5, FC8, FC10

Figure 64. 360 deg. rolls at FC1

Figure 65. Step α gusts

Figure 66. Gear-down transient at FC17

Figure 67. Gear-up transient at FC17

Figure 68. Wing-up transient at Mach 0.23, 3K

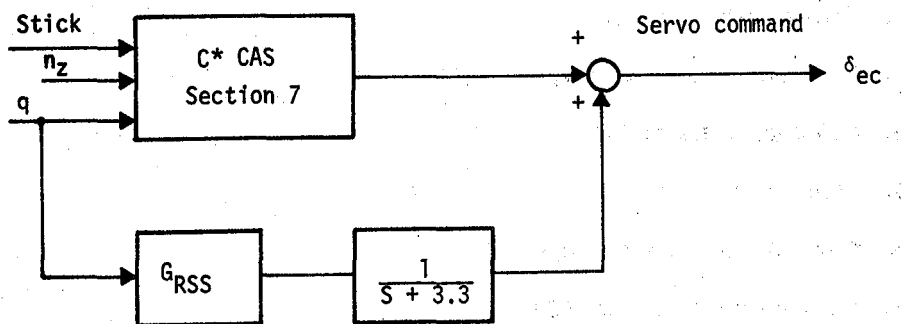
No surprises were found in these tests. The identifier performs properly for standard roll maneuvers and alpha gusts. It develops tolerable 30 percent errors (in $\bar{q}$) during 360 degree rolls, 10 to 20 percent errors during gear transitions, and larger 50 to 70 percent errors during wing transitions. None of these errors have major consequences on the controllability of the aircraft. In fact, when the wing transition was flown under

pilot control from the iron bird, the pilot judged control characteristics to be comparable to the CCV laws. This same judgement was also rendered for the adaptive controller's performance over the rest of the flight envelope.

Relaxed-Static-Stability Flights

The adaptive system was also flown with simulated aft motions of the c.g. For this purpose, the pitch control law was modified to include a low-passed pitch rate feedback path developed specifically for RSS flight in the CCV program.² This extra feedback is illustrated in Figure 21. It includes one additional adjustable gain, G_{RSS} , scheduled as a function of $\hat{M}_{\delta 0}$ and $\hat{M}_{\alpha}$. The identifier was changed only in the nominal model parameterization, Equations (36), where the M_{α} equation was replaced by $M_{\alpha} = (0.3 + 0.92 c_2) \cdot M_{\delta 0}$ with larger a priori limits on $c_2 (\pm 0.6)$.

Flights were made at FC1 with zero and -2.0 ft c.g. displacement. The latter is approximately neutrally stable. Transient traces are given Appendix B, Figure 69. The identifier is seen to have no difficulty with the large M_{α} change induced by the c.g. displacement.



$$G_{RSS} = 0.5 [1 - \hat{M}_{\alpha} / (0.61 \hat{M}_{\delta 0})]$$

Figure 21. Relaxed-static-stability control law.

MLE PERFORMANCE ISSUES

As shown by the nonlinear simulation tests above, the MLE adaptive control concept performs well throughout the flight envelope. Its performance is not ideal, however, and it is important to highlight basic limitations. These include:

- Tracking errors,
- Effects of fixed statistics in the Kalman filters, and
- Test signal requirements.

The tracking limitation has already been discussed in the likelihood filter design section. We need a likelihood accumulation time of three to five seconds in order to assure correct minima of L , yet this accumulation produces lags when the parameters vary due to flight transitions or configuration changes. These lags are responsible for the largest errors seen in the performance trials. They can be relieved somewhat by additional cues to the identifier. For example, we could tell the identifier that the wing is going up or that velocity is increasing (forward acceleration) and allow it to adjust accordingly. Further research is recommended along these lines.

Effects of the second limitation (fixed statistics in the Kalman filters) are evident in most of the accuracy runs. The estimates change somewhat between quiet or sensor-noise-only conditions and turbulence conditions. This is especially evident at high $\bar{q}$ (FC10), where the total change is as much as 50 percent. The problem is caused by filter gains which are designed for one condition (turbulence plus sensor noise) and then operated at another. It could be alleviated by providing additional data concerning operation conditions (i.e., turbulence estimates) to adjust the filter gains. Research is also recommended in this direction.

Limitations due to test signal requirements are less quantifiable at this point. Test signal levels are primarily determined by pilot considerations and noise levels in the actual flight control system, so they will not be fully known until we fly the airplane. For the simulations at Langley Research Center, a small random test signal was found adequate. This signal used the spectral shaping network shown in Section 6 (with $\omega = 6$, $\zeta = 1.25$) and was inserted at the C^* -command point. Its rms magnitude was adjusted to $C^* = 4.0 \text{ ft/sec}^2$ (1.22 m/sec^2), which produces the following approximate accelerations at the pilot station:

FC	Test signal acceleration	
	rms g's	m/sec ²
1	0.025	0.25
5	0.010	0.10
8	0.020	0.20
10	0.040	0.40

These signal levels can be lowered somewhat with tolerable (but noticeable) performance degradation. An example is shown in Figure 70 (Appendix B) which shows an accuracy test for FC1 with half the normal test signal. Potentially, they could be eliminated entirely if operation with rapid gain response to pilot commands proves acceptable. More experimentation is recommended to explore these possibilities.

SECTION 9

THE MODEL TRACKER DESIGN

This section presents the design of an adaptive gain adjustment procedure based on explicit parameter identification. The identified parameters are coefficients in a simple model of the linearized aircraft equations of motion. They are adjusted on-line to minimize a Liapunov error function. This section will present the theoretical framework of model trackers, discuss the design issues in a practical model tracker, and finally present the performance of a model tracker for the F-8C based on the pitching moment equation. The adaptive gain adjustment of the pitch and lateral CAS is made on the basis of an identified surface effectiveness parameter.

PROBLEM FORMULATION

The explicit model-tracking identification method is illustrated in Figure 22. The true system is represented in state variable form as

$$\dot{\mathbf{x}} = \mathbf{F}\mathbf{x} + \mathbf{G}u \quad (57)$$

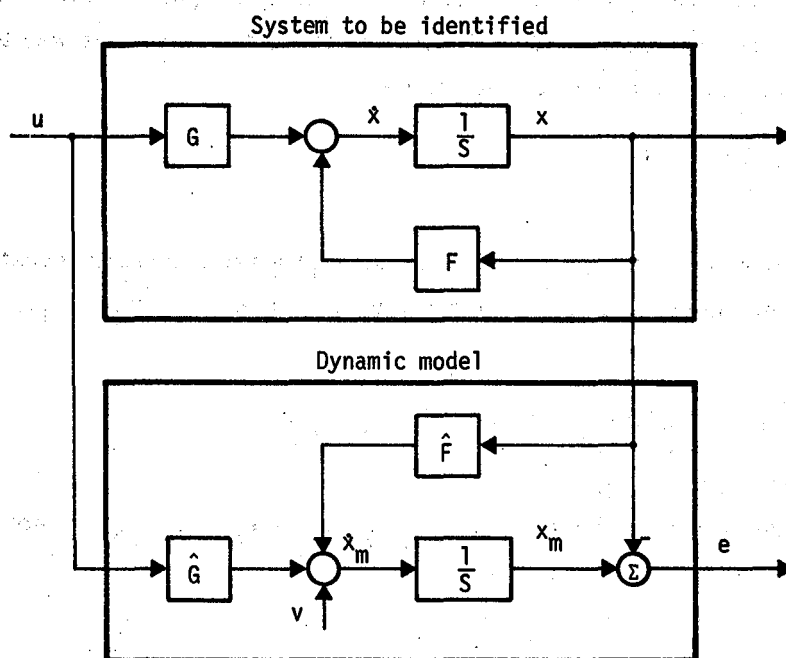


Figure 22. Model-tracking identification based on Liapunov function.

A model is chosen to have the same structure

$$\dot{\mathbf{x}}_m = \hat{\mathbf{F}}\mathbf{x} + \hat{\mathbf{G}}\mathbf{u} + \mathbf{v} \quad (58a)$$

where the elements of matrices $\hat{\mathbf{F}}$ and $\hat{\mathbf{G}}$ are adjustable. An error input to the model is denoted by $\mathbf{v}$. Identification is complete when $\hat{\mathbf{F}} = \mathbf{F}$, $\hat{\mathbf{G}} = \mathbf{G}$, and $\mathbf{v} = 0$. The procedure involves finding a way to adjust the parameters to force $\mathbf{v}$ to zero.

A measure of the identification error is the vector difference between the system and the model states, $\mathbf{e} = \mathbf{x}_m - \mathbf{x}$. A Liapunov design approach will provide expressions for $\dot{\hat{\mathbf{F}}}$, $\dot{\hat{\mathbf{G}}}$, and $\mathbf{v}$ which force convergence and guarantee stability.^{50, 68} The input $\mathbf{v}$ is chosen to be $\mathbf{v} = \mathbf{D}(\mathbf{x}_m - \mathbf{x}) = \mathbf{D}\mathbf{e}$, where $\mathbf{D}$ is a matrix of design parameters. The resulting expression for the model tracker can be written as

$$\dot{\mathbf{x}}_m = [\hat{\mathbf{F}} - \mathbf{D}]\mathbf{x} + \mathbf{D}\mathbf{x}_m + \hat{\mathbf{G}}\mathbf{u} \quad (58b)$$

The error vector ($\mathbf{e}$) defined above satisfies

$$\dot{\mathbf{e}} = \mathbf{D}\mathbf{e} + (\hat{\mathbf{F}} - \mathbf{F})\mathbf{x} + (\hat{\mathbf{G}} - \mathbf{G})\mathbf{u} \quad (59)$$

The differential equations for $\hat{\mathbf{F}}$ and $\hat{\mathbf{G}}$ are derived using Liapunov's direct method to provide asymptotic convergence of $\mathbf{e}$, $\hat{\mathbf{F}} - \mathbf{F}$, and $\hat{\mathbf{G}} - \mathbf{G}$ to zero. One way to achieve this goal is to take the Liapunov function to be

$$V = \frac{1}{2} [\mathbf{e}'\mathbf{P}\mathbf{e} + \sum_i \rho_i z_i^2] \quad (60)$$

with $\mathbf{P}$ as a positive definite symmetric matrix, $\{\rho_i\}$ a set of positive constants, and z_i components of the vector $\mathbf{z}$ of parameter differences; i.e., $\mathbf{z}' = (\hat{f}_{11} - f_{11}, \hat{f}_{12} - f_{12}, \dots)$. Then

$$\dot{V} = \frac{1}{2} \mathbf{e}'(\mathbf{P}\mathbf{D} + \mathbf{D}'\mathbf{P})\mathbf{e} + \sum_i [\rho_i \dot{z}_i + h_i(\mathbf{p}, \mathbf{e}, \mathbf{x}, \mathbf{u})] \dot{z}_i \quad (61)$$

where h_i is a sum of terms of the form $e_k p_{kj(i)} x_{q(i)}$ or $e_k p_{kr(i)} u_{s(i)}$. Choosing $\mathbf{D}$ such that $\mathbf{P}\mathbf{D} + \mathbf{D}'\mathbf{P}$ is negative definite and

$$\dot{z}_i = -\rho_i^{-1} h_i \quad (62)$$

yields a negative semidefinite $\dot{V}$, which is zero only if $e = 0$. Then sufficient variation in u and x will yield convergence of the vectors e and z to zero.

If P is a diagonal matrix, the h_i simplify, and then the parameter adjustment given in (62) becomes a function of only one error and either a state or an input. Therefore, the individual elements of $\hat{F}$ and $\hat{G}$ should be adjusted using

$$\dot{\hat{f}}_{ij} = \lambda_{ij} e_i x_j \quad (63)$$

$$\dot{\hat{g}}_{ij} = \omega_{ij} e_i u_j \quad (64)$$

where the "gains" λ_{ij} and ω_{ij} must be positive numbers.

Note that the Liapunov function could have been defined such that the e_i 's in Equation (63) are replaced by odd functions of the e_i 's such as $\text{sgn}[e_i]$, or e_i^3 , etc. This will be considered later.

Identification proceeds provided the system is being perturbed by some arbitrary input $u(t)$ such that the parameter adjustments in Equation (63) are not identically zero.

Recall the structure of Figure 22. By substituting the expression $v = De$, the form of the explicit model tracker shown in Figure 23 results.

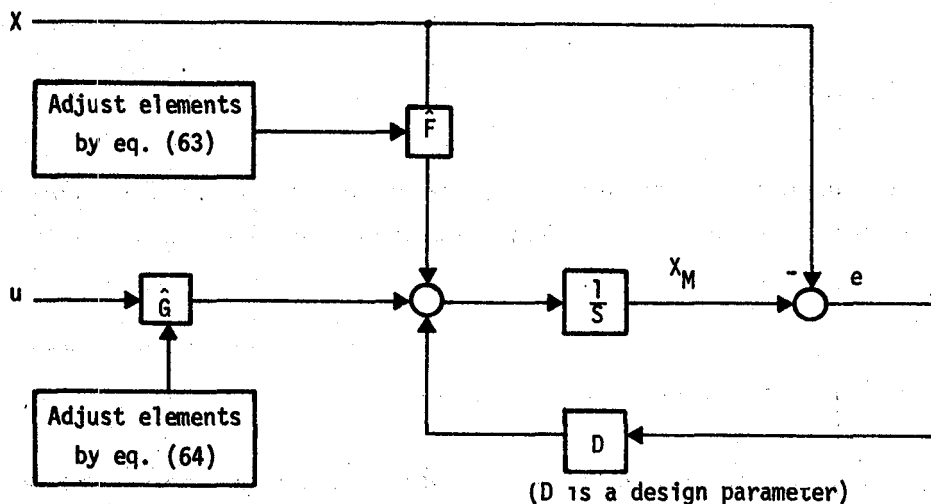


Figure 23. Model tracker structure.

DESIGN ISSUES

Implementing the tracker of Figure 23 requires specifying parameters D , λ_i , and ω_i . Also, the states x and inputs u must be measured for use in the parameter adjustment equations. This section will present the design issues related to:

- Modeling--definition of the states and inputs in terms of measurable quantities,
- Selection of D , λ , and ω parameters,
- Accommodating trim with the model tracker,
- Effects of turbulence on the model tracker, and
- Effects of measurement noise on the model tracker.

Each of the above considerations influences the final design of the model tracker. Finally, the final functional block diagram and its F-8 simulator performance are presented.

Modeling

The F-8C aircraft models presented in Section 5 were simplified for the design of the model tracker. The linear model was reduced to the short-period perturbation equations. The effects of trim values will be discussed later. These can be written as

$$\dot{q} = M_{\alpha} \alpha + M_q q + M_{\delta} \delta_e \quad (65)$$

$$\dot{\alpha} = q + Z_{\alpha} \alpha + \frac{V}{L} \alpha + Z_{\delta} \delta_e \quad (66)$$

Pitch rate, q , and normal acceleration, N_z , are the measured variables where N_z is positive down and measured "d" m forward of the c.g. and is given by

$$-N_z = Z_{\alpha} V \alpha + Z_{\delta} V \delta - d \dot{q} \quad (67)$$

(Here the notation follows the definitions of Section 6.)

To have the states be measured quantities, q and N_z are used as state variables. By differentiating Equation (67) and using Equations (65) and (66), the short period can be written as

$$\begin{bmatrix} \dot{q} \\ N_z \end{bmatrix} = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix} \begin{bmatrix} q \\ N_z \end{bmatrix} + \begin{bmatrix} g_{11} & 0 \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} \delta_e \\ \dot{\delta}_e \end{bmatrix} + \begin{bmatrix} 0 \\ c_1 \end{bmatrix} \alpha_g \quad (68)$$

where the elements of F and G are obtained using a linear transformation as in Reference 2. Values of f_{11} , f_{12} and g_{11} for the pitching moment equation are given in Table 21 for selected flight conditions spanning the F-8C flight envelope.

TABLE 21. PITCHING MOMENT PARAMETERS

FC	h (ft) (Km)	Mach	(psf) $\bar{q}$	(n/m ²)	f_{11}	f_{12}	g_{11}
1	6.1 K	0.67	305	14,603	-0.7105	0.3218	-11.400
5	6.1 K	0.40	109	5,219	-0.4446	0.4817	-5.024
6	6.1 K	0.90	551	26,382	-0.9972	0.3017	-21.980
7	12.2 K	0.70	134	6,416	-0.3591	0.3157	-6.790
8	12.2 K	1.20	395	18,913	-0.5466	1.6020	-19.070
9	3.0 K	0.80	652	31,218	-1.1260	0.2896	-23.680
10	0	0.70	725	34,713	-1.2650	0.3200	-25.230
11	0	0.30	133	6,368	-0.6204	0.3566	-6.032
12	0	0.53	416	19,918	-1.0060	0.3405	-15.860
16	12.2	0.90	222	10,629	-0.5704	0.5802	-11.040
17	P.A.	0.18	53	2,538	-0.3822	0.1668	-0.879

The components of the input vector u (δ_e and $\dot{\delta}_e$) can be determined by including the differential equation describing the primary actuator from Section 5.

$$\dot{\delta}_e = -12.5 \delta_e + 12.5 u_e \quad (69)$$

The secondary actuator dynamics are assumed to be unity. Then u_e is the command to the secondary actuator.

Parameter Adjustment Equation

The parameters D , λ , and ω in the model tracker were selected with the aid of a linear simulation of Equation (68). Since we wish the error to reach zero quickly, D was set equal to

$$D = \begin{bmatrix} -5.0 & 0 \\ 0 & -5.0 \end{bmatrix} \quad (70)$$

This is a time constant of 0.2 second.

The adjustment "gains" λ_{ij} and ω_{ij} were picked to result in good parameter tracking for step and ramp parameter changes. Values of λ_{ij} and ω_{ij} optimized for a step parameter change tended to be low for following a ramp parameter change. In both situations the values were dependent on the magnitude of the test signal (u_e).

Starting values for the parameter gains were determined using small test signals. Step parameter changes and ramp parameter changes were made to gather a data base for further design work. Ramp parameter changes were generated by linearly interpolating between FC5 and FC10. Although this does not reflect the actual way the parameter changes, this approximation is sufficient for "tuning" the model tracker.

Typical parameter adjustment gains are shown in Table 22. The smaller values worked well for step parameter changes; the larger values were better for tracking parameter ramp changes. These values were initial starting points. As each design issue is considered, these values are modified.

Some difficulty was experienced in achieving "good" (monotonic) convergence of all parameters simultaneously. This difficulty occurs even in the special case of tracking two elements in the first row of the system equations assuming all other parameters are known. This special case may be written in the form

$$\dot{x}_1 = f_{11}x_1 + f_{12}x_2 + v(x_3, \dots, x_n, u) \quad (71)$$

$$\dot{y}_1 = \hat{f}_{11}x_1 + \hat{f}_{12}x_2 + v(y_3, \dots, y_n, u) + d(y_1 - x_1) \quad (72)$$

$$\dot{x}_j = \sum f_{jk}x_k + \sum g_{jk}u_k, \quad j > 1 \quad (73)$$

where x is the true state and y is the model state.

TABLE 22. TYPICAL PARAMETER ADJUSTMENT GAINS^a

Parameter	Gain	Range
f_{11}	λ_{11}	$0.1(10)^6 - 0.1(10)^9$
f_{12}	λ_{12}	$0.1(10)^6 - 0.1(10)^8$
f_{21}	λ_{21}	$0.1(10)^4 - 0.5(10)^9$
f_{22}	λ_{22}	$0.1(10)^4 - 0.1(10)^7$
g_{11}	ω_{11}	$0.5(10)^8 - 0.1(10)^{11}$
g_{22}	ω_{22}	$0.1(10)^6 - 0.5(10)^7$

^a Sensors at c.g. for these runs.

$$\dot{y}_j = \sum f_{jk} x_k + \sum g_{jk} u_k, \quad j > 1 \quad (74)$$

$$e = y_1 - x_1, \quad z_1 = \hat{f}_{11} - f_{11}, \quad z_2 = \hat{f}_{12} - f_{12} \quad (75)$$

The tracker equations take the form

$$\dot{e} = De + x_1 z_1 + x_2 z_2, \quad d < 0 \quad (76)$$

$$\dot{z}_1 = -\rho_1^{-1} x_1 e, \quad \rho_1 > 0 \quad (77)$$

$$\dot{z}_2 = -\rho_2^{-1} x_2 e, \quad \rho_2 > 0 \quad (78)$$

This set of equations may be viewed as the following linear system with time varying coefficients:

$$\begin{bmatrix} \dot{e} \\ \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} \Phi \end{bmatrix} \begin{bmatrix} e \\ z_1 \\ z_2 \end{bmatrix} \quad (79)$$

with

$$\Phi = \begin{bmatrix} D & x_1 & x_2 \\ -\rho_1^{-1} x_1 & 0 & 0 \\ -\rho_2^{-1} x_2 & 0 & 0 \end{bmatrix} \quad (80)$$

The matrix A has two eigenvalues with negative real parts and a zero eigenvalue for each value of x_1 and x_2 . Thus, it is not surprising that for specific x_1 and x_2 time histories, the convergence of z_1 and z_2 is initial condition dependent. In some cases one parameter will converge essentially monotonically as the other exhibits initial divergence. To alleviate this difficulty, the following nonlinear tracking equations were studied:

$$\begin{bmatrix} \dot{e} \\ \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \Phi \begin{bmatrix} e \\ z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \mu_1(\hat{f}_{11} - f_{11}^0) \\ \mu_2(\hat{f}_{12} - f_{12}^0) \end{bmatrix} e^2 \quad (81)$$

where f_{11}^0 and f_{12}^0 denote nominal (constant) values of f_{11} and f_{12} , respectively. The convergence of this system is assured via the Liapunov function

$$V = e^2 + \rho_1 z_1^2 + \rho_2 z_2^2 \quad (81)$$

if $\rho_1 > 0$, $\rho_2 > 0$, $\mu_1 \leq 0$, $\mu_2 \leq 0$ and $D < [\rho_1 \mu_1 (f_{11} - f_{11}^0)^2 + \rho_2 \mu_2 (f_{12} - f_{12}^0)^2]/4$.

A priori bounds may be used for $(f_{11} - f_{11}^0)^2$ and $(f_{12} - f_{12}^0)^2$ in the constraining inequality on D.

After some experimentation, it appeared that only limited improvement was noted, and even this required f_{ij}^0 close to f_{ij} . As a result, no changes were made to the parameter adjustment equations.

Trim Considerations

As explained earlier, only perturbation equations have been considered. The trim values of the accelerometer and elevator surface will now be included. An additional parameter (M_0) representing the trim pitching moment was added to the model where N_z and δ_e now have non-zero trim values. The adjustment equation used for estimating M_0 was

$$\dot{\hat{M}}_0 = -\lambda e \quad (82)$$

Simulation runs were made to pick a λ such that $\hat{M}_0$ would account for changing flight conditions. Early results show that tracking M_0 degraded the tracking of the remaining parameters. The M_0 parameter was dropped, and high-pass filters were added to remove the trim values of N_z and δ_e to permit tracking the perturbation equations.

Early results with first-order high-pass filters were favorable. However, in tracking ramp parameter changes, large biases appeared as a result of the high pass "differentiating" the slowly varying elevator trim. This was alleviated by using second-order high passes. There is a trade-off in selecting the high-pass time constants. The time constants were chosen to compromise rapid trim changes (such as roll maneuvers) with good sensitivity. A one-second time constant is currently used. Finally, the same filter is applied to the pitch rate gyro even though its trim value is zero. This is done to maintain the proper phase relationship between q , N_z , and δ_e .

Effects of Turbulence

The effects of α_g (refer to Equation (68)) driving the true plant (but not measurable by the model) were determined using a linear simulation of the short-period dynamics. The angle-of-attack gusts were modeled using a first-order approximation to the Dryden spectrum as explained in Section 5.

A simple C* feedback controller (with fixed gains) defined $u_e(t)$, and no additional pilot inputs or extra test signals were used.

Simulation results indicated that identification of the first row of the model (f_{11} , f_{12} , and g_{11} in Equation (68)) proceeded satisfactorily, but the second row did not appear to converge or converged to wrong values.

This is not unexpected since the model does not contain a gust input. If angle-of-attack were measured, then this problem would be alleviated.²⁰ However, since we do not wish to use air-data measurements (such as α), it does not appear that good tracking during gust disturbances can be obtained for the coefficients of the normal force equation (the second row). An alternative would be to try to estimate the wind gust in the model. However, the identifiability results indicate that this cannot be estimated with sufficient accuracy. A possibility for tracking second-row elements would be to use thresholds to eliminate gust effects and to attempt to track only during large maneuvers. This was considered too restrictive for further consideration.

Measurement Noise Effects

Sensor noise was added to the q , N_z , and δ_e measurements, and errors in tracking the first-row parameters (f_{11} , f_{12} , and g_{11}) were studied. The sensor noise was defined in Section 5. Initially, sensor noise seriously degraded parameter estimation as usually results in Liapunov designs.⁶⁹ This is not surprising since the above errors are of the same order of magnitude as the vehicle motion induced by gust or small test signals.

Several modifications to the tracker were made to reduce errors caused by sensor noise. This included filtering the measurements and adding thresholds and limits to the error function.

First-order low-pass filters were added to remove as much of the high frequency energy as possible without degrading performance in the noise-free case. The time constant of the low pass was set equal to the time constant of the elevator primary actuator.

Next, threshold and amplitude limits were added to the error quantity to reduce the amount of parameter adjustment resulting from sensor noise. The nonlinear element shown in Figure 24 was added to the error equation. The parameters T_1 and T_2 were determined from simulation results. T_1 is the error threshold and was set to 0.5×10^{-3} rad/sec. Mismatch in pitch rate less than T_1 will not cause any parameter adjustment. The error is limited to $T_2 = 0.0055$ rad/sec to prevent large commands and disturbances from producing large parameter adjustments.

The parameter gains that work well were

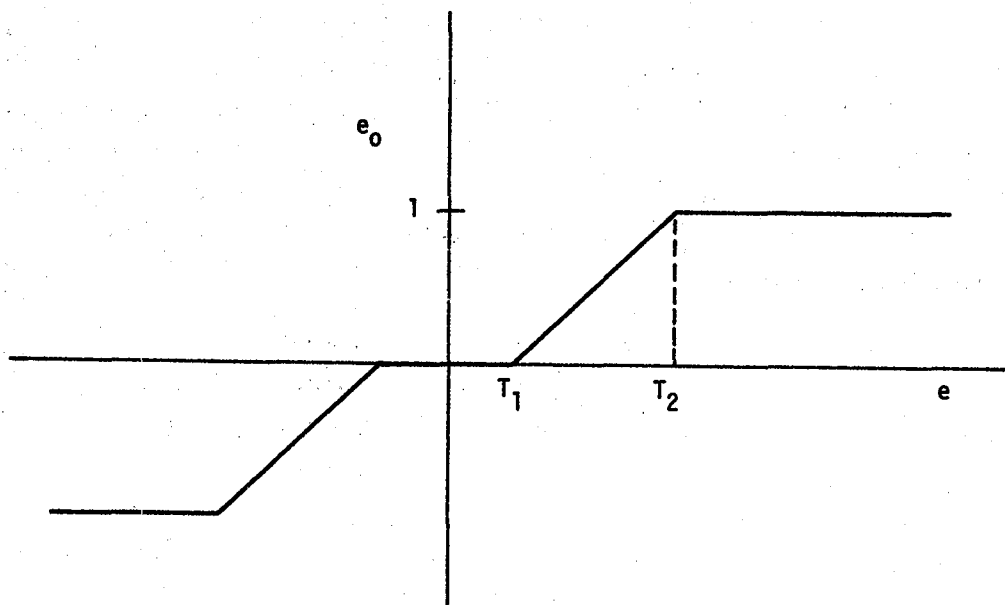


Figure 24. Nonlinear error characteristics.

$$\lambda_1 = 100$$

$$\lambda_2 = 5.$$

$$\omega_1 = 2. \times 10^5$$

(83)

Further reduction in sensitivity to error magnitudes (especially for thunderstorms and large commands) was achieved by adding a second threshold element to alter λ and ω . The parameter adjustment gains are reduced by a factor of 20 when the bandpassed elevator servo signal exceeds 0.002 radians.

Finally, rate limits were added to the three parameter adjustment equations to further equalize performance with small and large signal levels. They were chosen to provide adequate tracking rates for realistic maneuvers, but not to permit excessive parameter adjustment activity. The rate limits are

$$|\dot{f}_{11}| \leq 4.0$$

$$|\dot{f}_{12}| \leq 0.1$$

$$|\dot{g}_{11}| \leq 0.1$$

(84)

GAIN ADJUSTMENT

The G_{C*} gain used in the pitch CAS is determined from $\hat{g}_{11}$. A low-pass filter was added to smooth the estimate, and limits were added to constrain the gain values during large transients in $\hat{g}_{11}$. Figure 25 shows the variation of $\hat{g}_{11}$ with flight condition plotted as a function of dynamic pressure. When the corrections for flexibility are included, an approximate fit is

$$-\hat{g}_{11} = 0.0378 \bar{q} + 1.2 \quad (85)$$

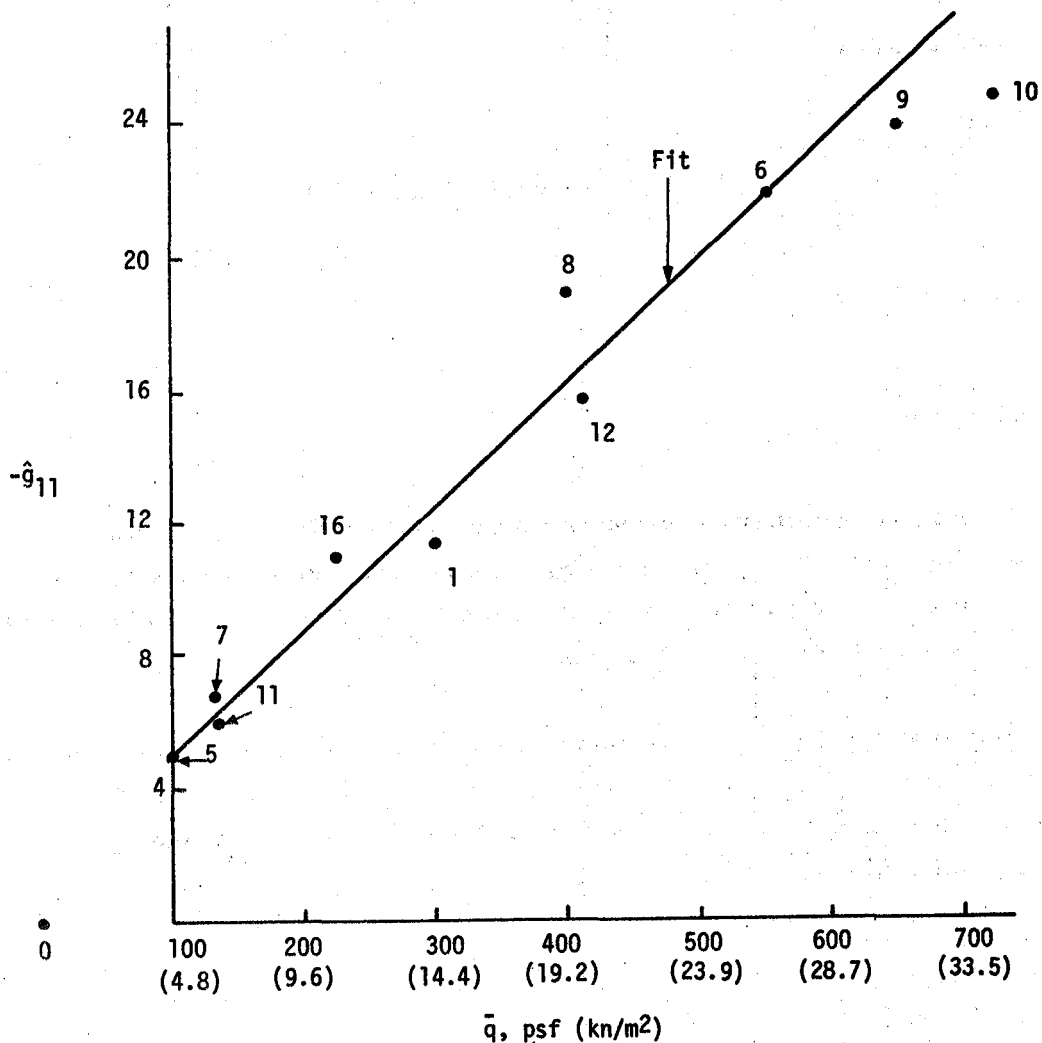


Figure 25. Parameter $\hat{g}_{11}$ versus dynamic pressure.

Using the estimate of $\hat{g}_{11}$, dynamic pressure was estimated from

$$\hat{q} = 26.455 (-\hat{g}_{11}) - 31.75 \quad (86)$$

The error in dynamic pressure

$$\frac{\bar{q} - \hat{q}}{\bar{q}} \quad (87)$$

is one of the time history plots included in the simulation data and used for accuracy evaluation only.

FUNCTIONAL BLOCK DIAGRAM

The final form of the pitch axis model tracker is shown in Figure 26. It contains a perturbation pitching moment equation with three parameters adjusted on the basis of a Liapunov error function. The filtering on q , N_z , and δ_e is shown.

The thresholds, nonlinear error functions, and rate limits previously discussed are defined. The specific values of these nonlinear elements were chosen on the basis of simulation responses. Finally, since the servo position rather than actual surface deflection is measured, a first-order linear model of the primary actuator has been incorporated.

DIGITAL MECHANIZATION

The tracker was designed using the continuous form of the equations and a Runge-Kutta integration method.

The tracker was converted to a 32 sps digital version for incorporation with the digital control laws presented in Section 6. The filters were converted to a discrete equivalent using Tustin's method. The $\hat{q}$, $\hat{f}_{11}$, $\hat{f}_{12}$, and $\hat{g}_{11}$ nonlinear equations are integrated using an Adams-Moulton formula.

$$x_{k+1} = x_k + \frac{\Delta}{6} (10\dot{x}_k - 5\dot{x}_{k-1} + \dot{x}_{k-2})$$

with $\Delta = 1/32$ second.

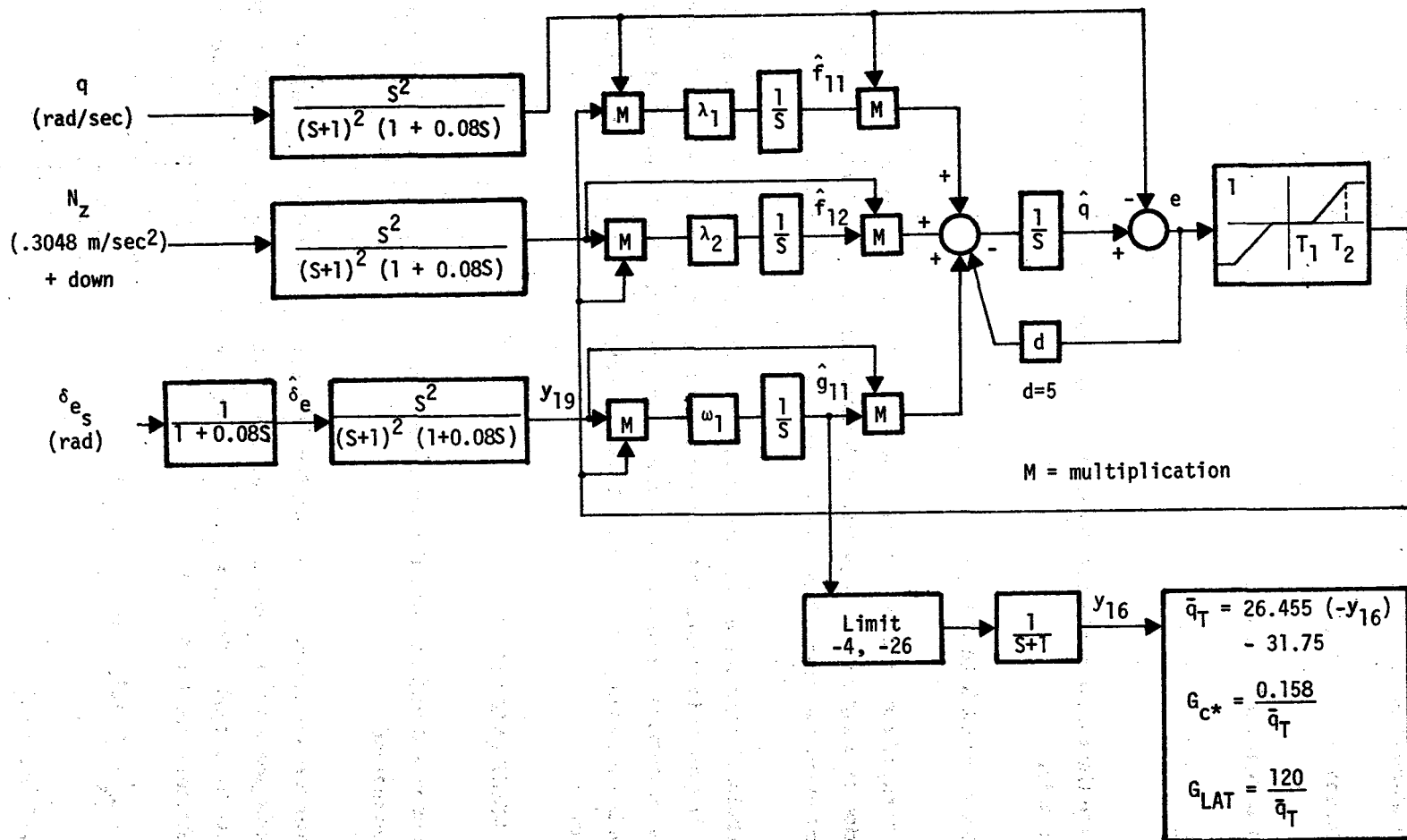


Figure 26. Pitch axis model tracker.

A flow chart of the digital model tracker is shown in Figure 27.

PITCH AXIS PERFORMANCE

The model tracker was evaluated against three "measures of goodness:"

1. Accuracy at several fixed flight conditions including effects of servo hysteresis, sensor noise, turbulence, and pilot commands,
2. Convergence properties, and
3. Tracking varying flight conditions.

The major performance measure is the accuracy of identifying the g_{11} surface effectiveness parameter. Time histories from the NASA/LRC F-8 simulator for the conditions summarized below are contained in Appendix C.

Accuracy at Fixed Flight Conditions

The results of the accuracy runs are tabulated in Table 23. A standard pitch sequence consisting of periods of quiet, gusts, and C*-doublet commands was used as system inputs.

The accuracy in identifying g_{11} varies from five to 40 percent for pilot command and turbulence. Sensor noise (only) causes errors to grow as large as 80 percent.

Convergence Properties

Convergence characteristics of parameter estimates are important in the model tracker. This test determines how well initial condition errors are removed when turbulence or pilot commands are applied. Two cases were studied. First, the algorithm was initialized at FC10 with the aircraft at FC5. Second, the algorithm was initialized at FC5 with the aircraft at FC1. Two separate inputs were applied for each case. The first input was turbulence of the Dryden form. The second input was C* pilot commands. Results are summarized in Table 24.

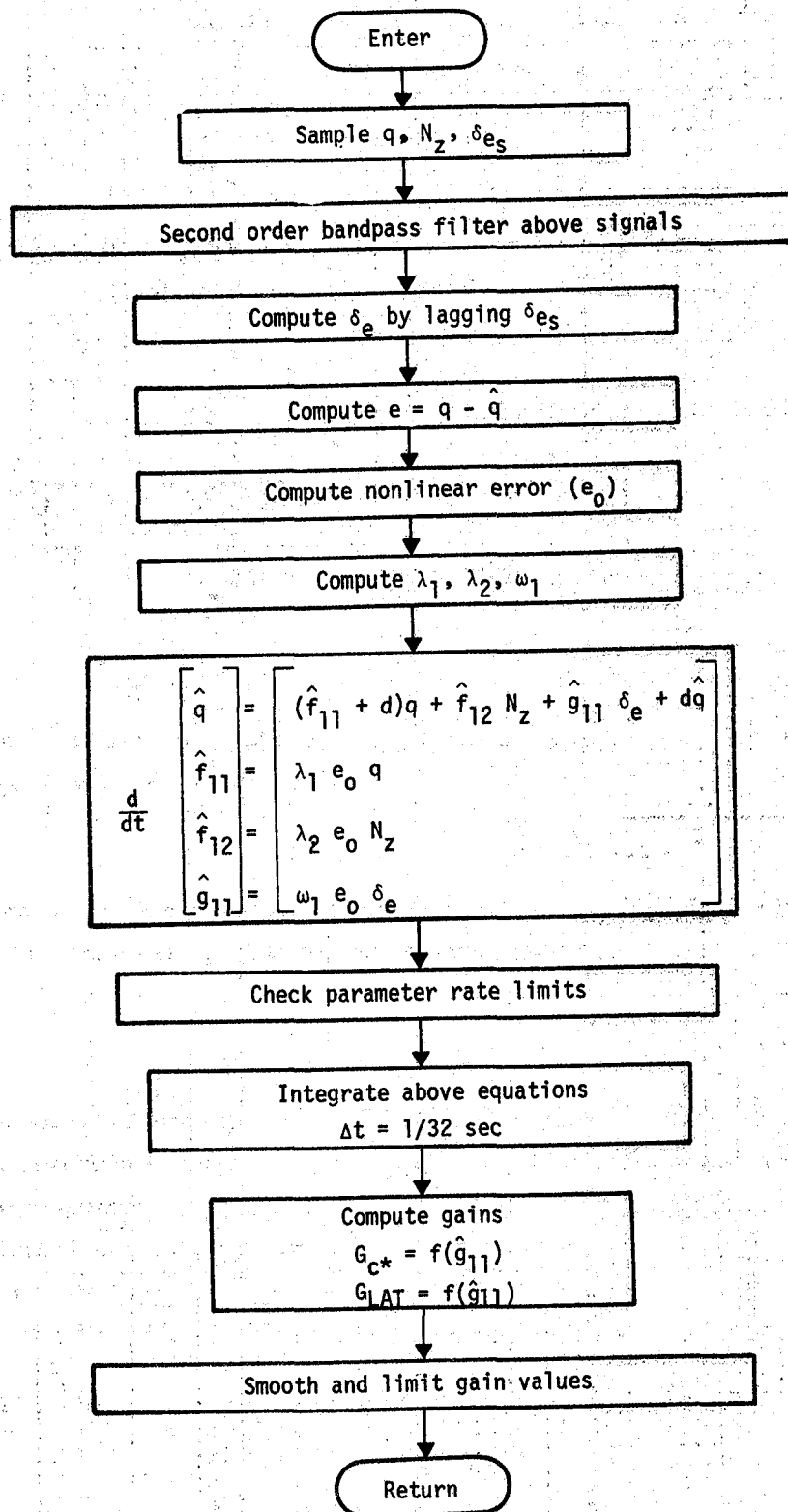


Figure 27. Model tracker flow chart.

TABLE 23. PERFORMANCE AT FIXED FLIGHT CONDITIONS

Input condition	Error in $\hat{g}_{11}$ (percent)				Comments
	Flight condition				
	1	5	8	10	
Quiet	0	0	0	0	During quiet periods, no parameter adjustment is made; so if initial conditions are correct, no errors will occur for no test signal case.
Pilot doublet	5	20	30	5	-
Turbulence	10	20	40	5	-
Doublet plus turbulence	15-20	10	10	5	-
Sensor noise	20	70-80	25	5	Sensor noise causes parameters to drift off.
Doublet plus sensor noise	5	10	5	5	Doublet provides information that reduces errors caused by sensor noise only.
Turbulence plus sensor noise	20-25	20	40	5	Sensor noise effects dominate.
Doublet plus turbulence plus sensor noise	10	10-20	10	5	-
Servo hysteresis	70	-	-	-	-

TABLE 24. CONVERGENCE RESULTS

Test condition	Gusts	Commands
Case 1 IC = FC10 Plant = FC5	$\hat{g}_{11}$ to 50% of steady state in 2 sec $\hat{g}_{11}$ to 80% of steady state in 4 sec	Same as gusts
Case 2 IC = FC5 Plant = FC1	$\hat{g}_{11}$ to 80% of steady state in 4 sec	Same as gusts

Parameter Tracking

Two profiles for changing flight conditions were run. The first started at FC5 ($\bar{q} = 5.03$ kn/m^2) and accelerated at full power holding a constant value of C^* (no pilot input). The aircraft accelerates from 61 m/sec to 152 m/sec and climbs 1524 m in about a minute. This profile was repeated for a quiet case, turbulence, and turbulence plus sensor noise.

The second profile consisted of a deceleration from FC10. Thrust is reduced to idle, and the speed brake is extended. Altitude remains nearly constant, and airspeed drops off. Results are summarized in Table 25. Because of the thresholds in the tracker, some error buildup is necessary before any parameters can change.

LATERAL AXIS PERFORMANCE

The lateral-directional CAS used the structure described in Section 7 with the loop gain adjustment slaved to the pitch axis tracker. The gain G_{LAT} is computed from estimated dynamic pressure, Equation (86). Performance of the lateral-directional CAS in terms of turn coordination and Dutch roll damping is similar to the performance of the MLE lateral-directional CAS, so time history traces will not be repeated in this section. Performance also compares with the results presented for the high-gain limit-cycle design in the next section.

TABLE 25. PARAMETER TRACKING ACCURACY

	Quiet (small test signal)	Turbulence	Turbulence plus sensor noise
Acceleration	g_{11} errors 20-25%	g_{11} errors, 20%	g_{11} errors 20-25%
Deceleration	Peak errors in $g_{11} \sim 50\%$	g_{11} errors, 20%	g_{11} errors 20%

SECTION 10

HIGH-GAIN LIMIT-CYCLE DESIGN

This section discusses the design and performance of the limit-cycle gain changer (LCGC) applied to the F-8C. The relationship of LCGC to the aircraft CAS is shown in Figure 28. The gain changer is a closed-loop controller that adjusts the loop gain of the CAS based on secondary servo activity. The objective of the gain changer is to keep the CAS operating at the highest practical gain. In the absence of gusts or measurement noise, it will maintain a small amplitude limit cycle on the secondary servo. Adjustment of the CAS loop gain by this method is, then, an implicit adaptive process. This concept is similar to a model reference implicit concept since the objective of both approaches is to have the bandwidth of the aircraft's control system sufficiently high so that it can follow commands generated by an explicit model. In the case of the F-8C, the pitch axis uses an explicit second-order C*-model, and the roll axis uses an explicit first-order model of roll rate.

The LCGC concept is more restrictive than implicit model reference approaches since the structure is limited to adjustment of a single gain parameter (i. e., "loop gain"). However, for the F-8C, a single loop gain provides sufficient adaptation.

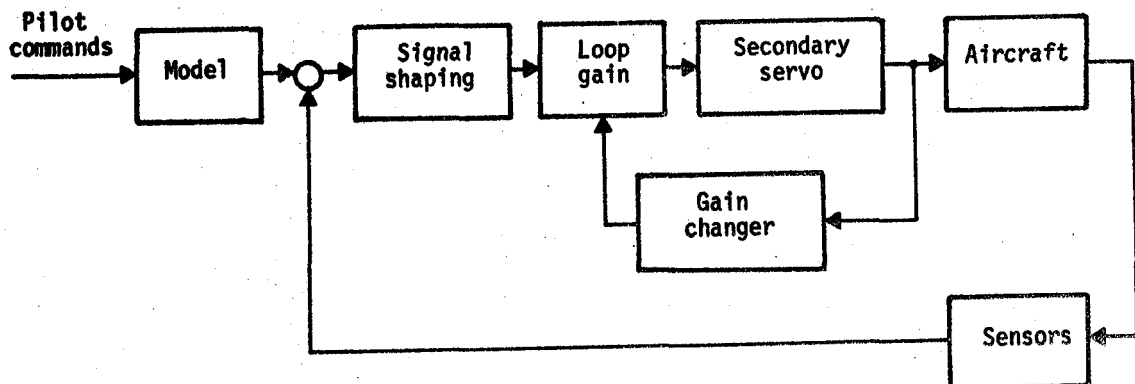


Figure 28. LCGC operation.

EVOLUTION OF THE LIMIT-CYCLE ADAPTIVE CONCEPT

The limit-cycle concept evolved from early work (1957-1958) in adaptive controls sponsored by the Wright Air Development Center (Reference 70). This work studied application of a bi-stable element (relay) to high-bandwidth feedback control loops. The results were generally unsatisfactory because acceptable limit-cycle amplitudes could not be achieved with adequate control authority. The concept of varying a linear gain to maintain the limit cycle at a fixed small amplitude was then conceived, thereby avoiding a constraint on control authority. The limit cycle was sensed via a bandpass filter, compared to a set magnitude reference, and the resulting error applied to a proportional-plus-integral function to vary the control-loop gain. The first application of this approach was an experimental system in an F-101A airplane (1958-1959), involving adaptive gain control in pitch and roll control augmentation systems (Reference 15). This was followed by the design, build, and flight test of a three-axis augmentation system for one of the X-15 aircraft (References 8 through 14) in 1964. The X-15 application used the adaptive gain control to maintain high-bandwidth loops for the aerodynamic controls and also used the resulting gain level to signal engagement of reaction controls when the normal surfaces became ineffective.

Both the F-101 and X-15 applications encountered higher frequency structural modes which reduced attainable gains below initial design levels. Notch filters were added in each case to adequately restore loop bandwidths. Some reduction in limit-cycle frequencies occurred as a result of the added filtering, necessitating some changes in parameters of the adaptive gain controls (e.g., bandpass filter frequencies).

Another problem common to the adaptive approach described above is the effect of system disturbances with significant signal content around the limit-cycle frequency (e.g., turbulence, sensor noise, high frequency pilot inputs).

The gain control can interpret these as excessive limit-cycle amplitude and drive the loop gain down. Although generally satisfactory operation can be achieved for a stable vehicle, an unstable vehicle will generate added oscillations as the gain reduces, causing complete loss of control. To correct this problem and to improve gain regulation for a stable aircraft, a low frequency signal path phased to cause a gain increase was added to the adaptive controller. This new approach was never flight tested but was extensively studied, and hardware was built for launch boosters (Reference 71) and other aircraft (Reference 16). The design for the F-8C includes this "up-logic" feature.

A significant benefit of the above adaptive developments was experience gained with high-bandwidth control systems. By accommodating much of the airframe variations through normal feedback control, avoidance of any control parameter changes is sometimes possible or requires, at most, use of simple air-data schedules. Ironically, this situation eliminates some of the original motivation for adaptive development.

GAIN CHANGER OPERATION

The basic elements of the gain changer as applied to the pitch and lateral-directional CAS are shown in Figure 29. In operation the set point will increase the loop gain until the secondary actuator begins to oscillate at the limit-cycle (or "crossover") frequency. The down-logic portion of the gain changer senses the resultant servo motion and decreases the gain until a stable limit cycle is established at the "linear critical gain." Some proportional gain is required to stabilize the gain changer loop; otherwise it will tend to oscillate about the critical gain. However, if the proportional gain is too high, excessive gain changer activity results. Hence a compromise must be reached. In a quiescent environment, damping of the loop gain response resulting in an overshoot of 100 percent in the servo limit-cycle envelope is generally acceptable. In actual operation, usual noise levels will essentially eliminate this overshoot. The limit-cycle amplitude is determined by the ratio of set point to down-logic gain. The down-logic gain is adjusted to give the desired limit-cycle servo amplitude (0.0035 rad peak-to-peak was used for the F-8C). Subsequent adjustment of limit-cycle amplitude may be made with the set point parameter.

DESIGN ISSUES

The major issues involved in the design of a limit-cycle gain changer concern:

- Selection of limit-cycle frequency,
- Selection of limit-cycle amplitude,
- Design of down-logic and up-logic bandpass filters,
- Requirements for nonlinear LCGC response,
- Definition of the set point,
- Rate limits on gain adjustment, and
- Amplitude limits on gain adjustment.

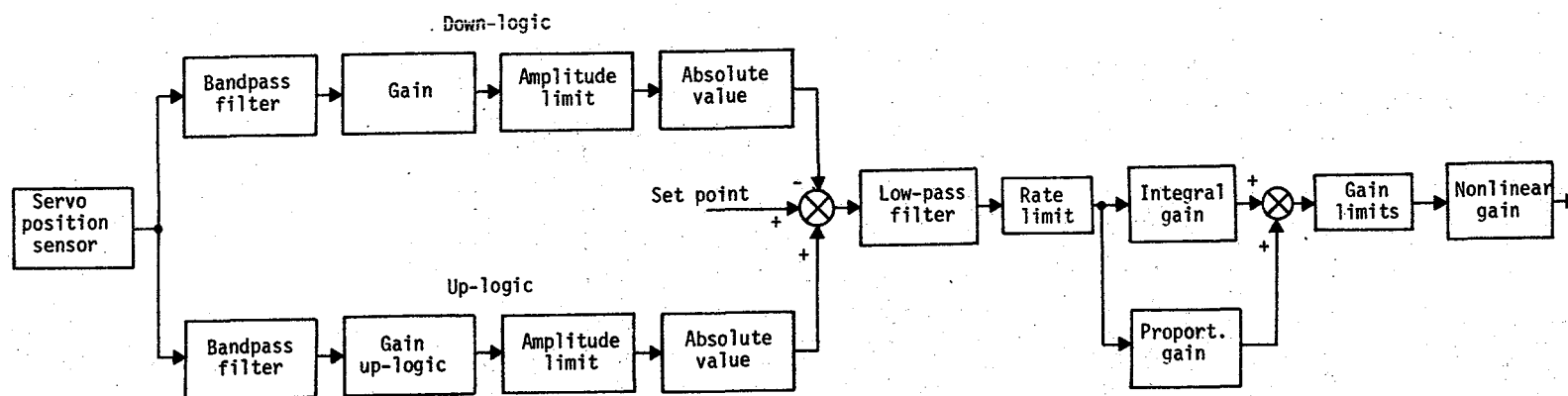


Figure 29. Basic elements of limit-cycle gain controller (LCGC).

Each of these topics will be individually discussed. Next the functional block diagram of the LCGC as evaluated on NASA/LRC's F-8C simulator will be presented. Finally a summary of the performance of the pitch axis LCGC and the roll axis LCGC will complete this section.

Selection of Limit-Cycle Frequency

The limit-cycle frequency is selected primarily to reflect the vehicle property being compensated for by the adaptive gain adjustment. Other secondary criteria include minimum interaction with normal control dynamics (a limit-cycle frequency higher than dominant frequencies), small variations due to flight condition and component tolerances, and acceptable amplitudes (avoidance of low-damped resonances).

The main object of the gain adjustment applied to high-bandwidth aircraft control loops is to compensate for varying surface effectiveness (fuselage rotational acceleration per unit control deflection). Measurement of this quality with a single frequency signal must be performed at frequencies above aerodynamic roots (e.g., short period, dutch roll, or roll subsidence), but below structural modes (flexure). Fortunately, this also generally satisfies the secondary criteria listed above. With a limit-cycle frequency so selected (generally around three to four Hz in a fighter-type airplane), significant system dynamics usually include the rigid aircraft (a pure inertia with angular rate feedback), the power actuator (a first-order lag), the secondary servo (a second-order lag), and applicable system filtering (perhaps a notch for flexure mode attenuation). These dynamics are relatively stable, resulting in a predictable limit-cycle frequency. Applied to a pitch rate loop with no added filtering, the applicable transfer function is

$$\frac{q}{\delta_e} = \frac{M_{\delta_e}}{s} \left(\frac{1}{\tau_a s + 1} \right) \left(\frac{\omega^2}{s^2 + 2\zeta\omega s + \omega^2} \right)$$

where

q = pitch angular rate,

M_{δ_e} = pitch surface effectiveness,

t = $\frac{d}{dt}$,

τ_a = actuator time constant (e.g., 0.05 sec),

$\zeta\omega$ = servo damping and frequency (e.g., 0.7 and 60 rad/sec), and

δ_e = elevator deflection.

Note for this example, a limit-cycle frequency of about 27 rad/sec would occur. The critical control-loop gain (K_c) would vary inversely with M_{δ_e} , approximated by

$$\frac{K_c M_{\delta_e}}{\tau_a (27)^2} \approx 1$$

Limit Cycle Amplitude Considerations

The amplitude of the servo limit cycle is designed to be twice the assumed value of flow hysteresis in the secondary actuator. Depending on the actual level of hysteresis or other control system nonlinearities, it may be necessary to adjust the limit-cycle amplitude with the set point parameter. The assumed amount of hysteresis will reduce the limit-cycle frequency by about 10 percent, which will change the limit-cycle amplitude. Final adjustment with actual hardware may be required.

For the F-8C, the nominal limit-cycle amplitude is set to provide a peak-to-peak secondary actuator amplitude of 0.0035 rad. This is equivalent to 0.0021 rad elevator position and 0.17 degrees aileron position with the assumed power actuator characteristics. At FC1 ($\bar{q} = 14603 \text{ n/m}^2$), this size of limit cycle produces only 0.047 n/m^2 (rms) normal acceleration.

Down-logic and Up-logic Frequency Characteristics

For the F-8C, the roll axis crossover frequency was intentionally set to be identical to the pitch axis crossover frequency at a value near 20 rad/sec (three Hz). The bandpass filters for the pitch axis are shown in Figure 30 and for the roll axis in Figure 31. There are only minor differences in the gain value.

Rapid attenuation outside the frequencies of interest is desirable to minimize the effects of wide spectrum noise or disturbances on gain changer operation. Disturbances usually result in a reduction of loop gain which may degrade the performance of the CAS. The bandpass networks were implemented as third-order linear filters, resulting in a 60 db/decade attenuation outside of the bandpass frequency.

The down-logic bandpass filter is peaked slightly above the crossover frequency in each axis. This assures limit cycling on the positive slope of the bandpass, where phase lead will contribute to stability of the limit cycle and give more positive control of the limit-cycle amplitude.

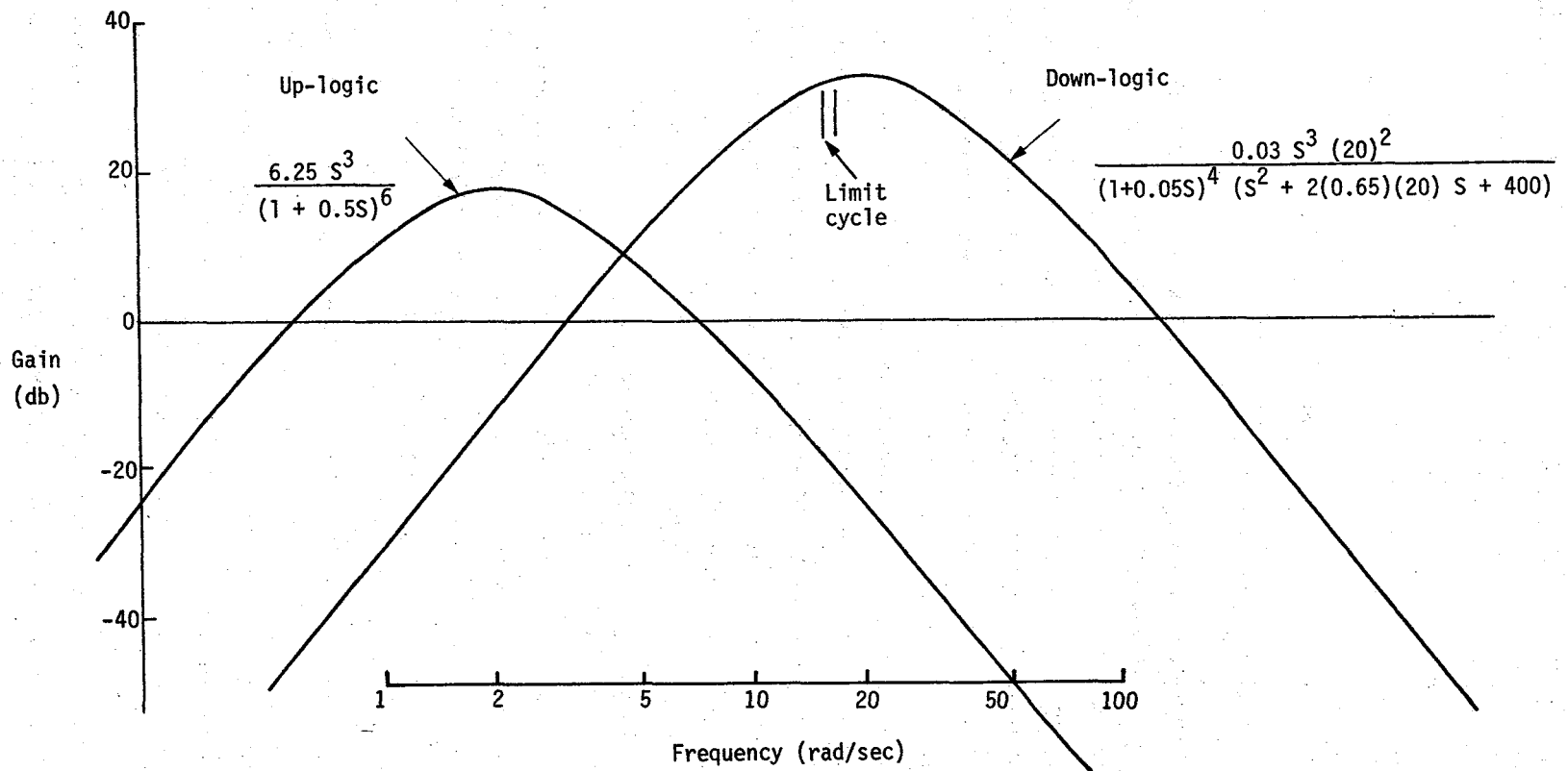


Figure 30. Pitch axis bandpass filters.

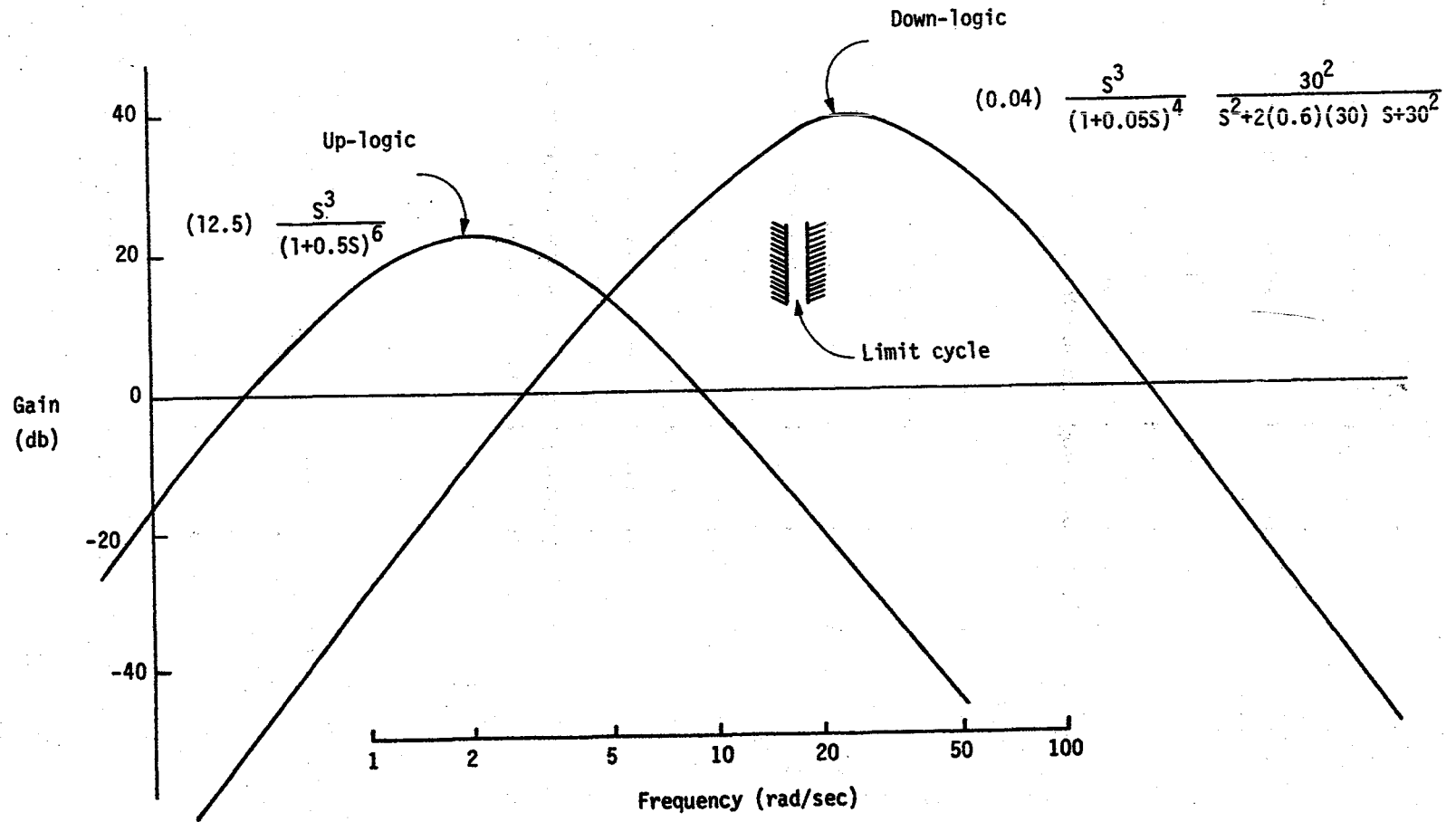


Figure 31. Roll axis bandpass filters.

The up-logic counters the tendency of the loop gain to reduce during gusts and other disturbances including pilot commands. The up-logic bandpass and gains maintain an energy balance during wide spectrum disturbances. Based on past experience, the up-logic frequency in this application was set a decade lower than the down-logic. If a low frequency instability exists (as in some previous limit-cycle applications), then this frequency would govern. Up-logic gain is set to maintain an average six-db gain margin during turbulence or pilot command inputs. Further optimization of the limit-cycle concept will require close evaluation of the up-logic mechanization, particularly during pilot inputs. Availability of spectral density information on pilot flying tasks would allow more accurate analytical definition of up-logic parameters.

Nonlinear Gain Element

The purpose of the nonlinear element is to maintain constant gain changer dynamics at all flight conditions. Near critical gain, a given percentage change in loop gain results in a proportional rate of change in limit-cycle amplitude at any flight condition. Therefore, the resulting feedback (i.e., the sensed limit-cycle amplitude) to the gain changer should produce a fixed percentage change in control gain. Since an exponential gain element has a slope proportional to the output (which is the control gain level), the above requirement for a nonlinear element is satisfied with an exponential function. For the F-8C, the exponential function has a 50-to-1 dynamic range and was scaled to produce a maximum value of unity. A 50-to-1 range in loop gain will allow the gain changer to maintain a limit cycle throughout the flight envelope, except power approach. At power approach, the maximum value gain is insufficient to sustain a limit cycle. The maximum value of G_{c*} is limited to 0.01.

Set Point Definition

Starting from a stable gain, the initial rate of gain adaptation is determined by the set-point and the integral gain. The set point and the integral gain are set such that their product will result in the driving of the nonlinear element from maximum to minimum (and vice versa) in a length of time consistent with the aircraft flight condition change capability. In this F-8C application, 40 seconds was selected as the nominal value. From other considerations, the set point and integral gain may require redistribution, but their product has been defined.

The integral rate may be further optimized when the system is subjected to disturbances to avoid excessive gain activity, particularly when the error limits in the system are reached.

Rate Limits on Gain Adjustment

Limits are imposed within the gain changer in order to define and bound gain changer operation when inputs are sufficient to saturate various elements. An important consideration is gain changer response to pilot inputs. A system operating at critical gain will obviously oscillate when disturbed. Ideally, a system with six-db gain margin provides the highest practical gain without excessive control surface oscillations (ringing). The rate limit should be set such that the product of the rate limit and proportional gain will produce a six-db change in the output of the nonlinear element. Thus, a rapid gain reduction will occur for pilot or disturbance inputs. Selection of the rate limit must also consider initial stabilization of a statically unstable vehicle if initialized at an unstable gain.

A second vital consideration in specifying the rate limit is related to pilot activity at the down-logic bandpass frequency. If the pilot concentrates an input at this frequency, the gain could be driven down from maximum to minimum in about 12 seconds if the rate limit is saturated. This level of determined, sustained effort is not felt to be probable but should receive investigation in pilot-simulation studies effecting not only the rate limits, but the up-logic bandpass and gain.

Amplitude Limits on Gain Adjustment

Down-logic is given twice the authority of the up-logic in the adjustment of the autopilot gain (refer to Figure 32). The down-logic authority limit is set equal to the rate limit. For saturation in the up- or down-logic, the rate limits will determine gain adjustment. With both up- and down-logic saturated, the relative authorities assure that the gain will be reduced. Without this assurance, extreme noise environment could conceivably drive the gain above critical values. In the present studies, no single disturbance could create this situation; but possibly multiple disturbances, including pilot inputs, may although the possibility is considered remote.

FUNCTIONAL BLOCK DIAGRAMS

The final design of the pitch axis high-gain adaptive controller is shown in Figure 32. The majority of the parameter trade-offs in the design of the high-gain limit cycle were made using a two-degree-of-freedom analog simulation. The digital implementation of LCGC used the Tustin equivalent of the bandpass filters and signal shaping. For this concept, the C*-model was changed slightly by adding a 0.1 second first-order lag to the pitch stick shaping to reduce elevator "ringing" in response to stick commands.

In verifying the design on the Langley nonlinear simulation, it was necessary to add a small lag ($\tau = 0.02$ sec) ahead of the elevator servo to obtain the desired pitch axis limit-cycle frequency. It is felt that this extra phase lag compensates for approximations in the digital integration used in Langley's simulation. It probably would not be needed in the real hardware system and is not shown in Figure 32.

The roll axis of the high-gain limit-cycle adaptive system is shown in Figure 33. The gain G_{roll} is set at $5.0 \cdot G_{LAT}$ and the roll axis is held near critical gain by adjusting G_{LAT} with the lateral LCGC. The yaw axis gain used G_{LAT} directly, and hence this adjustment is slaved to the roll axis. The G_{LAT} gain range for the yaw controller provides good performance and sufficient stability margins over the flight envelope.

PITCH AXIS PERFORMANCE

The high-gain limit-cycle design was evaluated against three areas related to adaptive performance:

1. Accuracy at fixed flight condition (consider effects of servo hysteresis, sensor noise, turbulence, and pilot commands),
2. Convergence properties, and
3. Tracking varying flight conditions.

Each of these areas will now be discussed.

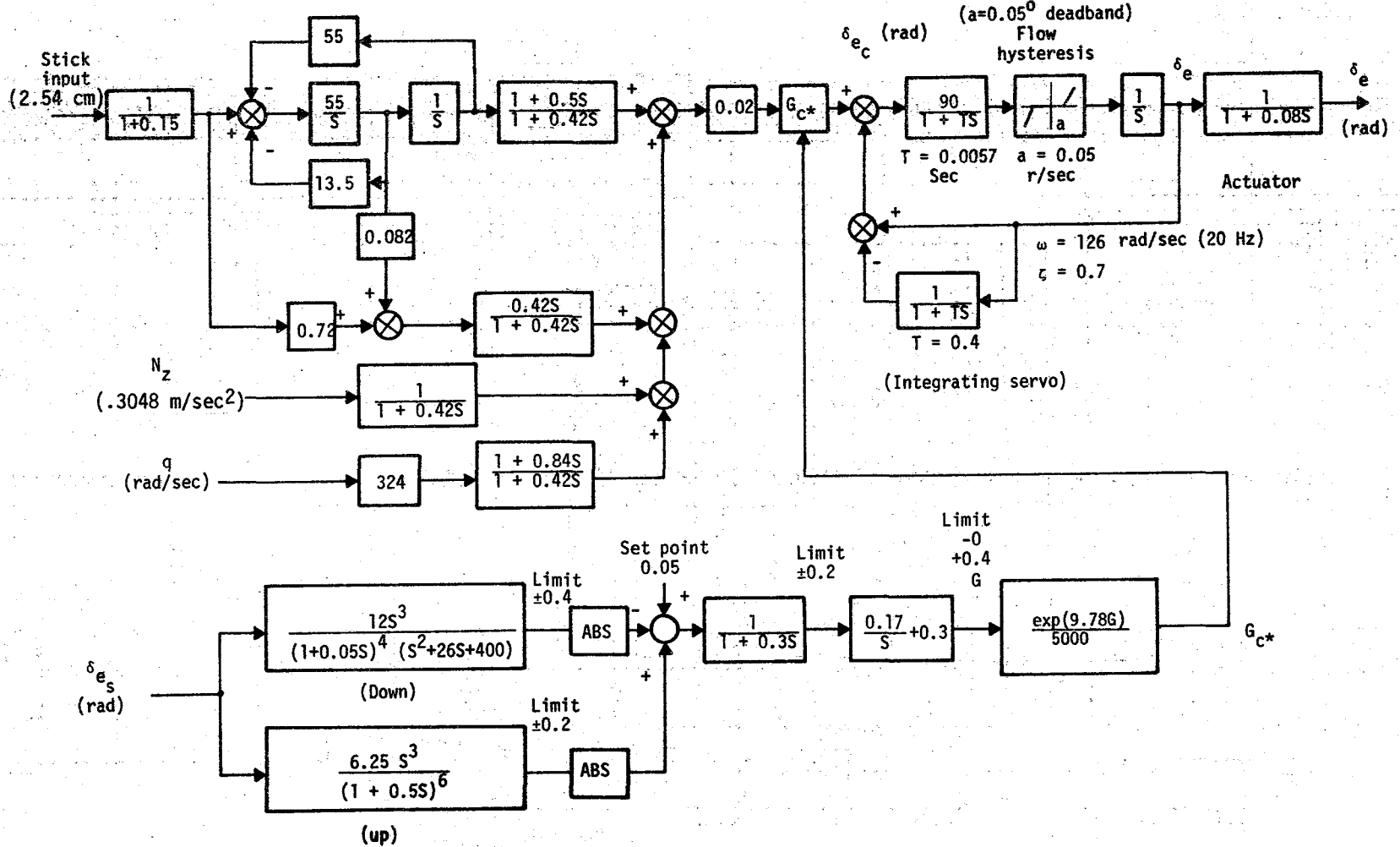


Figure 32. F8-C high-gain limit-cycle pitch axis.

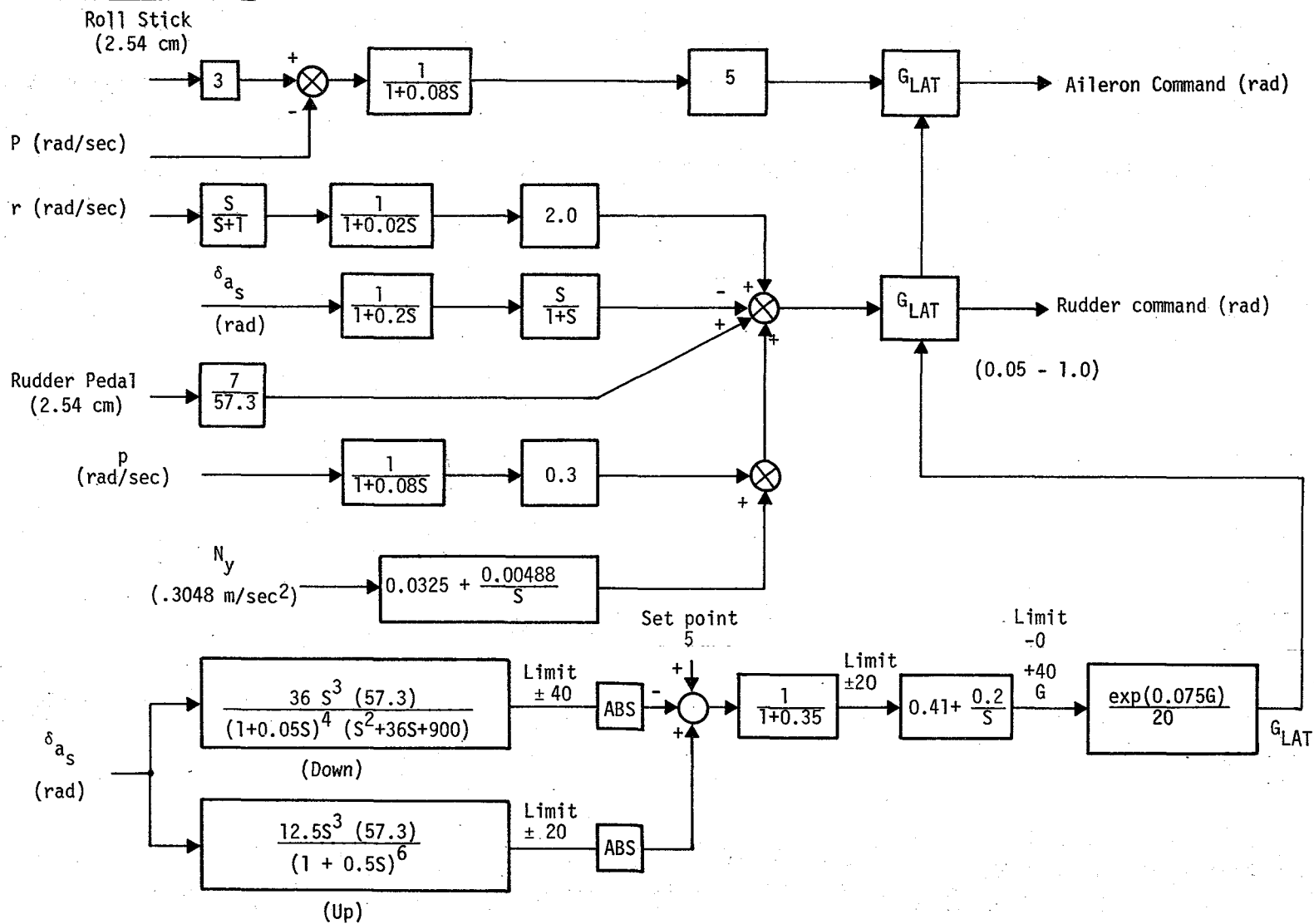


Figure 33. F-8C high-gain limit-cycle lateral-directional axes.

Accuracy at Fixed Flight Conditions

The standard pitch sequence consisting of periods of quiet, turbulence, and C*-doublet commands were used to measure the performance of LCGC and Flight Conditions 1, 5, 8, 10, and 17.

The time histories are contained in Appendix D. They were obtained from the Langley nonlinear simulation and were consistent with traces obtained on a linear two-degree-of-freedom analog simulation at Honeywell. The accuracy results are summarized in Table 26. Overall summary comments are provided together with the gain values (expressed as percent of critical) as functions of input conditions.

Convergence Properties

The concept of "converging" to a gain value does not apply to the LCGC in the same way it does to the explicit identifiers. In normal operation the gain changer will not be at a gain higher than critical. If the system starts at a value below critical, the rate of gain increase is largely determined by the integral gain. For example, if the LCGC is initialized at FC10 with the aircraft at FC5, it takes on the order of 10 seconds (in gusts) to reach "steady state" (which is six db below critical). Pilot commands do not speed up "convergence" since their effect tends to decrease the gain.

Adaptation to Changing Flight Conditions

In the case of increasing and decreasing M_{δ_e} , the gain changer tracked the critical gain (quiet case) or tracked the "turbulence" gain value.

Deceleration profiles starting from FC10 by reducing thrust to idle and extending the speed brake show that the limit-cycle amplitude remains constant.

Sensor Noise Effects

The LCGC was evaluated with the measurement noise defined in Section 5 for the pitch rate gyro, normal accelerometer, and servo position transducer. While gain changer activity increased in the presence of sensor noise, only the pitch rate gyro noise affected the LCGC to any significant degree. The gain changer will attempt

TABLE 26. PERFORMANCE AT FIXED FLIGHT CONDITIONS

Input condition	Gain (percent of critical)				Comments
	Flight condition				
	1	5	8	10	
Quiet	100	100	100	100	(1) Typically, the set point increases the gain over its critical value until the servo oscillation builds up. Then the gain is reduced to the critical value with a limit-cycle overshoot of about 100 percent.
Pilot doublet	60-63	60-75	60-71	46-48	(2) Gain is self-adjusting to achieve a nominal six-db gain margin during pilot inputs. Responses have typical six-db "ring." Up-logic prevents successive inputs from driving gain to minimum, establishing an average gain higher than six db.
Turbulence	75	56	81	69	(3) Gain is self-adjusting to provide control surface energy, induced by turbulence, to be essentially equivalent to the nominal limit-cycle energy (slightly higher due to up-logic bandpass). The net effect is a desirable ride softening as a function of turbulence.
Doublet plus turbulence	75	56	81	69	(4) Pilot inputs during noisy operation will cause gain to establish an average value about equal to sensor noise only case. Pilot input will initially drive gain down, wherever the starting point is.
Sensor noise	30	20	43	48	(5) Same as Note 3 except gyro noise reduced gain more than turbulence.
Doublet plus sensor noise	30	20	43	48	(6) See Note 5.
Turbulence plus sensor noise	30	20	43	48	(7) See Note 3.
Doublet plus turbulence plus sensor noise	30	20	43	48	(8) See above Notes 3, 5.
Servo hysteresis	74	68	67	80	(9) Frequency reduced 10 percent.

to adjust the gain downward until the noise-induced servo motion matches the equivalent limit-cycle energy in the down-logic network. The noise sensitivity was studied by simulating three different levels and bandwidths of gyro noise (white noise, first-order filter).

- A. 0.0087 rad/sec rms with a two Hz bandwidth. (This value was suggested by NASA from flight test recording data.)
- B. 0.026 rad/sec rms with a 16 Hz bandwidth. (This is a conservative value defined for these studies.)
- C. 0.0035 rad/sec peak-to-peak with a 16 Hz bandwidth. (Obtained from measurement of actual Honeywell GNAT gyro in a pristine environment.)

It is felt that actual gyro characteristics will fall somewhere between B and C above, in which case, the adaptive gain will tend to be between six and 12 db below critical. Significantly, this gain level is still greater than the original CAS $\bar{q}$ -schedule so autopilot performance should remain at acceptable levels.

Turbulence Effects

Thunderstorm turbulence will produce about the same gain reduction as gyro noise. At the 99 percent turbulence level (exceeded four days/year), the gain reduction is about three to six db, the greater reduction occurring at low $\bar{q}$. Up-logic plays a role in maintaining the gain level. Actually, the gain reduction, especially for high turbulence, produces a favorable "ride softening" effect.

Effect of Pilot Inputs

Since previous limit-cycle systems have been prone to excessive gain reduction,⁽¹⁵⁾ special note is made of gain levels during periodic pilot inputs. Initial response to a pilot input is a six-db gain reduction, a desirable and necessary feature of gain changer operation. However, without some up-logic capability, successive pilot inputs tend to further reduce the gain. The resultant interplay of the up- and down-logic results in considerable gain activity, but the average gain is generally slightly less than 50 percent below critical. When the up-logic was removed as an experiment during the pilot input sequence, the gain quickly reduced to 20 percent of critical.

The effects of sensor noise, turbulence, and hysteresis on the autopilot gain are summarized in Figure 34 as a function of dynamic pressure.

LATERAL-DIRECTIONAL PERFORMANCE

All preceding pitch axis time histories were obtained with the roll axis gain changer engaged. The variation in G_{LAT} in response to pilot commands and turbulence exactly parallel the characteristics of the pitch gain G_{C*} . The following summary will be limited to the performance of the lateral-directional CAS in conjunction with the gain changer.

Yaw Axis Gain Changer Limitations

Significant analysis effort was expended in attempting to apply the limit cycle to the yaw axis (rudder servo). The relatively low level of rudder effectiveness resulted in extremely high yaw rate gain as well as correspondingly high crossfeed gains. The yaw axis parameters--surface effectiveness, servo rate limit, servo loop gain, desired limit-cycle frequencies, and expected noise levels--were such that the loop could not be operated at critical gain levels without encountering excessive noise amplification resulting in severe servo rate saturation. This problem is not evident in pitch or roll due to higher combinations of surface effectiveness and servo rate. Consequently, the aileron servo was used as the basic input to the gain changer, and the yaw axis gains slaved to the aileron gain.

Roll Axis Gain Changer Operation

The gain changer in the roll axis is nearly identical to the pitch axis, and they probably could be made identical if any advantage would accrue.

As in pitch, sensor noise will cause gain changer activity with the most significant gain reduction occurring for the assumed roll rate gyro noise. (During earlier yaw axis adaptive analysis, noise in all the yaw axis sensors and crossfeeds, coupled with high gains, would drive the adaptive gain to extremely low levels.) Figure 35 is a summary of the adaptive gain level with yaw rate gyro, lateral accelerometer, roll rate gyro noise, and aileron servo hysteresis inputs as a function of flight condition.

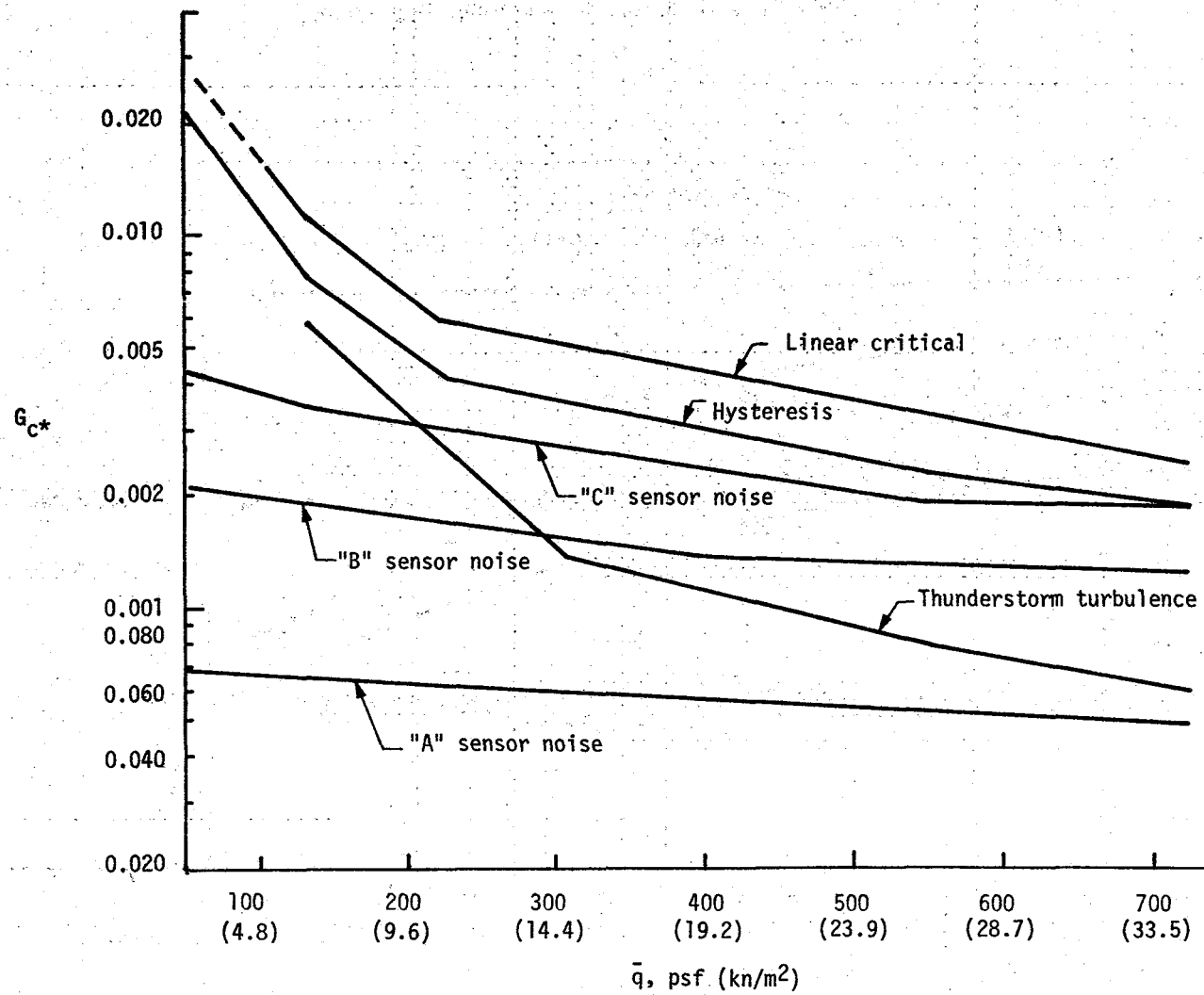


Figure 34. Summary of effects of disturbances on pitch axis gain.

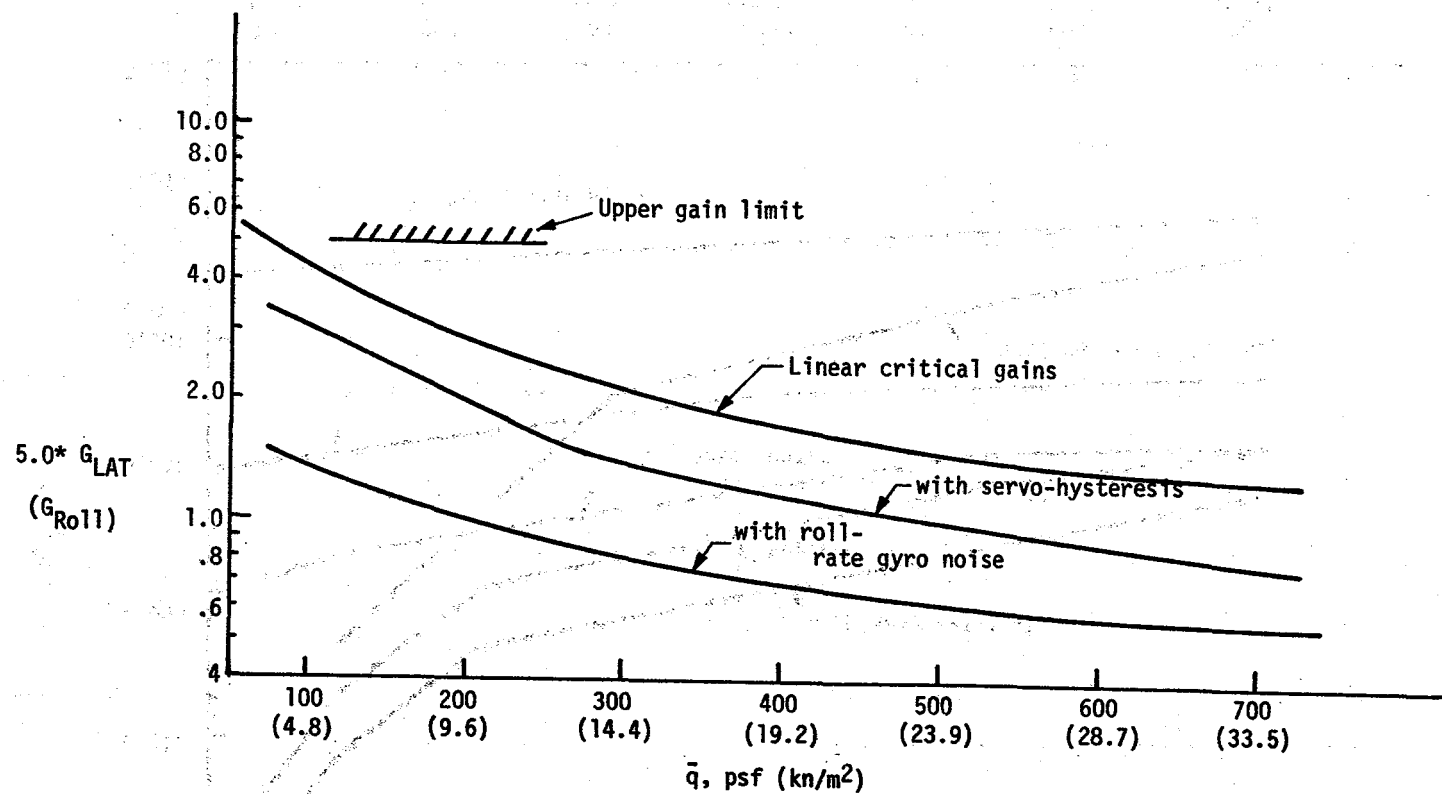


Figure 35. Summary of effects of disturbances on roll axis gain.

Appendix E contains a set of time histories showing the response of the lateral-directional CAS with LCGC. The adaptive gains increase the damping of a step beta gust response from 0.15 (free aircraft) to about 0.25 with little change in frequency. The turn coordination properties are satisfactory at all flight conditions.

SECTION 11

CONCEPT COMPARISON AND SELECTION

Three factors were used to compare candidate adaptive concepts:

- Performance,
- Growth potential, and
- Computer capacity requirements.

On this basis the MLE concept was selected for further refinement and recommended for eventual flight test. Its strength lies primarily in growth potential. It also has a slight edge in performance relative to other concepts.

PERFORMANCE

The performance of each concept was evaluated against three "measures of goodness:"

- Accuracy at fixed flight conditions (including effects of pilot commands, turbulence, sensor noise, and actuator hysteresis),
- Convergence properties, and
- Tracking varying flight conditions.

Tables 27 through 30 summarize the performance characteristics of each concept at four fixed flight conditions using a standard test sequence. For this test the initialization was at the proper flight condition. The test sequence consisted of:

1. Quiescent condition,
2. Pilot doublet followed by four seconds with no input,
3. Thirty seconds of 99 percent turbulence (Dryden model),
4. Pilot doublet with 99 percent turbulence (Dryden model), and
5. Repeat of (1) through (4) with sensor noise.

Flight conditions 1, 5, 8 and 10 are presented in Tables 27 through 30, respectively.

TABLE 27. PERFORMANCE CHARACTERISTICS AT FC1

[20,000 ft, $M = 0.67$, $\bar{q} = 14603 \text{ n/m}^2$]

Excitation	Tracker % error in $\hat{g}_{11}$	Limit cycle % of critical gain	MLE Max. % errors in			
			M_δ	M_α	V	$\alpha_{\omega L}^a$
Doublet	5	63	4	2	6	
Turbulence	10	75	7	2	8	
Sensor noise	20	30	4	10	15	1
Turbulence plus sensor noise	25	30	7	2	1	
Servo hysteresis with 0.3 ft/s turbulence	0	74	4	2	6	

^a Angle-of-attack referenced to water-line.

TABLE 28. PERFORMANCE CHARACTERISTICS AT FC5

[20,000 ft, $M = 0.4$, $\bar{q} = 5219 \text{ n/m}^2$]

Excitation	Tracker % error in $\hat{g}_{11}$	Limit cycle % of critical gain	MLE Max. % errors in			
			M_δ	M_α	V	$\alpha_{\omega L}^a$
Doublet	20	75	5	7	8	
Turbulence	20	56	5	8	2	
Sensor noise	70	20	5	7	17	8
Turbulence plus sensor noise	20	20	5	7	3	

^a Angle-of-attack referenced to water-line.

TABLE 29. PERFORMANCE CHARACTERISTICS AT FC8

[40,000 ft, $M = 1.2$, $\bar{q} = 18913 \text{ n/m}^2$]

Excitation	Tracker % error in $\hat{g}_{11}$	Limit cycle % of critical gain	MLE Max. % errors in			
			M_δ	M_α	V	$\alpha_{\omega L}$
Doublet	30	63	0	2	4	
Turbulence	40	81	6	6	1	
Sensor noise	25	43	0	2	4	4
Turbulence plus sensor noise	40	43	12	2	4	

TABLE 30. PERFORMANCE CHARACTERISTICS AT FC10

[Sea level, $M = 0.7$, $\bar{q} = 34713 \text{ n/m}^2$]

Excitation	Tracker % error in $\hat{g}_{11}$	Limit cycle % of critical gain	MLE Max. % errors in			
			M_δ	M_α	V	$\alpha_{\omega L}$
Doublet	5	48	6	9	29	
Turbulence	5	69	23	9	23	
Sensor noise	5	48	29	5	29	3
Turbulence plus sensor noise	5	48	23	9	23	

From these tables it is evident that sensor noise is more degrading to the tracker and limit-cycle concepts than to the MLE concept. MLE exhibits better accuracy than the model tracker. The limit-cycle design shows gain reductions from critical for pilot commands and turbulence as desired.

The convergence characteristics of the three concepts are summarized in Table 31. The adaptation was determined for two types of inputs: 1) a sequence of pilot doublets (square wave C*-commands of 6.1 m/sec^2 with a six-second period) with no turbulence, and 2) 99 percent turbulence with no pilot inputs. Two conditions were examined. For the first one, the aircraft was at FC1, but the algorithms were initialized at FC5. The MLE design converges faster than the model tracker. This test was not run for the limit cycle since it would imply initializing above critical gain which is not a realistic test. The second condition shows the aircraft at FC5 but initialized at a much higher dynamic pressure condition (FC10). Again, MLE has a faster response than the model tracker. During pilot commands, the limit-cycle design does not increase the loop gain to the proper value for this flight condition. During turbulence, the action of the set point does slowly return the gain to the appropriate value.

TABLE 31. ADAPTATION CHARACTERISTICS WITH INITIALIZATION AT DIFFERENT FLIGHT CONDITIONS

Condition	Input	Time(Sec) to 80% Steady sec^2		
		Tracker	Limit cycle	MLE
At FC1 initialized FC5	Doublet sequence	4.0	N.A.	<1
	99% turbulence	4.0	N.A.	<1
At FC5 initialized FC10	Doublet sequence	7.0	--	<1
	99% turbulence	7.0	10	<1

Table 32 shows the tracking characteristics during an acceleration at maximum power from Flight Condition 5 ($h = 6096 \text{ m}$, $M = 0.4$) to 7620 m and $M = 1.1$. Three different conditions (quiet, turbulence, and turbulence plus sensor noise) are presented. For the limit-cycle concept, it is desirable to maintain at least 50 percent of linear critical gain. For the two explicit concepts, it is desirable to accurately estimate surface effectiveness parameters.

TABLE 32. TRACKING CHARACTERISTICS FOR VARYING FLIGHT CONDITION

Excitation	Tracker % error in $\hat{g}_{11}$	Limit cycle % of critical gain	Peak MLE % error in $\hat{M}_{\delta 0}$
Quiet case	25 ^a	100	^a 31
Turbulence	20	75	28
Turbulence plus sensor noise	20	36	28

^a Test signal required.

GROWTH POTENTIAL

Growth potential is necessarily a qualitative notion. It is judged to be low for the limit-cycle system because the concept itself is restricted to a single (implicit) loop gain adjustment.

In theory, the tracker provides greater growth potential because it can estimate several parameters as well as states. As discussed in Section 9, however, this potential is not realized in practice due to severe limitations imposed by sensor noise and disturbances. This leaves the MLE concept as the only approach with capability to achieve complex multidimensional gain adjustments. Its $M_{\delta 0}$ estimate can be used to adjust loop gain to compensate for changing surface effectiveness, while the M_{α} estimate is useful for reduced-static-stability control laws. As discussed in Section 8, the M_{α} estimate can adjust an additional gain on a lagged pitch rate feedback to compensate for the changing stability properties of the airframe. This is especially important in transitions from subsonic to supersonic flight due to the shift in the center of pressure.

COMPUTER REQUIREMENTS

To estimate flight computer requirements, the adaptive subroutines were compiled and run on a CDC-6600. The resulting time and memory requirements are listed in

Table 33. The data are given for FORTRAN generated code that is fairly efficient for the given algorithm (no indexing used). A comparison of operation times for the CDC-6600 and the AP-101 flight computer is given in Table 34. This table suggests a speed ratio of 7:1 for typical multiply-add operations. Hence, the MLE control package which requires 1.1 msec per control cycle on the CDC-6600 should consume 7.7 msec per control cycle on the AP-101. The frame time for the F-8's digital system is 20 msec, so the control law plus adaptation will take well below half of the time available. However, this does not include time for redundancy management or mode switching which may be part of the total control repertoire.

RECOMMENDED CONCEPT

The concept comparison is summarized in Table 35. Overall, the MLE concept is judged to be the most promising and is recommended for refinement for flight test.

TABLE 33. COMPUTER REQUIREMENTS BY SUBROUTINE

Subroutine	Memory	CDC 6600 Cycle time (msec)
1. Pitch and lateral controller (no adaptive computation)	570 (1072 octal)	0.3
2. Limit (up- and down-logic, both axes)	546 (1042 octal)	0.4
3. Track	272 (420 octal)	0.2
4. MLE	2146 (4142 octal)	0.8

TABLE 34. COMPUTER OPERATION SPEEDS

Operation ^a	CDC-6600 ^b	IBM AP-101 ^c
Memory access (μsec)	0.1	0.9
Add (μsec)	0.4	2.4
Multiply (μsec)	0.7	5.4
Divide (μsec)	1.0	10.0 - 10.5

^a Floating point arithmetic, register to register.

^b CDC-6600 CYBERNET applications group data.

^c IBM specification sheet data.

TABLE 35. OVERALL COMPARISON OF CONCEPTS

Quality	Model tracker	High-gain limit cycle	Maximum likelihood estimation
Performance	Acceptable to good	Good	Good to excellent
Growth potential	Low - limited to single variable explicit gain schedule	Low - limited to single variable implicit adaptation	High - multiple parameter gain adjustment possible
Computer requirements ^a	Minimal	Low	Medium

^a These requirements are relative to AP-101 capacity.

SECTION 12

CONCLUSIONS AND RECOMMENDATIONS

CONCLUSIONS

Simulator quality designs for the F-8C DFBW application were successfully developed from three different adaptive concepts: maximum likelihood, model tracking, and high-gain limit-cycle control. Performance, growth potential, and computer requirements were used as criteria for selecting the most promising of these candidates for further refinement. The maximum likelihood concept was selected primarily because it offers the greatest potential for identifying several aircraft parameters and, hence, for better control performance in future aircraft applications. In terms of identification and gain adjustment accuracy, the MLE concept proved slightly superior to the rest, but, as expected, this increment has no significant effects on the control performance achievable with the F-8C aircraft.

The MLE design is based on standard maximum likelihood estimation theory but uses a parallel channel implementation in order to avoid on-line data management and minimization iterations. It identifies a reduced set of three parameters from pitch axis data only. This structure was selected on the basis of separate theoretical identifiability studies which indicate that only limited identification potential exists for the F-8C under the ground rules imposed in Section 4. Identifiers which look for more parameters and/or use additional lateral-directional data cannot be expected to perform substantially better. Actual identification accuracies of the MLE concept are 10 to 20 percent for the critical $M_{\dot{\alpha}}$ parameter. This is sufficient to schedule gains in both the pitch and lateral-directional control laws.

The model-tracking design is based on modified Liapunov-stable model-following theory. Because this theory ignores process and measurement noise, it produces sensitive designs with large identification errors in applications which include

such disturbances. Hence, the F-8C model tracker relies on bandpass filters, thresholds, and limiters to minimize these problems. Like MLE, the tracker was set up to identify three parameters. However, only the surface effectiveness estimate proved reasonable in the presence of gusts and sensor noise.

The high-gain limit-cycle design is based on established flight tested control technology. It is applicable whenever the control structure has a single-loop gain as its variable parameter. Within this restriction, the design performed well. Its up-logic successfully alleviates the excessive gain reductions during pilot commands which were characteristic of previous flight systems, and there is no adverse coupling between simultaneous pitch and roll limit-cycle gain changers.

RECOMMENDATIONS

The maximum likelihood concept is recommended for flight test as part of the second phase of NASA's F-8C DFBW program. Several refinements of this design are also suggested. These include gust level estimation to fine-tune filter gains and automatic data length adjustment on the basis of aircraft maneuvers to reduce tracking errors. We further recommend development of a special purpose lateral identifier to extract improved velocity estimates during large rolling maneuvers. These estimates could be used to implement an inertially coordinated lateral CAS² without accurate air data. Advantages of inertial coordination are good dutch roll damping and good turn coordination at all angles-of-attack. These are not simultaneously achievable with the present reduced measurement lateral control law.

Flight demonstrations of the MLE concept will mark an important step in the development of modern adaptive flight control technology. They will serve to verify quantitative predictions of identifier and control-loop performance and to establish qualitative properties of the overall control system. The latter include test signal requirements and their subjective effects on the pilot, performance during large maneuvers and configuration changes, the feasibility of quiescent operation without test signal excitation, and effects of modeling limitations associated with transonic flight, static and dynamic flexibility, real sensors, and actuators, etc. Positive flight test results will contribute substantially to making adaptive control a viable alternative design approach for future aircraft applications.

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APPENDIX A

SCATTER PLOTS OF PITCH AXIS MODEL COEFFICIENTS

x = Subsonic flight condition

s = Supersonic flight condition

Data does not include quasistatic
aircraft flexibility.

Figure 36. M_q versus M_δ .



Figure 37. M_α versus M_δ .

Figure 38. V versus M_δ .

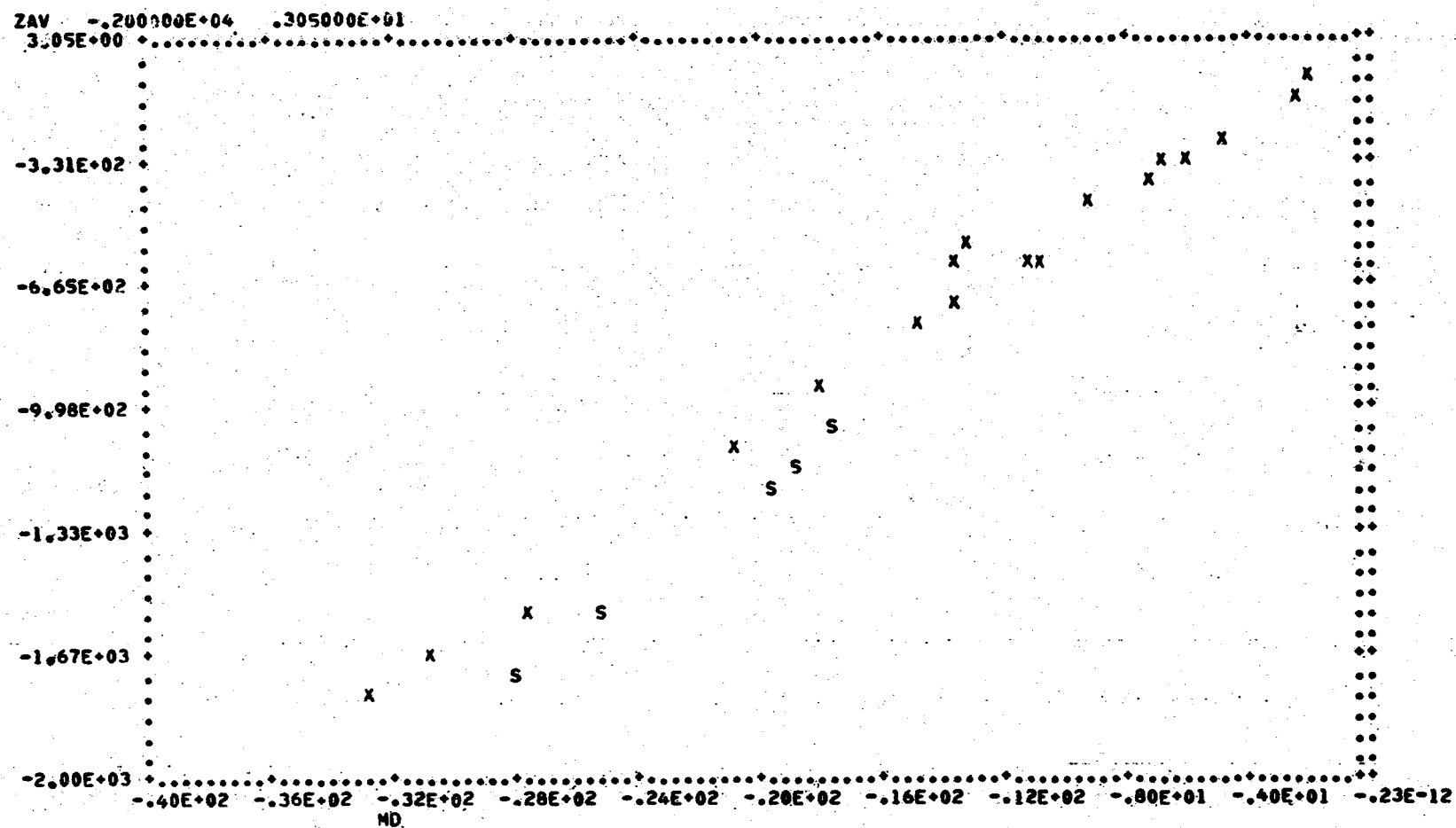
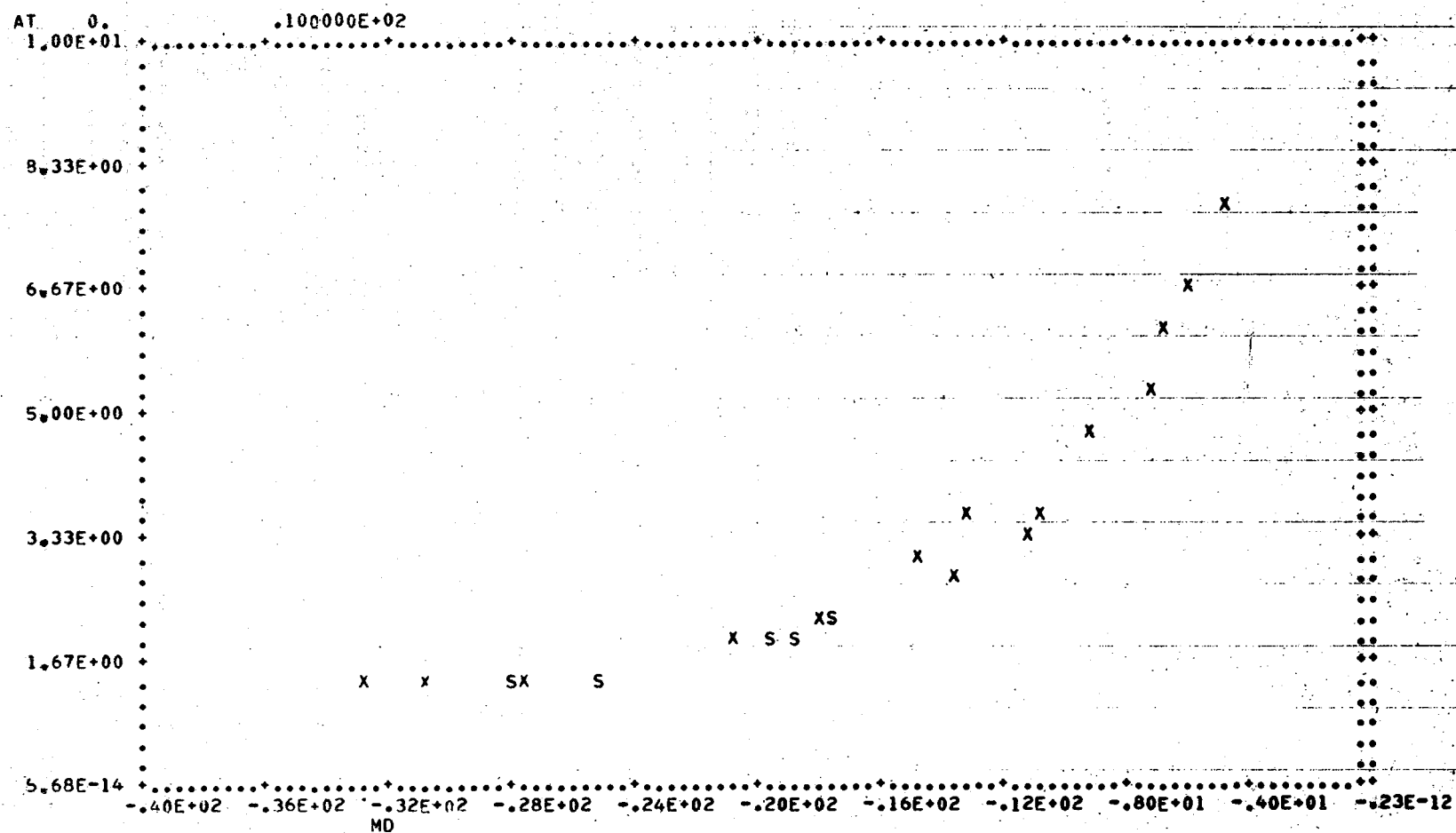
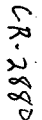


Figure 39. $(Z_{\alpha} V)$ versus M_{δ} .

Figure 40. $(Z_{\delta} V)$ versus M_{δ} .

Figure 42. α_{trm} versus n



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MDELTA	QBAR	MALPHA	MQ	ZDV	ZALPHA	V	DT(DEG)	AT(DEG)	MO	ZAV
-.1375E+02	.3054E+03	-.6340E+01	-.6165E+00	-.1090E+03	-.1049E+01	.6941E+03	-.2398E+01	.2892E+01	-.2555E+00	-.7283E+03
-.1469E+02	.3054E+03	-.6781E+01	-.6165E+00	-.1098E+03	-.1091E+01	.6914E+03	-.3701E+01	.2985E+01	-.5955E+00	-.7544E+03
-.1380E+02	.3054E+03	-.1209E+02	-.6165E+00	-.1148E+03	-.9228E+00	.6798E+03	-.8162E+01	.4435E+01	-.1030E+01	-.6274E+03
-.1321E+02	.3054E+03	-.7673E+01	-.5784E+00	-.8232E+02	-.8271E+00	.6933E+03	-.3071E+01	.3658E+01	-.2179E+00	-.5735E+03
-.4852E+01	.1089E+03	-.3188E+01	-.3622E+00	-.3853E+02	-.6373E+00	.4102E+03	-.4409E+01	.7708E+01	.5556E+01	-.2614E+03
-.2771E+02	.5511E+03	-.1266E+02	-.8728E+00	-.2154E+03	-.1652E+01	.9333E+03	-.1850E+01	.1455E+01	-.5732E+00	-.1542E+04
-.6838E+01	.1343E+03	-.2854E+01	-.3125E+00	-.4869E+02	-.4962E+00	.6736E+03	-.3945E+01	.6094E+01	-.1673E+00	-.3343E+03
-.1746E+02	.3946E+03	-.2972E+02	-.3112E+00	-.1133E+03	-.9050E+00	.1161E+04	-.3373E+01	.2119E+01	.7138E+01	-.1051E+04
-.3087E+02	.6517E+03	-.1338E+02	-.9901E+00	-.2456E+03	-.1959E+01	.8622E+03	-.1714E+01	.1341E+01	-.6103E+00	-.1689E+04
-.3289E+02	.7255E+03	-.1521E+02	-.1099E+01	-.2716E+03	-.2251E+01	.7818E+03	-.1664E+01	.1305E+01	-.6085E+00	-.1760E+04
-.5898E+01	.1333E+03	-.2876E+01	-.5309E+00	-.4744E+02	-.9127E+00	.3323E+03	-.3147E+01	.6577E+01	.6148E+02	-.3032E+03
-.1812E+02	.4159E+03	-.8503E+01	-.8663E+00	-.1495E+03	-.1577E+01	.5915E+03	-.1847E+01	.2274E+01	-.2468E+00	-.9328E+03
-.1077E+02	.2450E+03	-.5795E+01	-.5405E+00	-.8602E+02	-.9562E+00	.6210E+03	-.2821E+01	.3516E+01	-.1747E+00	-.5938E+03
-.2063E+02	.4355E+03	-.8914E+01	-.7648E+00	-.1612E+03	-.1363E+01	.8294E+03	-.1963E+01	.1912E+01	-.4093E+00	-.1138E+04
-.9045E+01	.1754E+03	-.3937E+01	-.3775E+00	-.6271E+02	-.5738E+00	.7720E+03	-.3159E+01	.4612E+01	-.1818E+00	-.4430E+03
-.1104E+02	.2220E+03	-.8591E+01	-.4477E+00	-.8476E+02	-.6977E+00	.8698E+03	-.2747E+01	.3424E+01	-.1612E+01	-.6069E+03
-.1860E+01	.5289E+02	-.6217E+00	-.3537E+00	-.1696E+02	-.5012E+00	.2092E+03	-.1432E+02	.1992E+02	-.2488E+00	-.1048E+03
-.2413E+01	.7101E+02	-.1622E+01	-.4098E+00	-.2124E+02	-.6181E+00	.2444E+03	-.1287E+02	.1403E+02	-.1451E+00	-.1510E+03
-.1367E+02	.3054E+03	-.6395E+01	-.6165E+00	-.1132E+03	-.1045E+01	.6948E+03	-.1793E+01	.2820E+01	-.1130E+00	-.7262E+03
-.1358E+02	.3054E+03	-.6445E+01	-.6165E+00	-.1177E+03	-.1041E+01	.6953E+03	-.1120E+01	.2732E+01	.4175E+01	-.7235E+03
-.1952E+02	.5372E+03	-.3327E+02	-.3274E+00	-.1288E+03	-.8942E+00	.1355E+04	-.3100E+01	.1852E+01	.1977E+01	-.1212E+04
-.1866E+02	.4632E+03	-.3177E+02	-.3206E+00	-.1219E+03	-.9050E+00	.1258E+04	-.3188E+01	.1962E+01	.4921E+01	-.1139E+04
-.2810E+02	.6332E+03	-.4775E+02	-.3983E+00	-.1782E+03	-.1419E+01	.1194E+04	-.2014E+01	.1300E+01	.9591E+01	-.1695E+04
-.2518E+02	.5320E+03	-.4057E+02	-.3797E+00	-.1796E+03	-.1401E+01	.1095E+04	-.1926E+01	.1428E+01	.1650E+00	-.1534E+04
-.7019E+01	.1583E+03	-.3698E+01	-.3824E+00	-.5678E+02	-.6489E+00	.5940E+03	-.3805E+01	.5348E+01	-.1209E+00	-.3854E+03

Figure 44. References.

APPENDIX B

MLE ADAPTIVE SYSTEM TRANSIENT RESPONSES

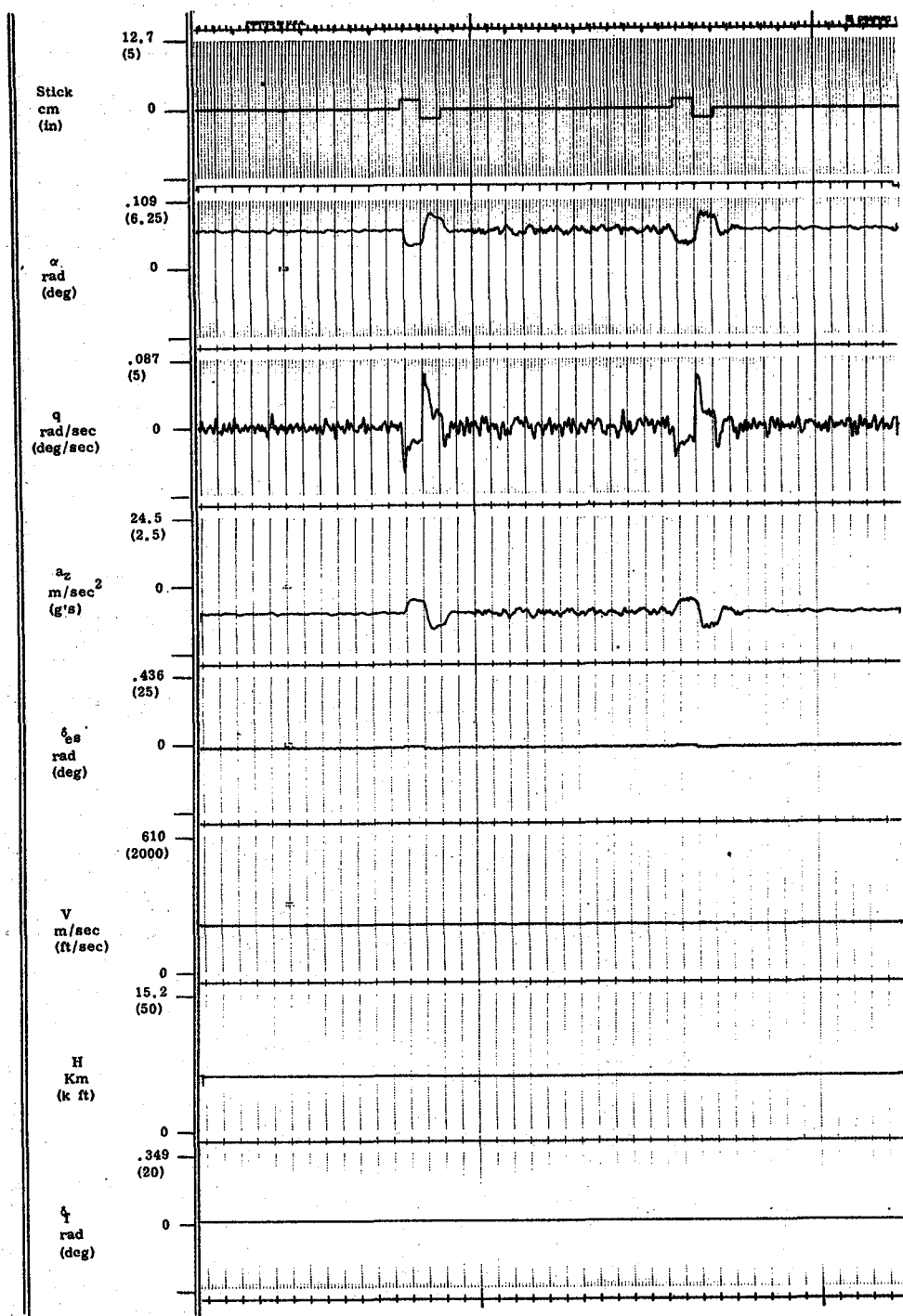


Figure 45. MLE accuracy test, FC1, no sensor noise.

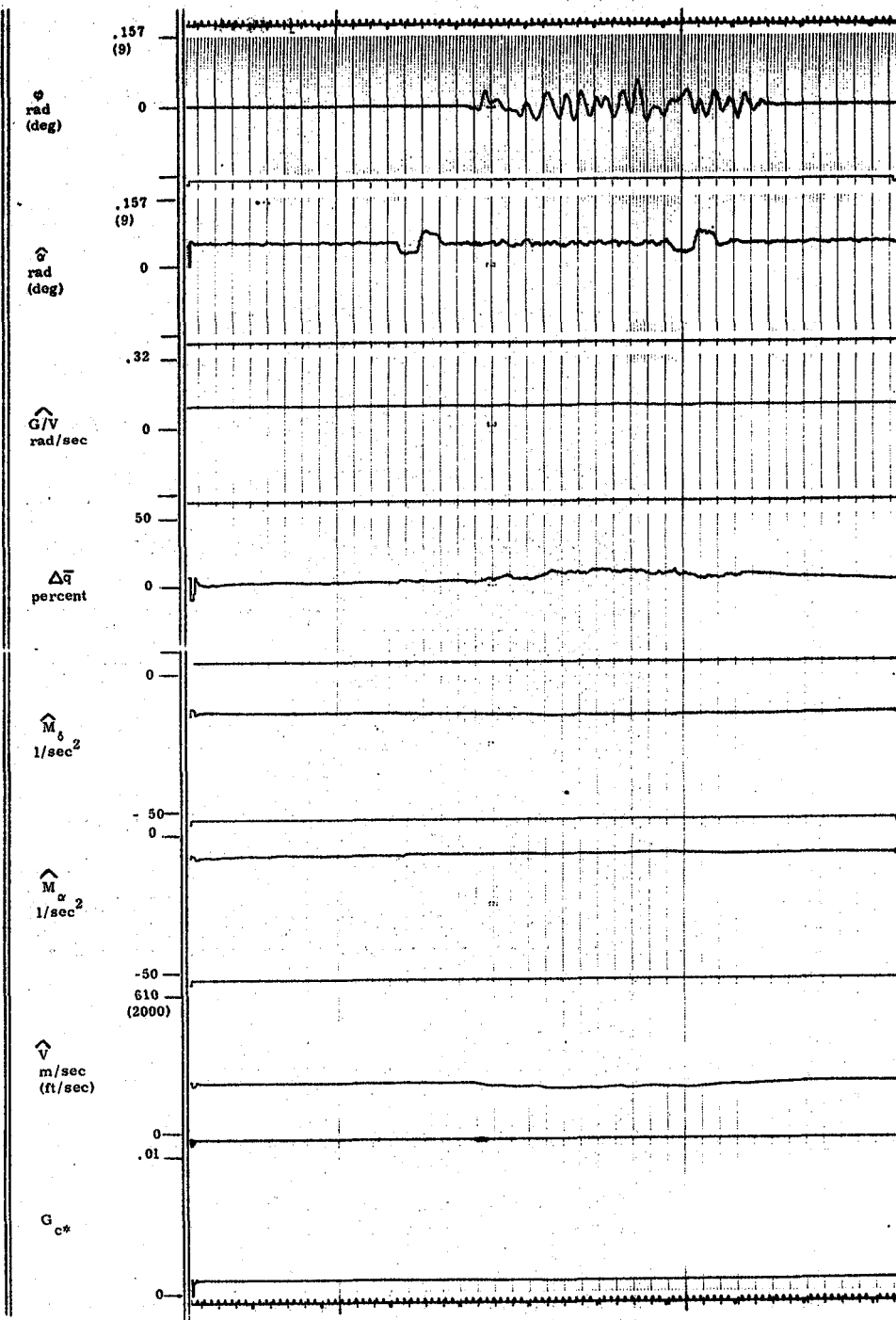


Figure 45. Concluded.

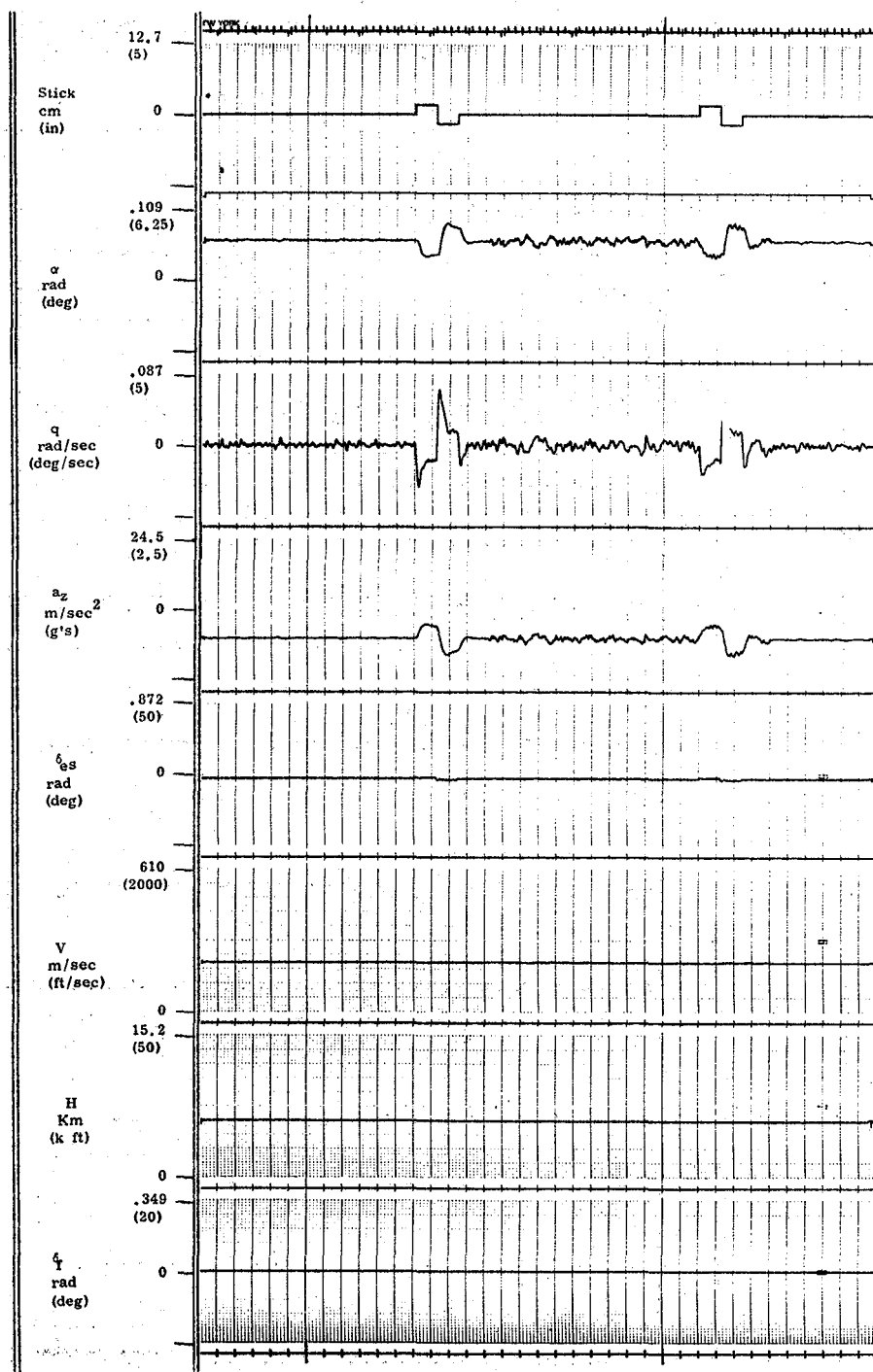


Figure 46. MLE accuracy test, FC1, with sensor noise.

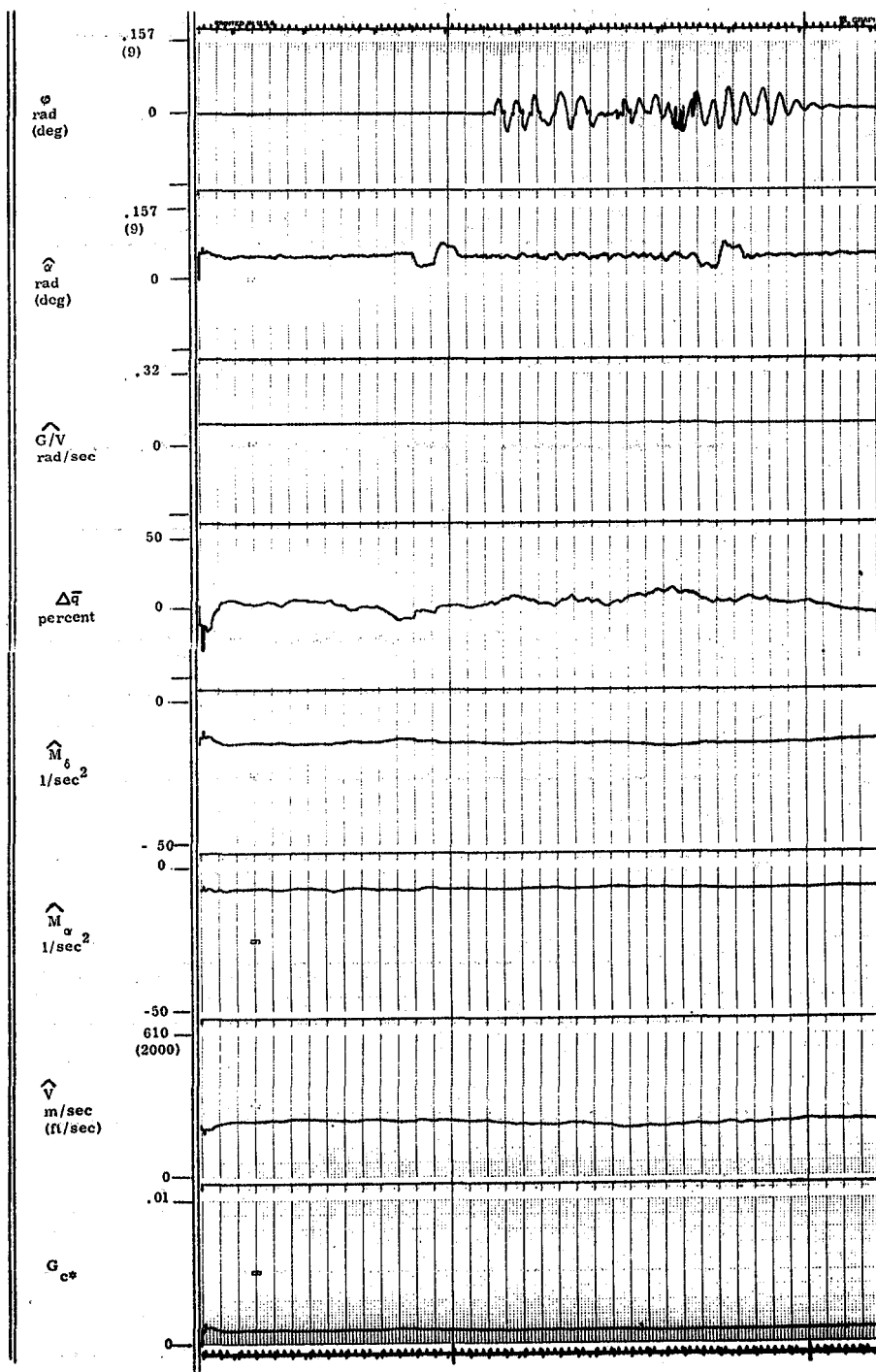


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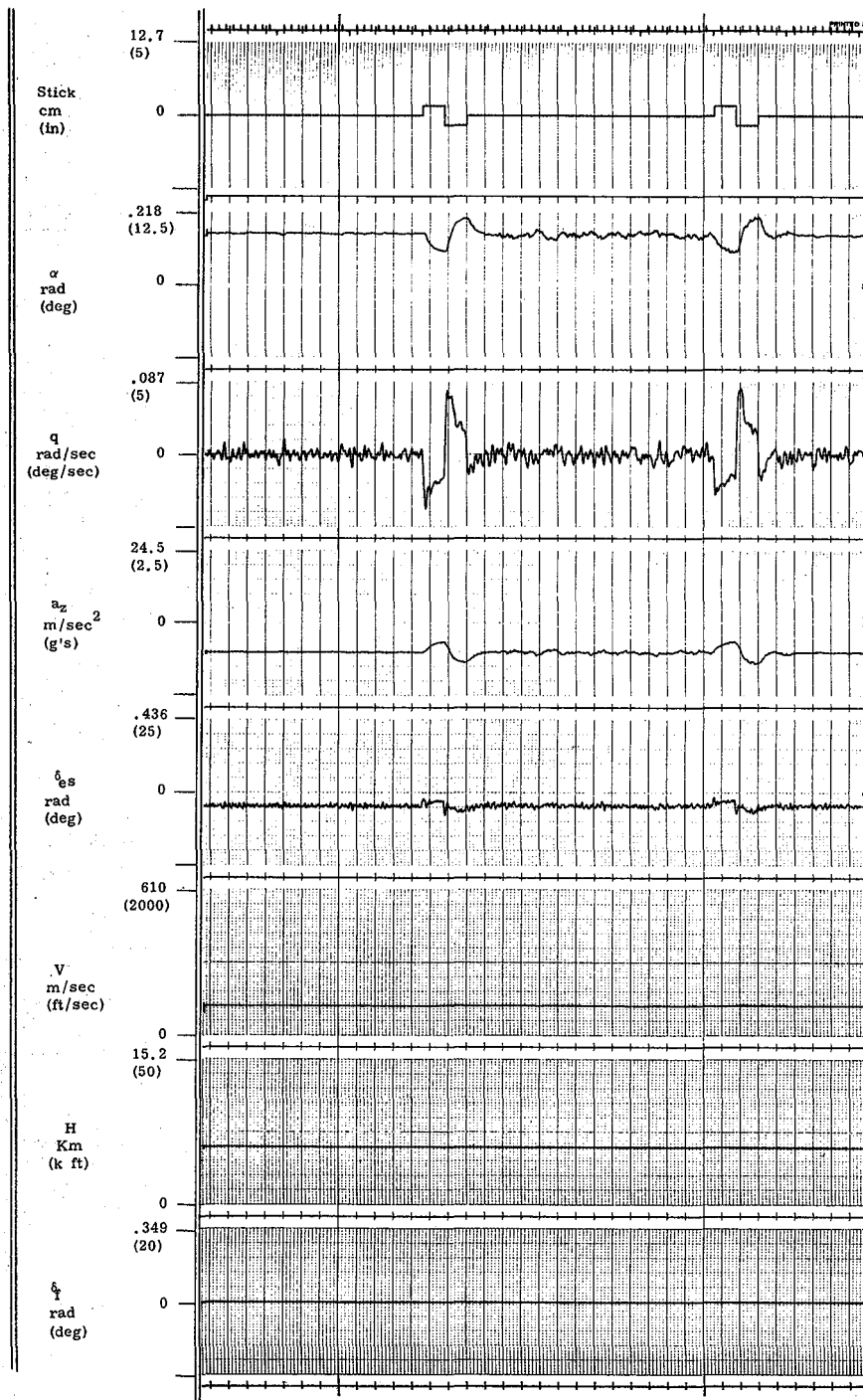


Figure 47. MLE accuracy test, FC5, no sensor noise.

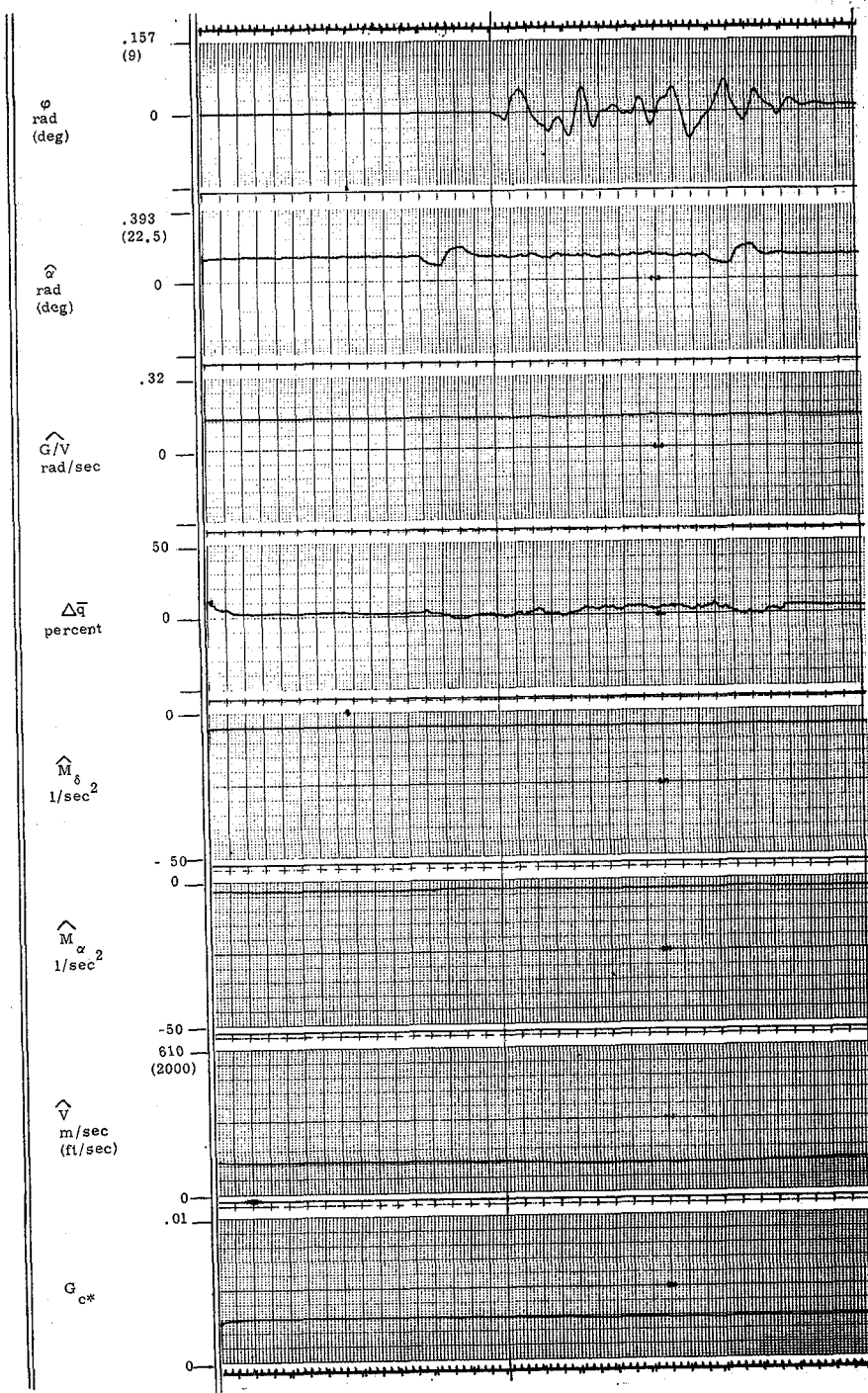


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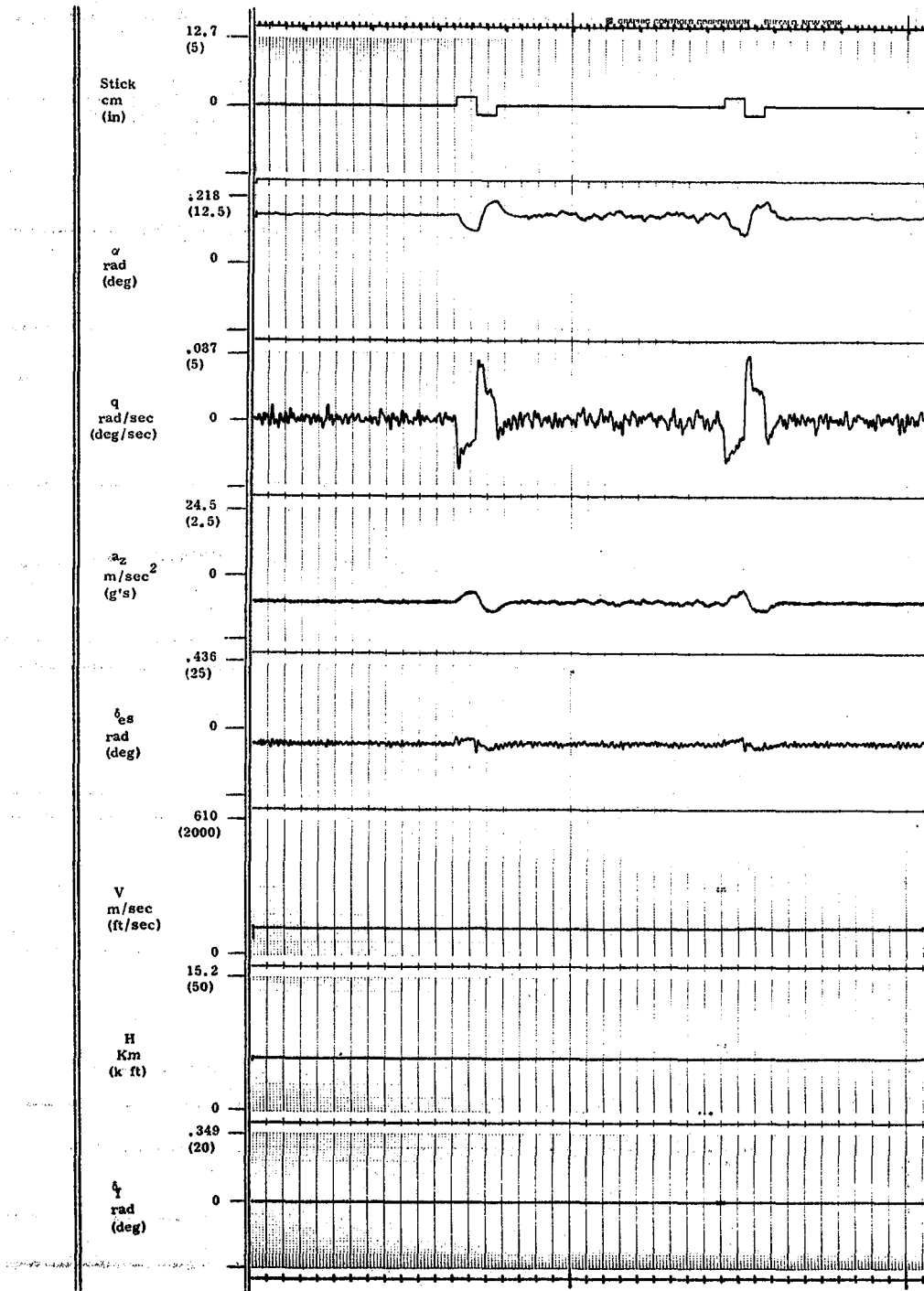


Figure 48. MLE accuracy test, FC5, with sensor noise.

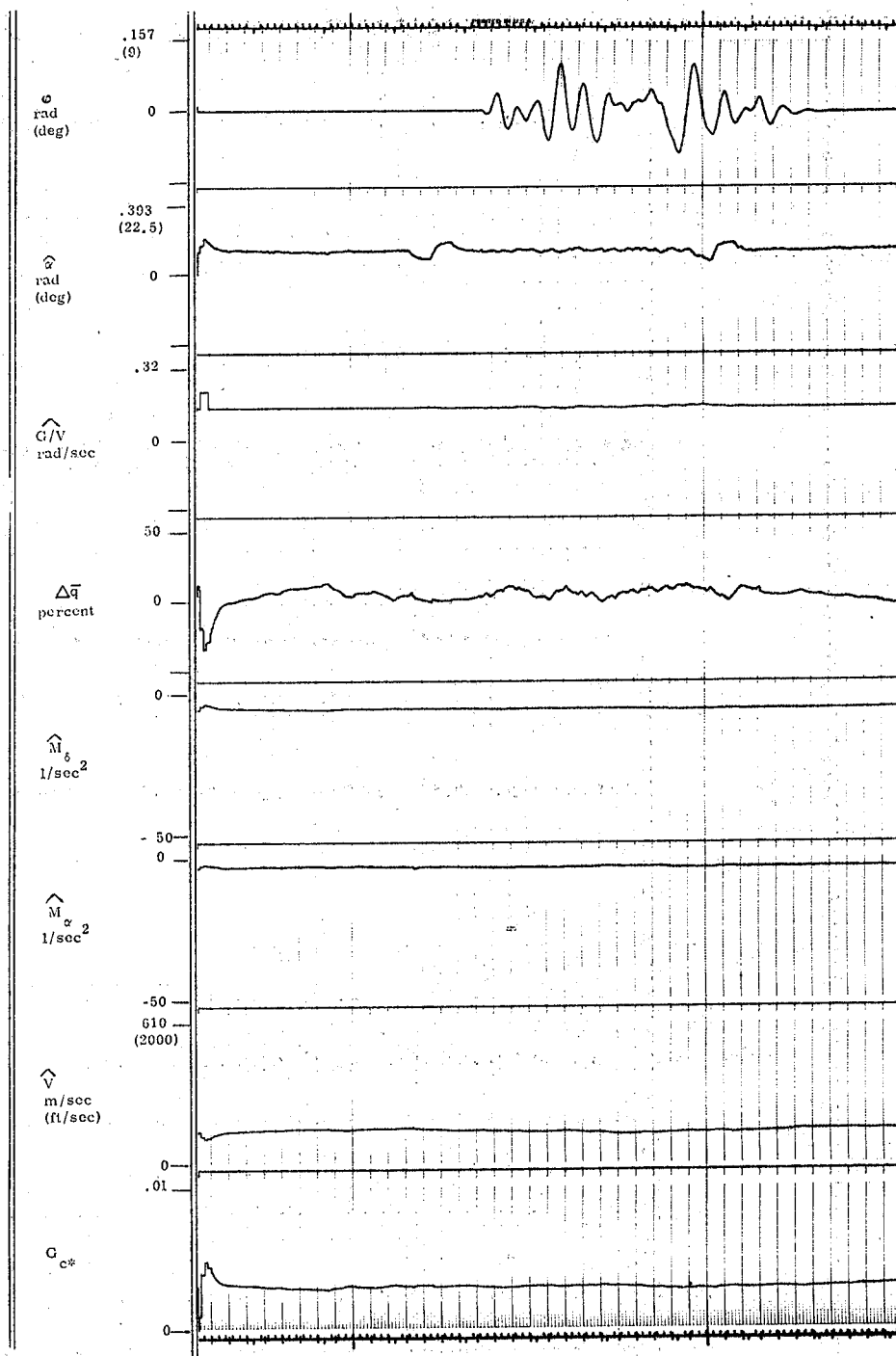


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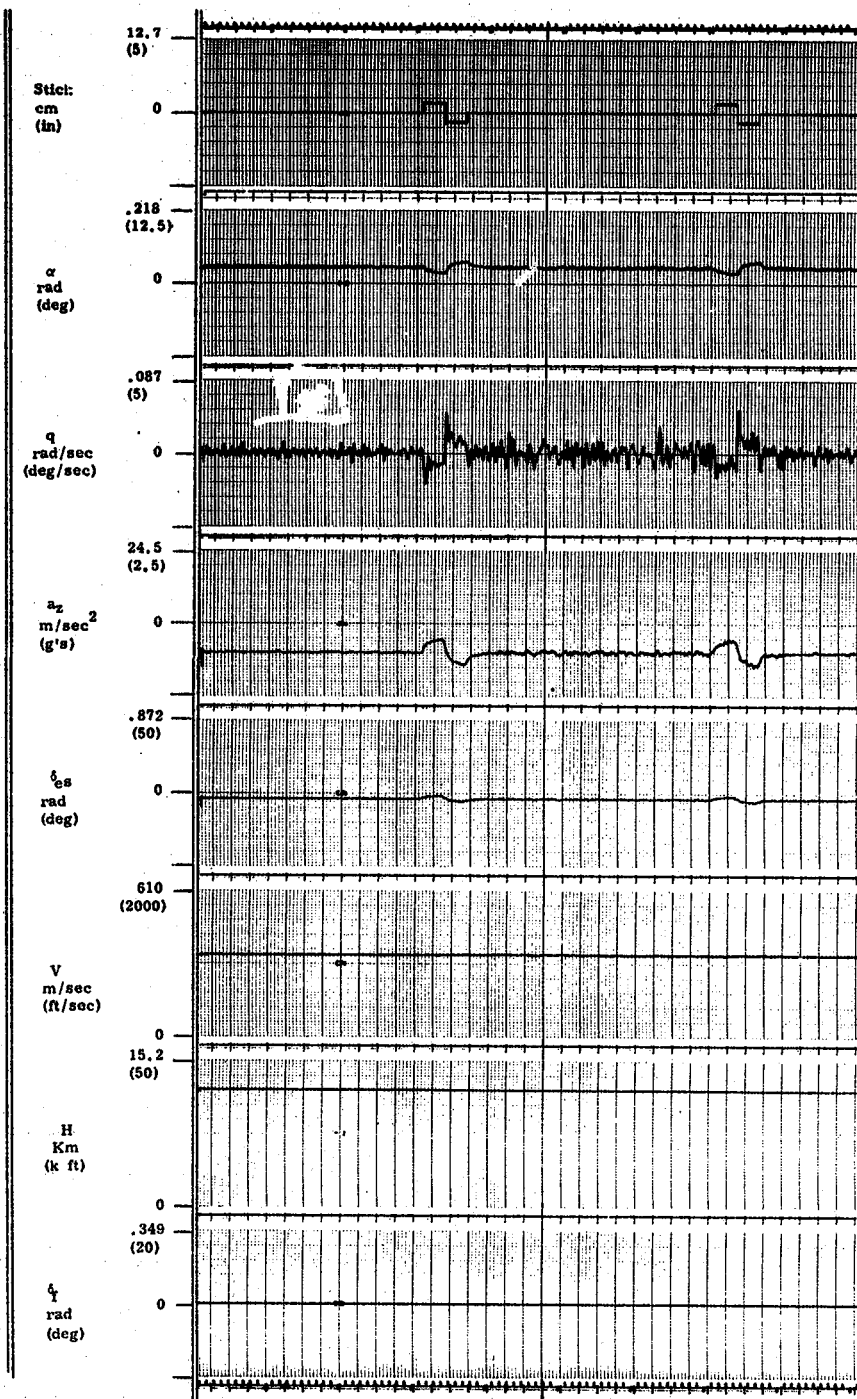


Figure 49. MLE accuracy test, FC8, no sensor noise.

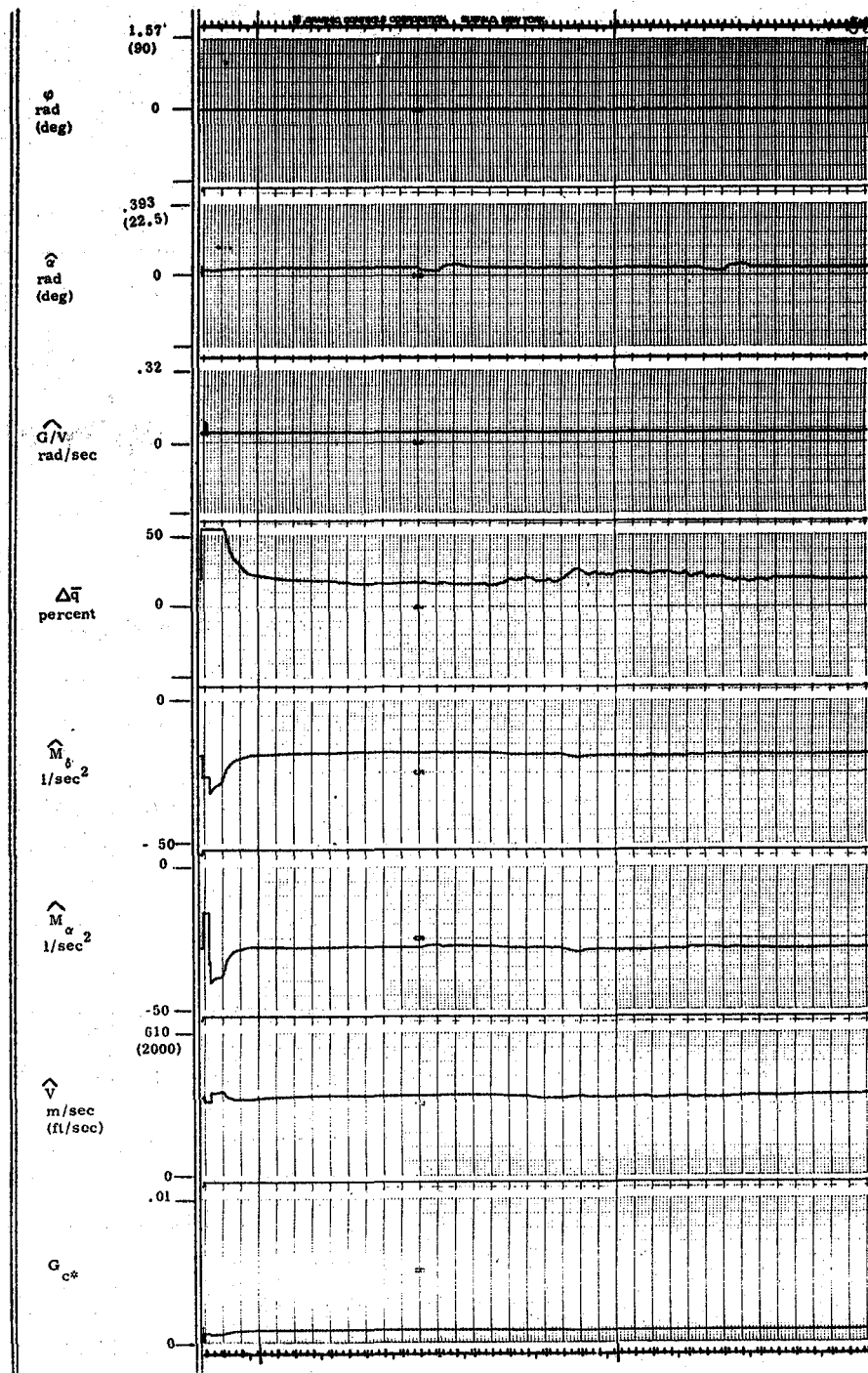


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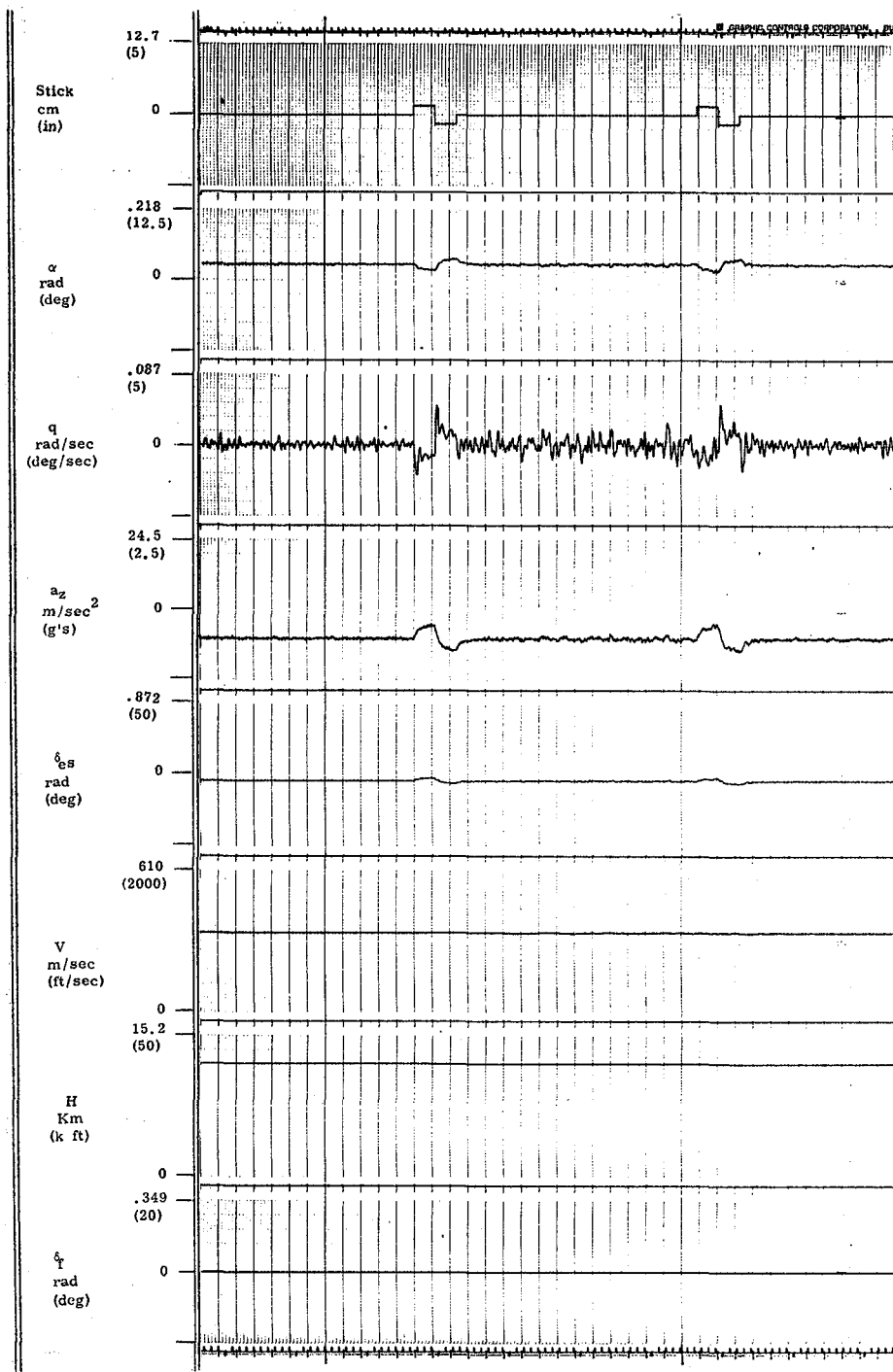


Figure 50. MLE accuracy test, FC8, with sensor noise.

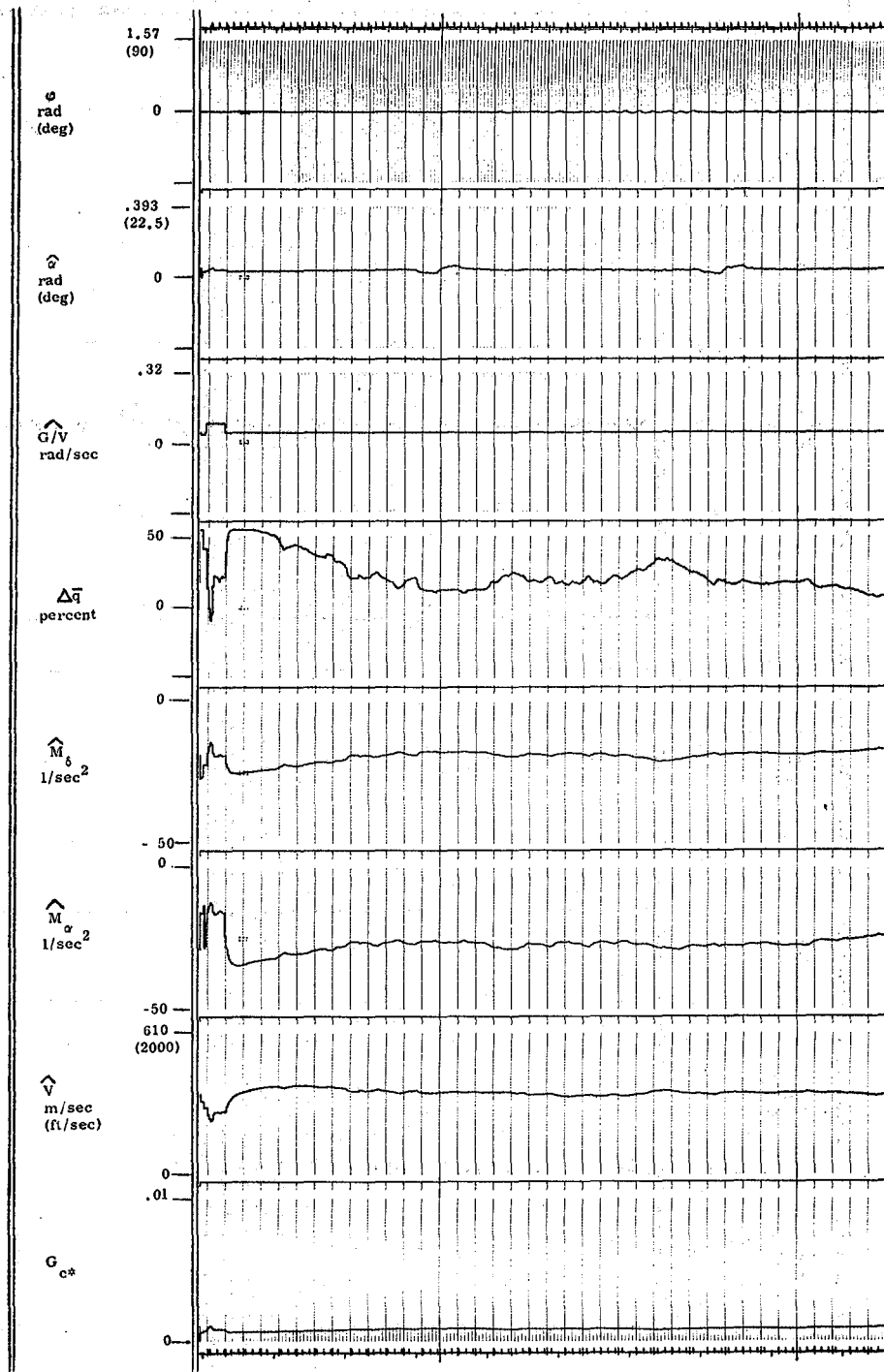


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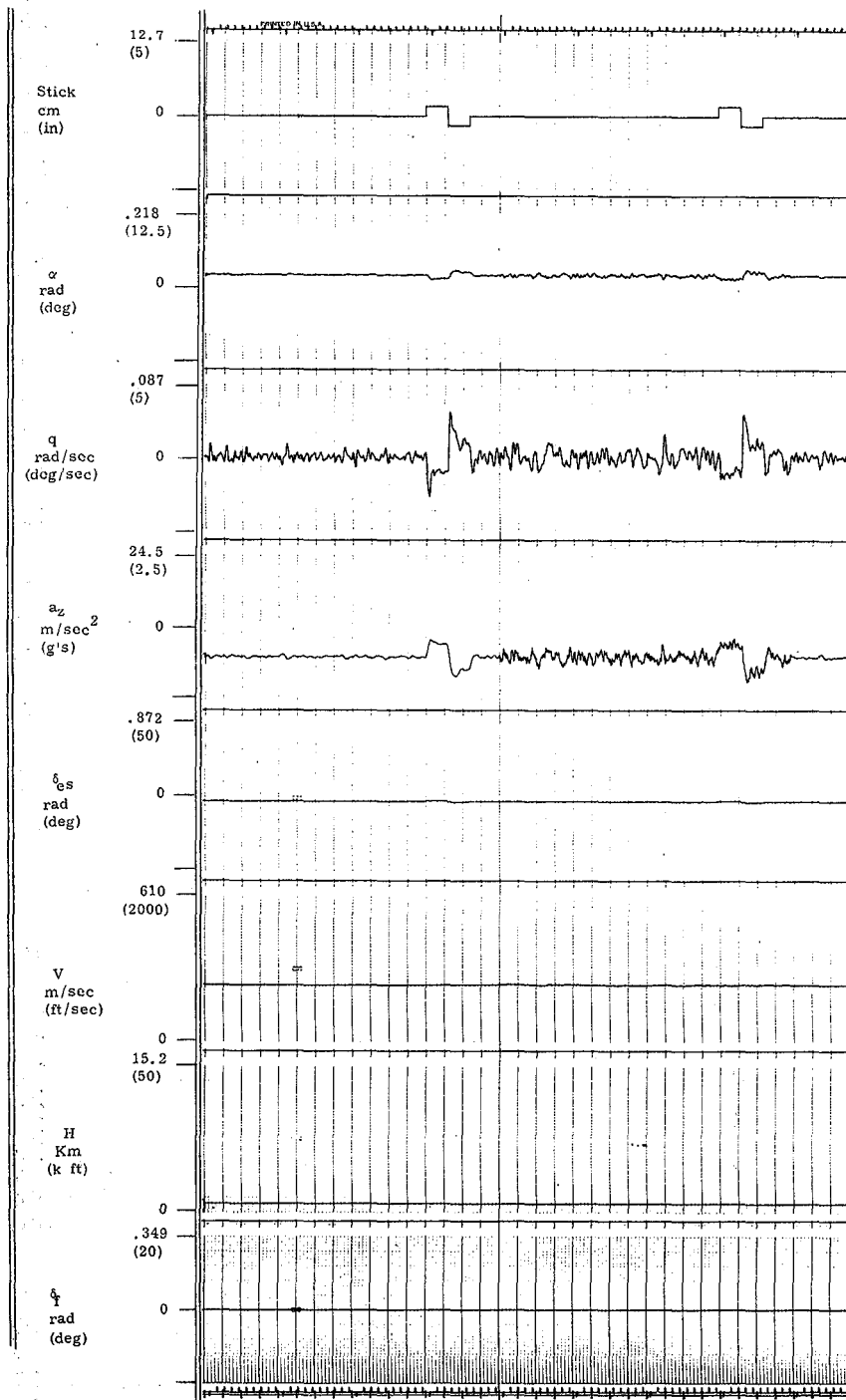


Figure 51. MLE accuracy test, FC10, no sensor noise.

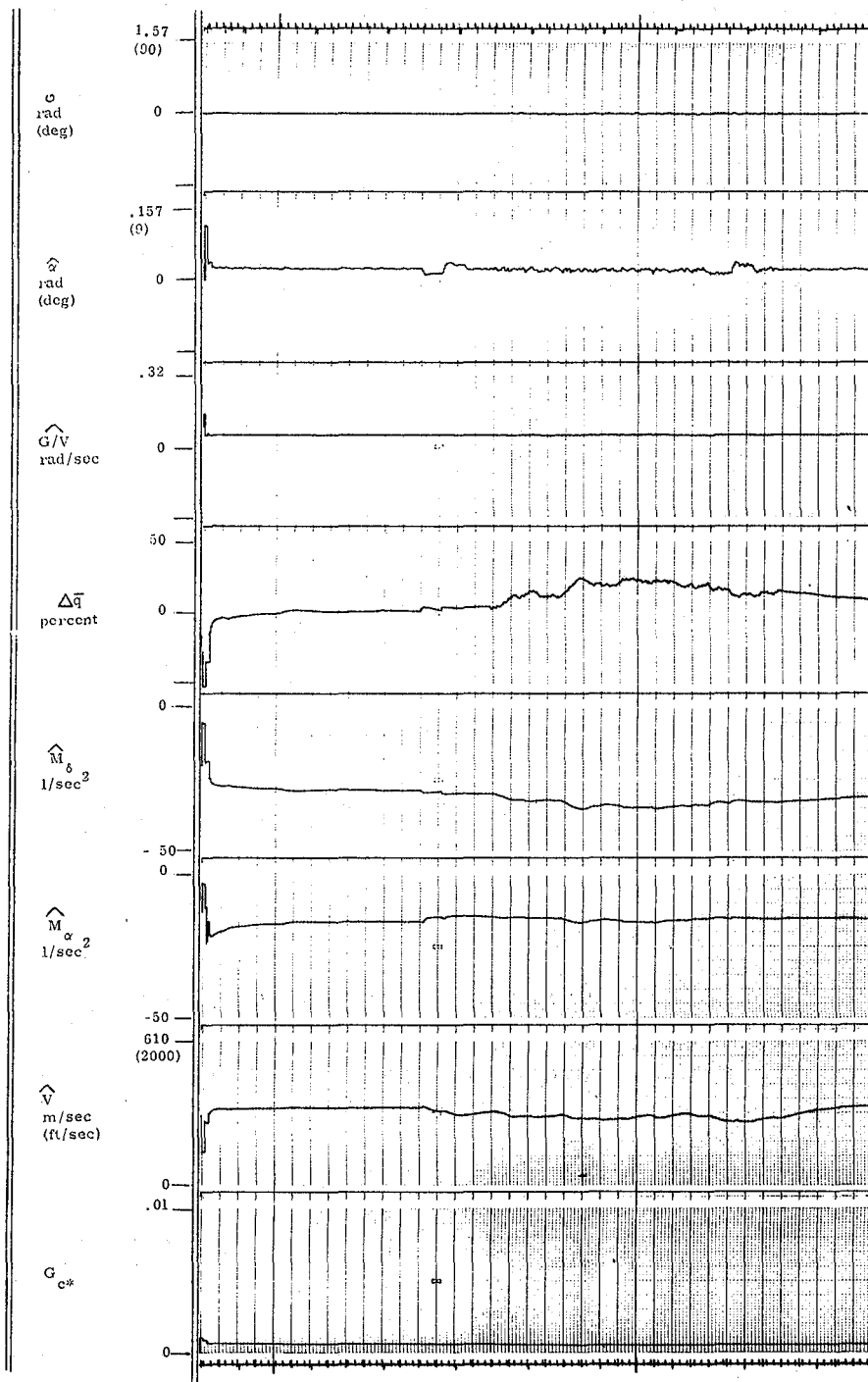


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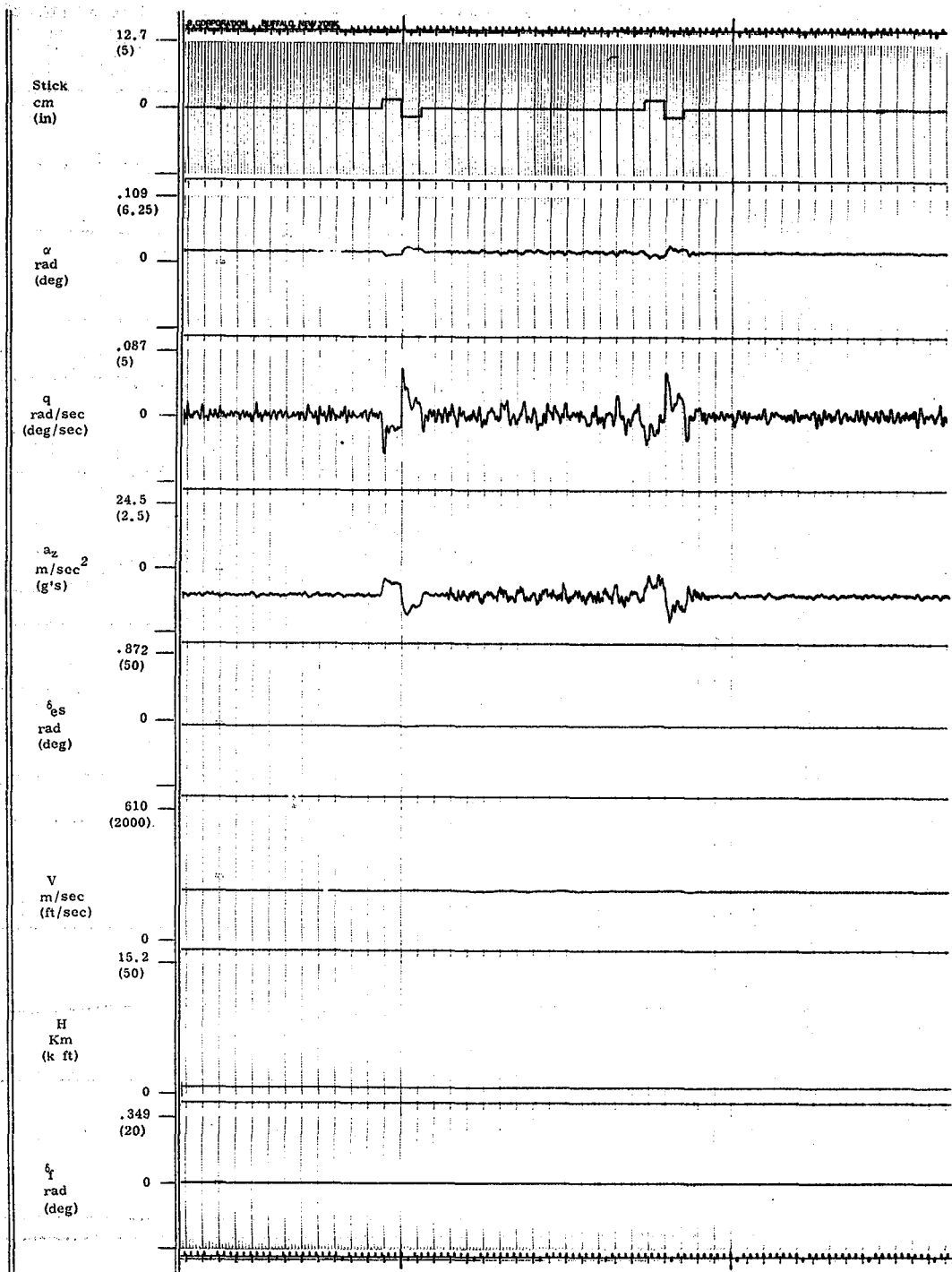


Figure 52. MLE accuracy test, FC10, with sensor noise.

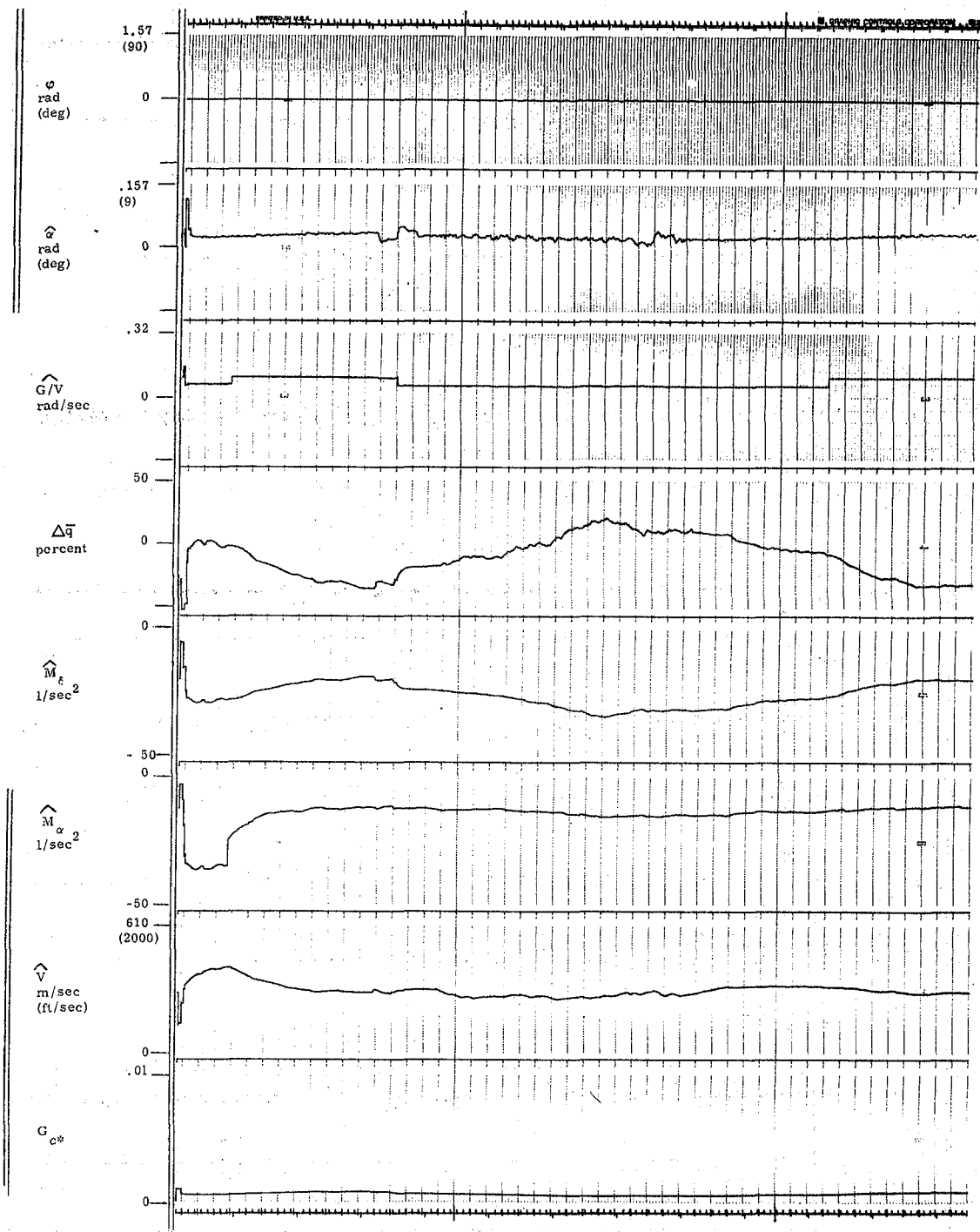


Figure 52. Concluded.

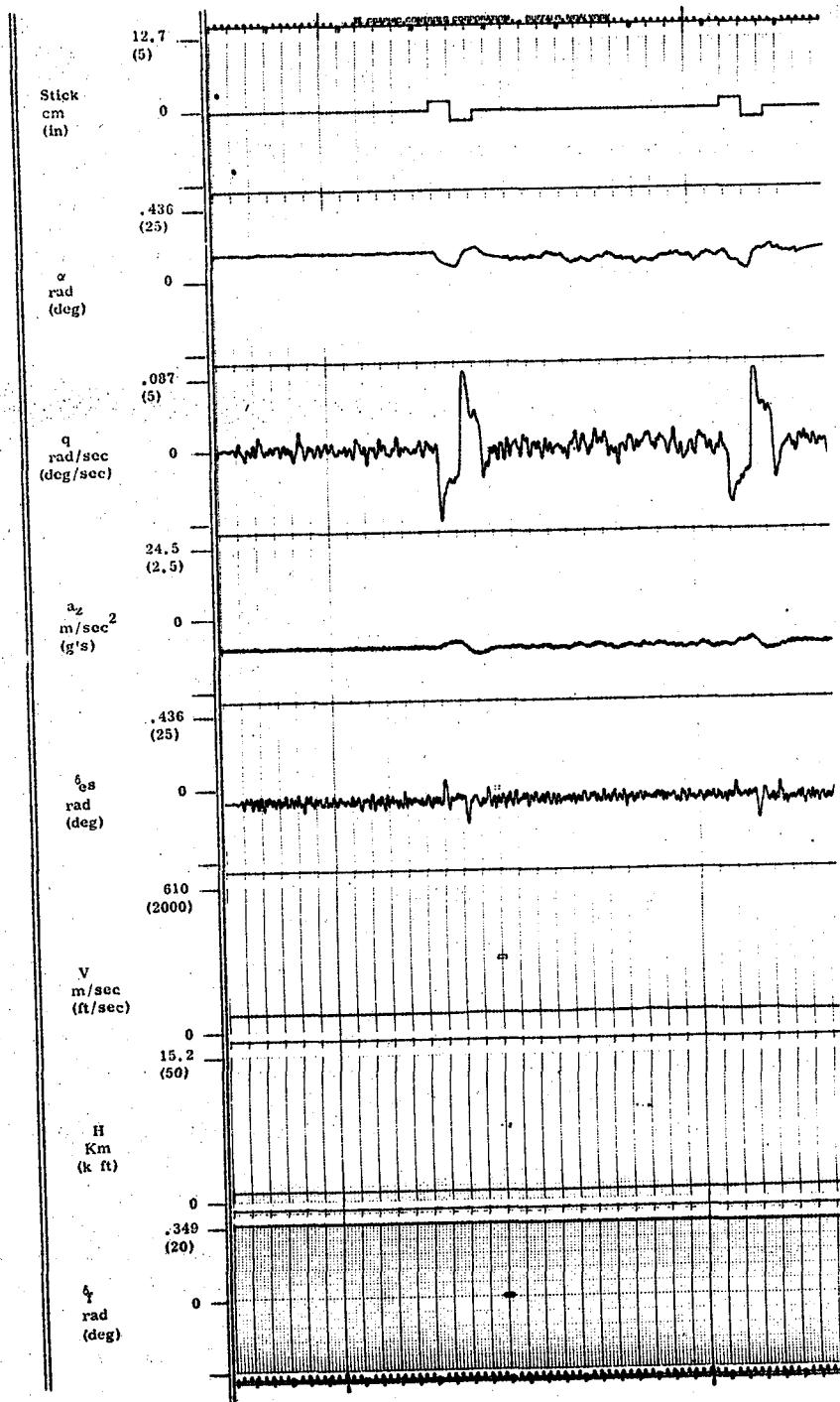


Figure 53. MLE accuracy test, FC17, no sensor noise.

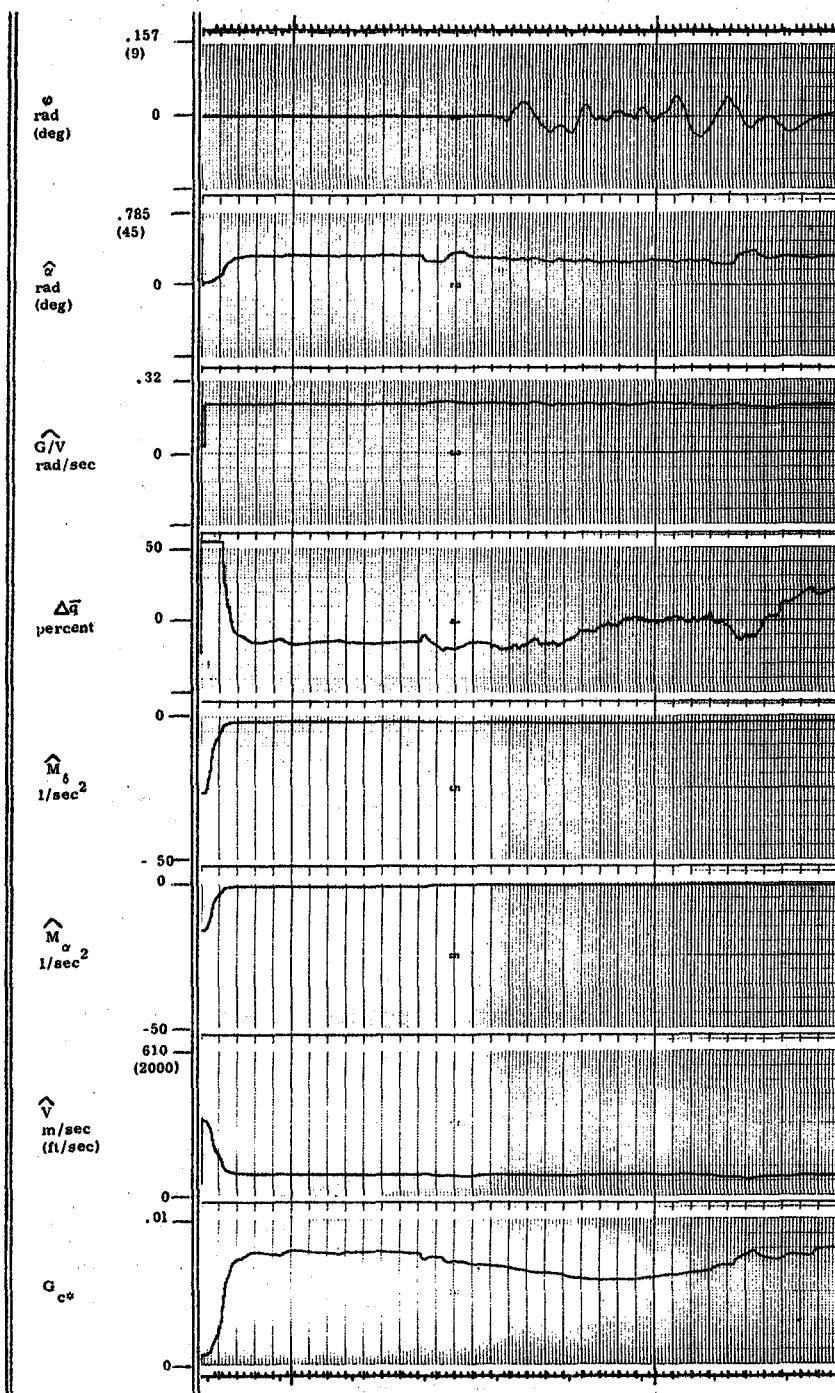


Figure 53., Concluded.

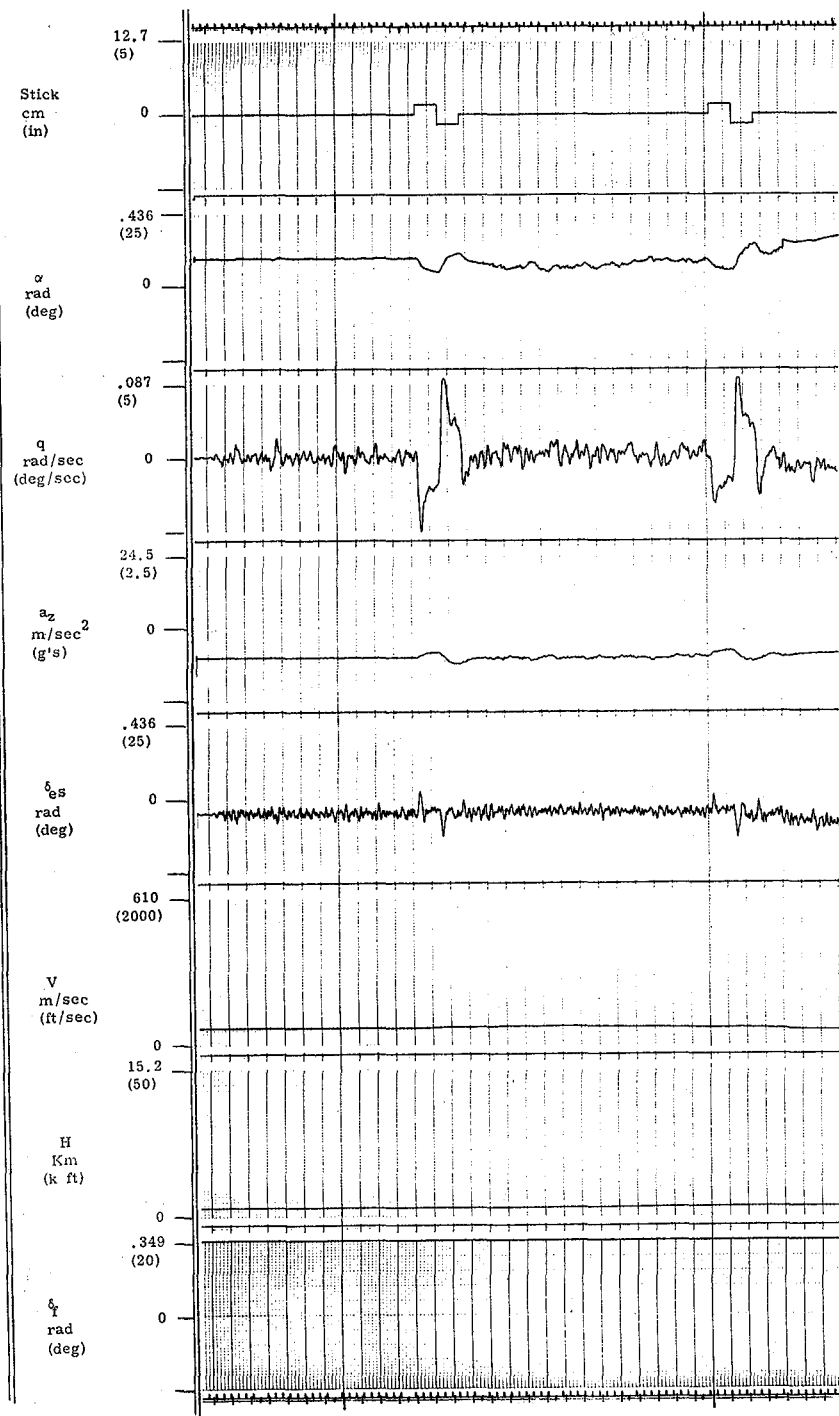


Figure 54. MLE accuracy test, FC17, with sensor noise.

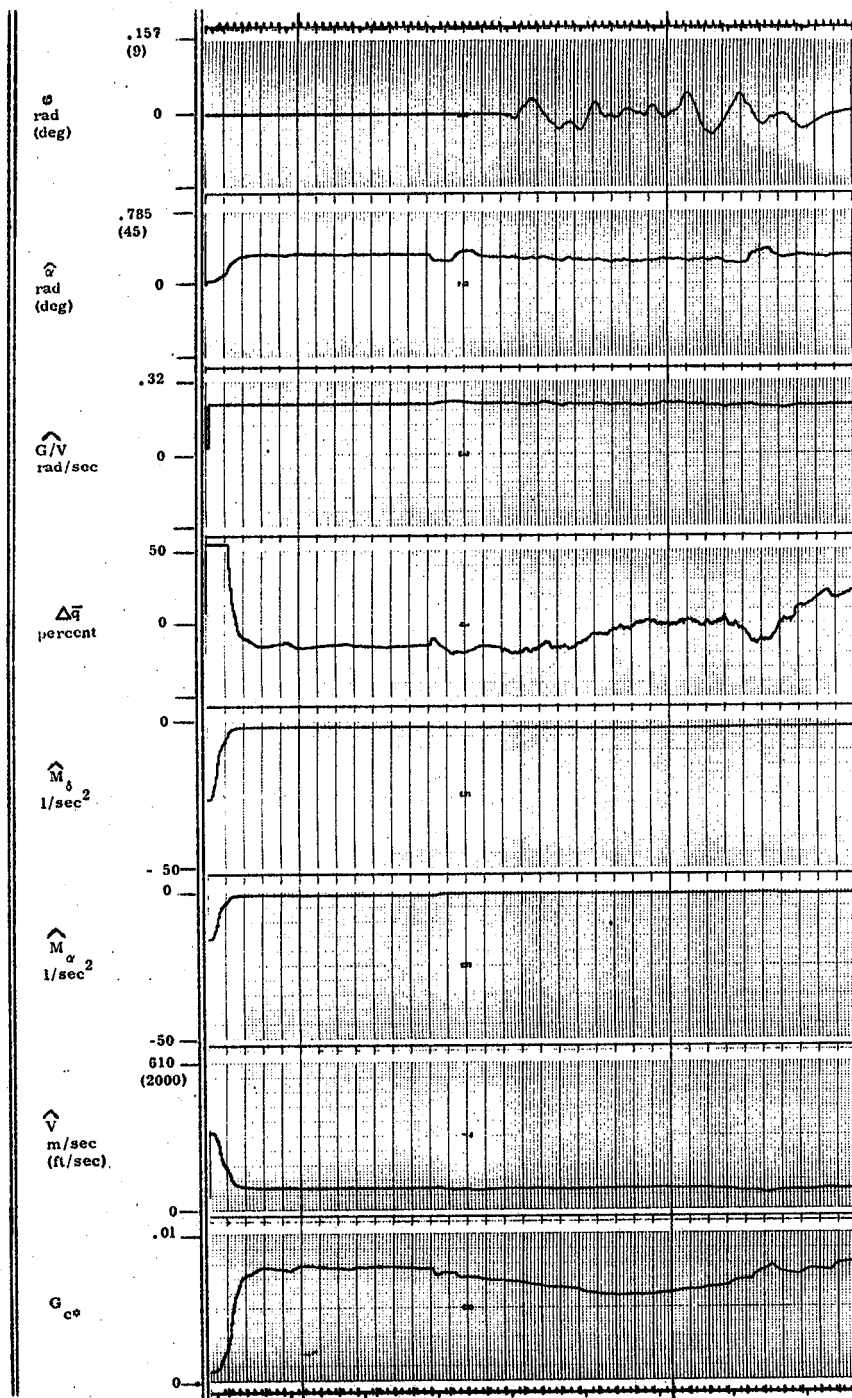


Figure 54. Concluded.

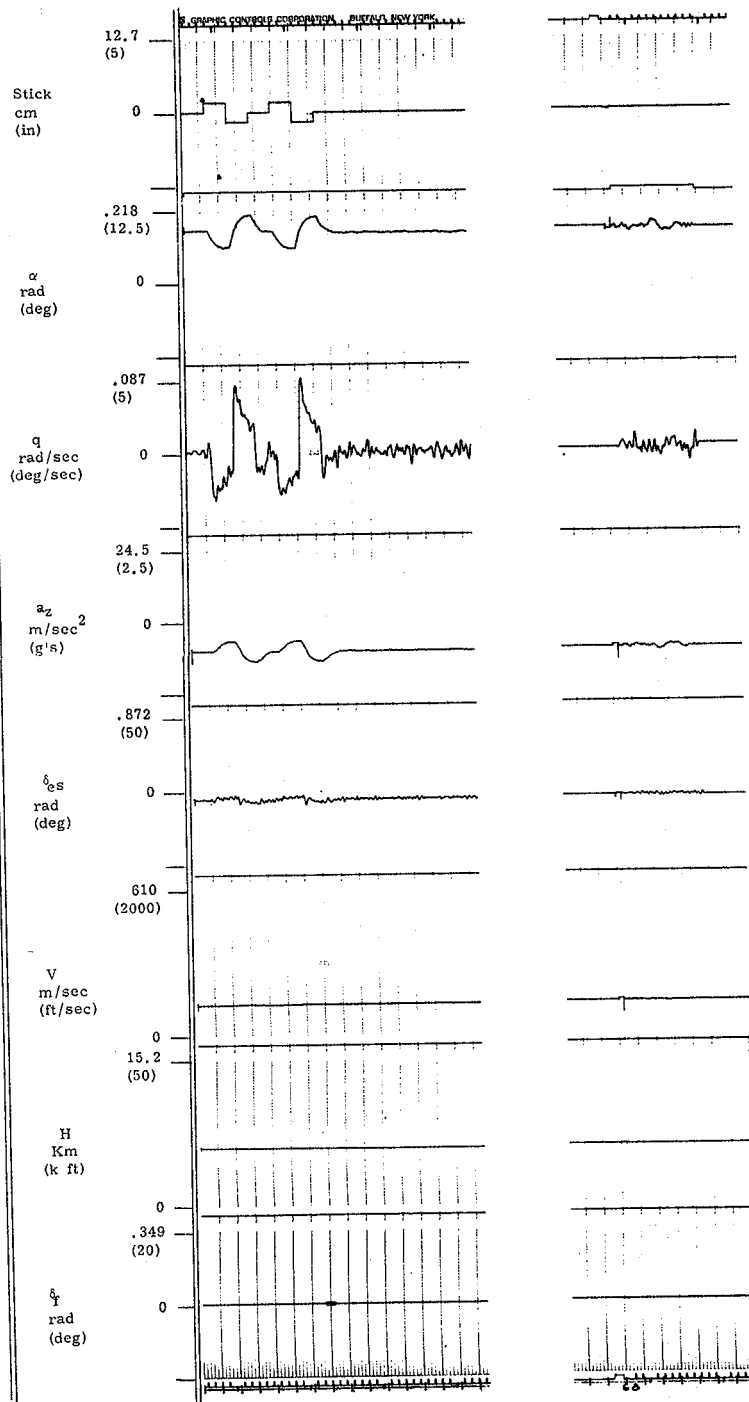


Figure 55. MLE convergence runs to FC5.

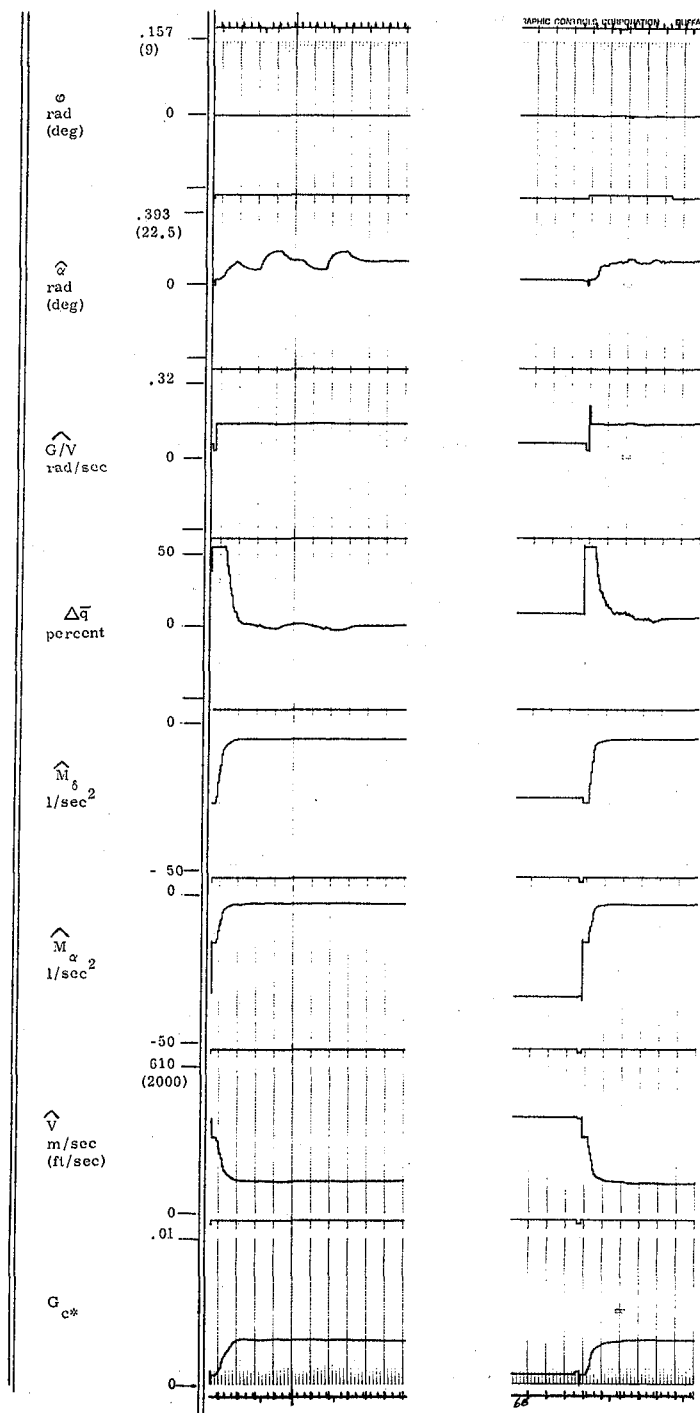


Figure 55. Concluded.

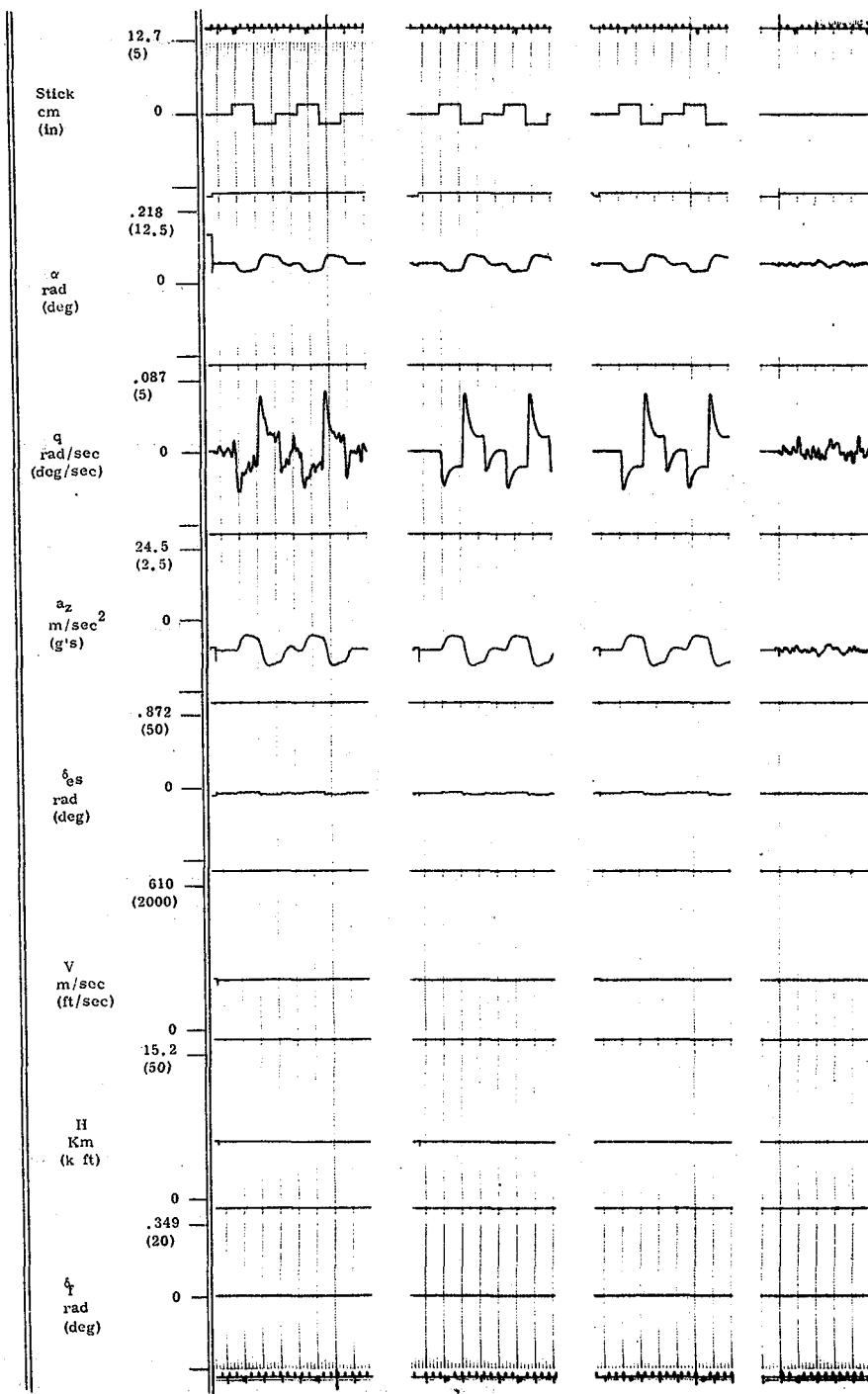


Figure 56. MLE convergence runs to FC1.

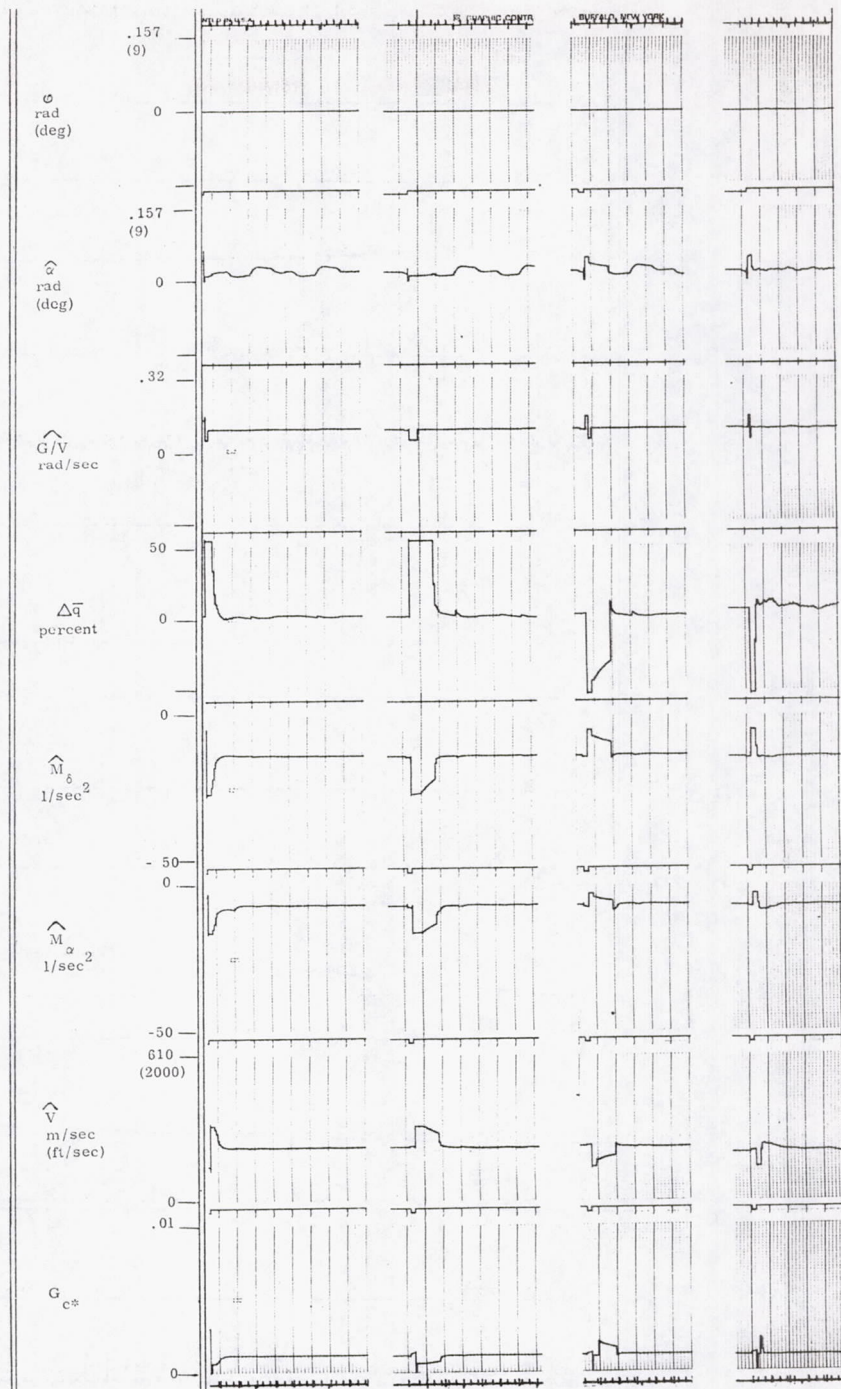


Figure 56. Concluded.

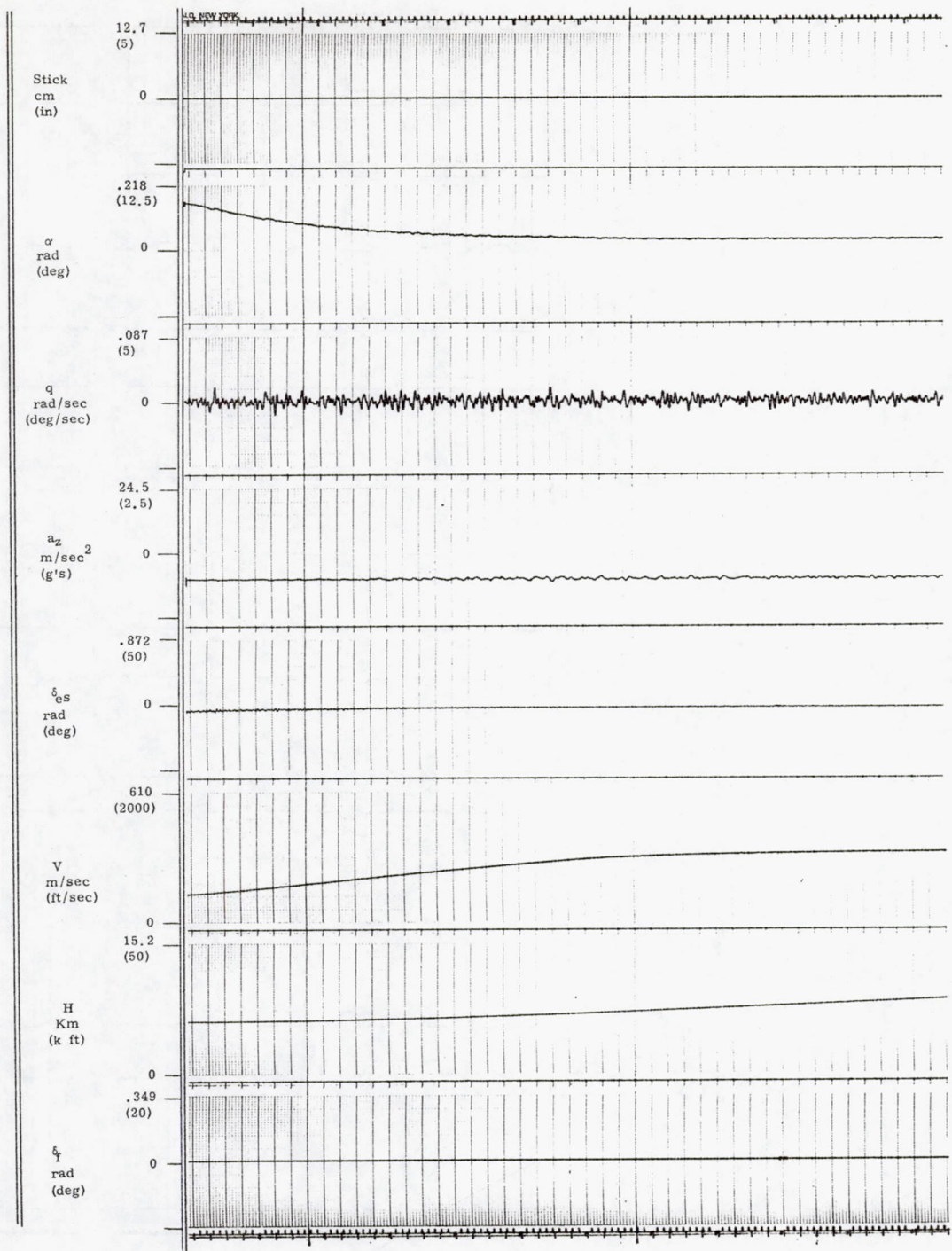


Figure 57. MLE tracking test, Transition 1.

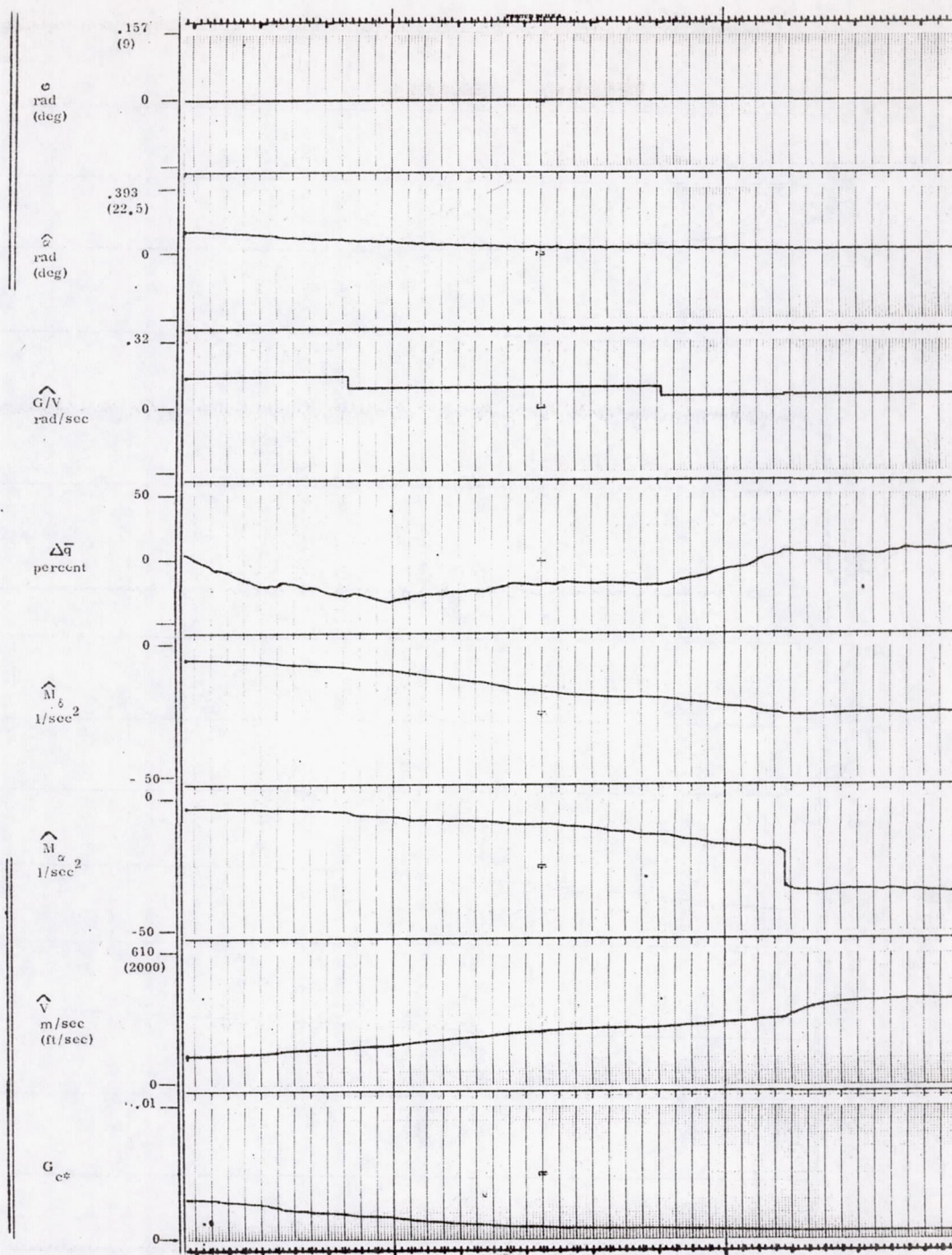


Figure 57. Concluded.

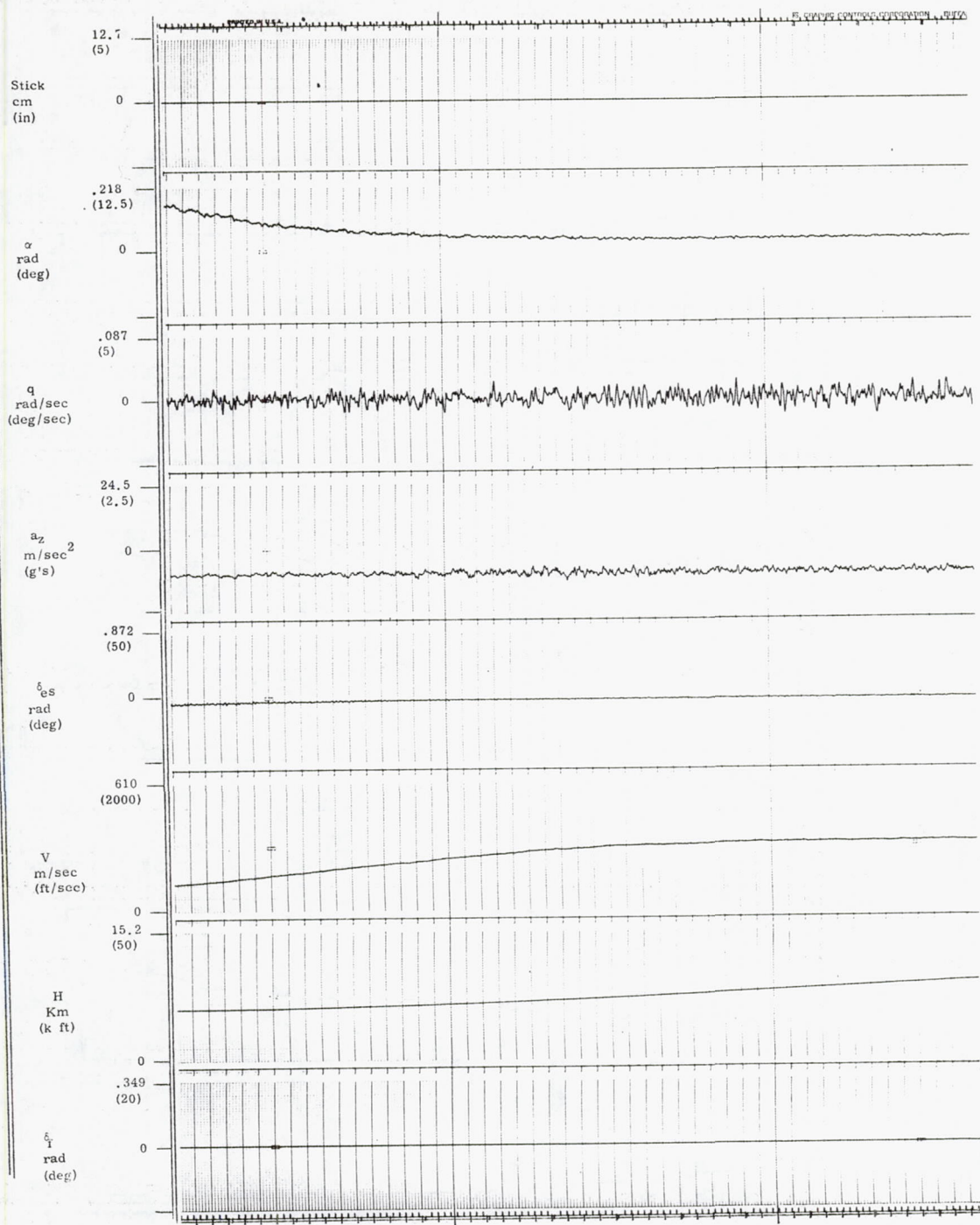


Figure 58. MLE tracking test, Transition 1 in 99 percent turbulence.

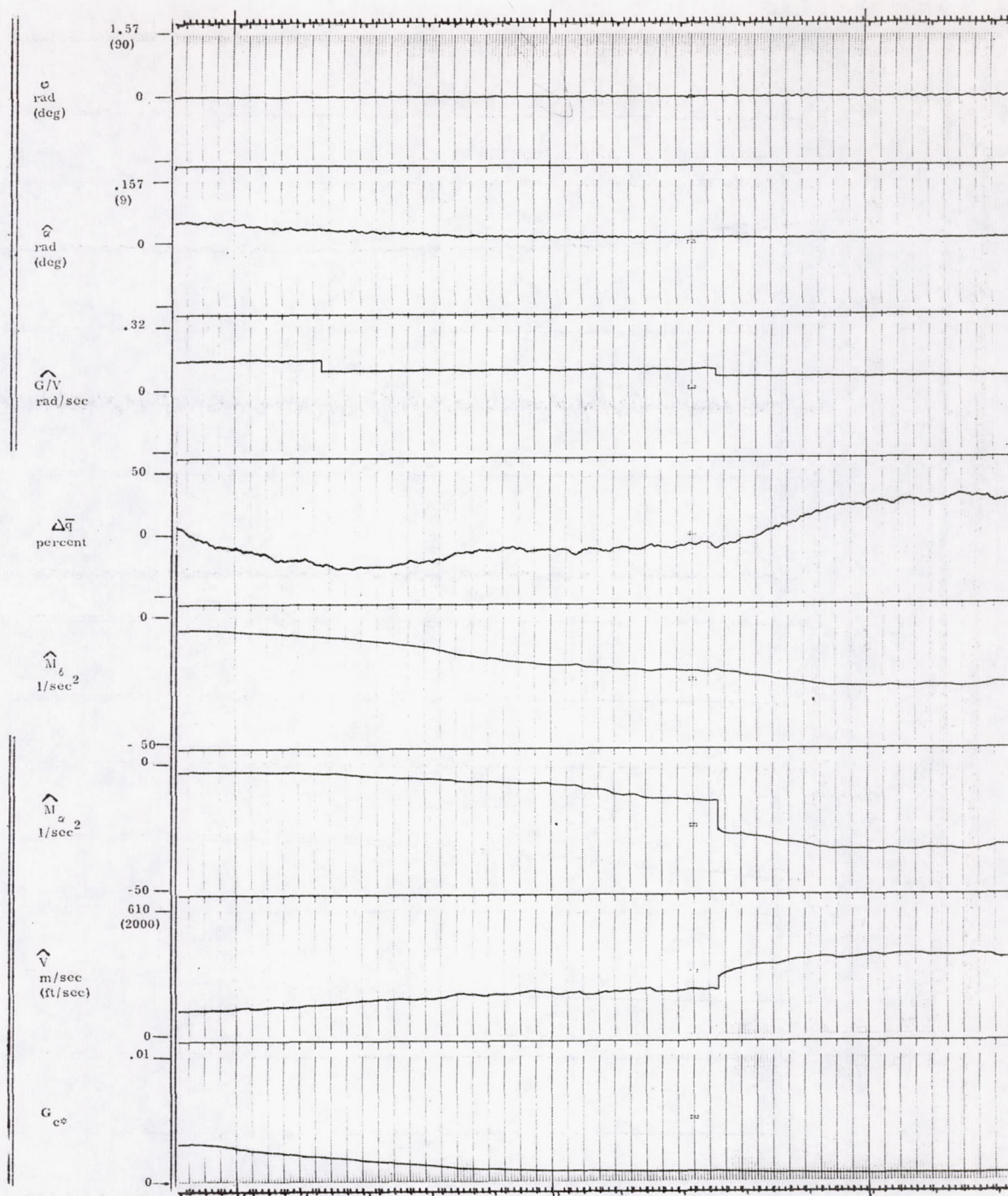


Figure 58. Concluded.

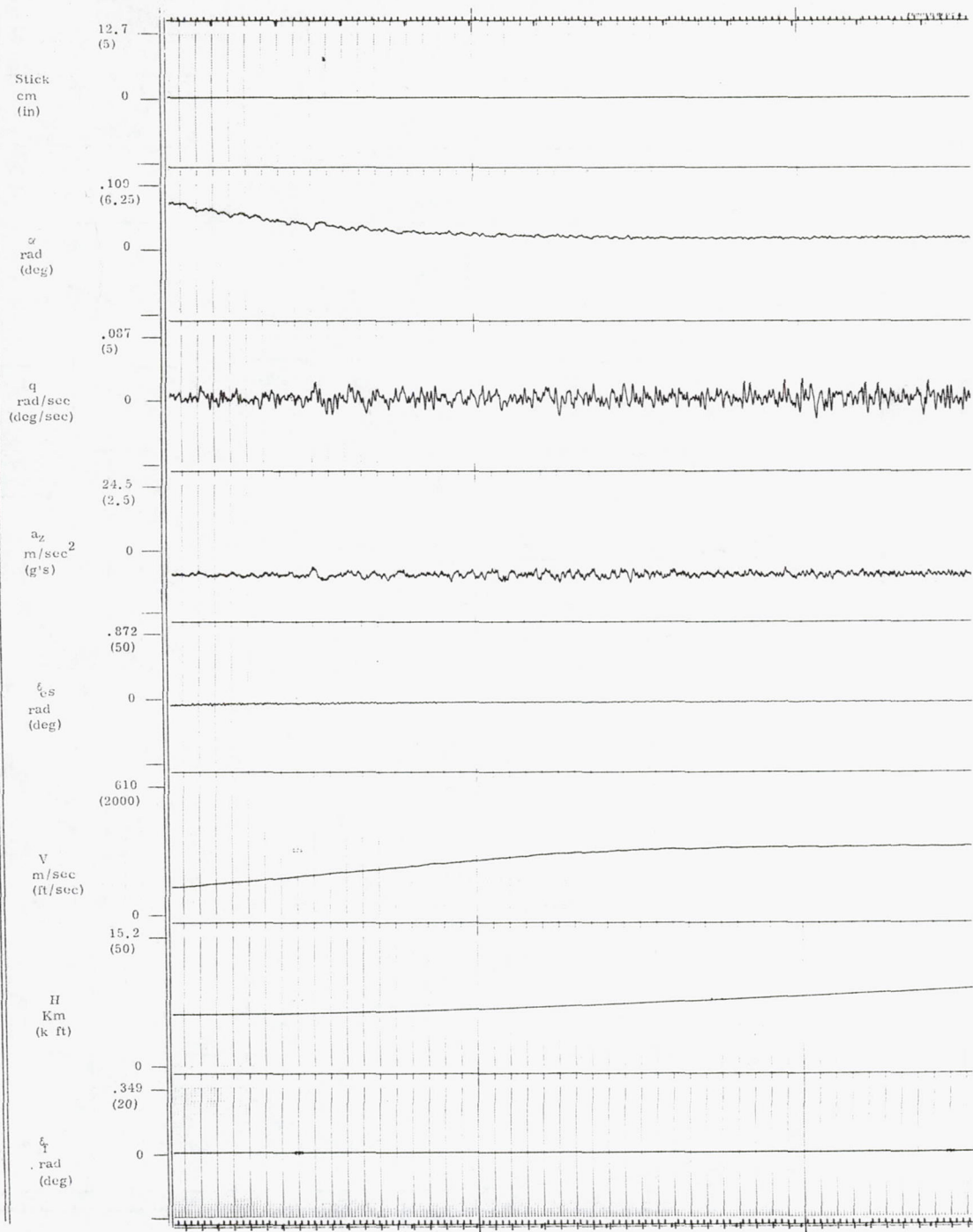


Figure 59. MLE tracking test, Transition 1 in 99 percent turbulence plus sensor noise.

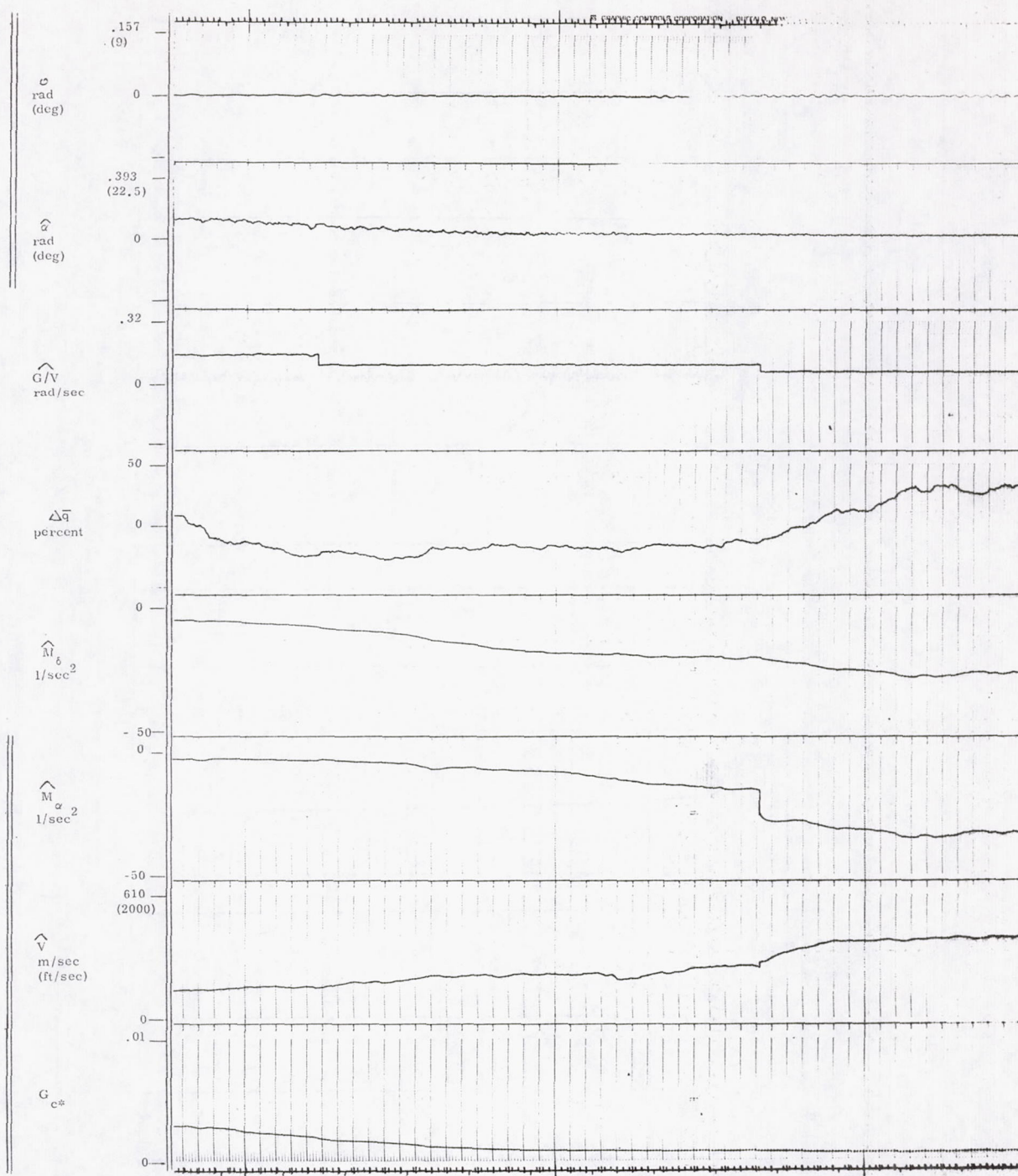


Figure 59. Concluded.

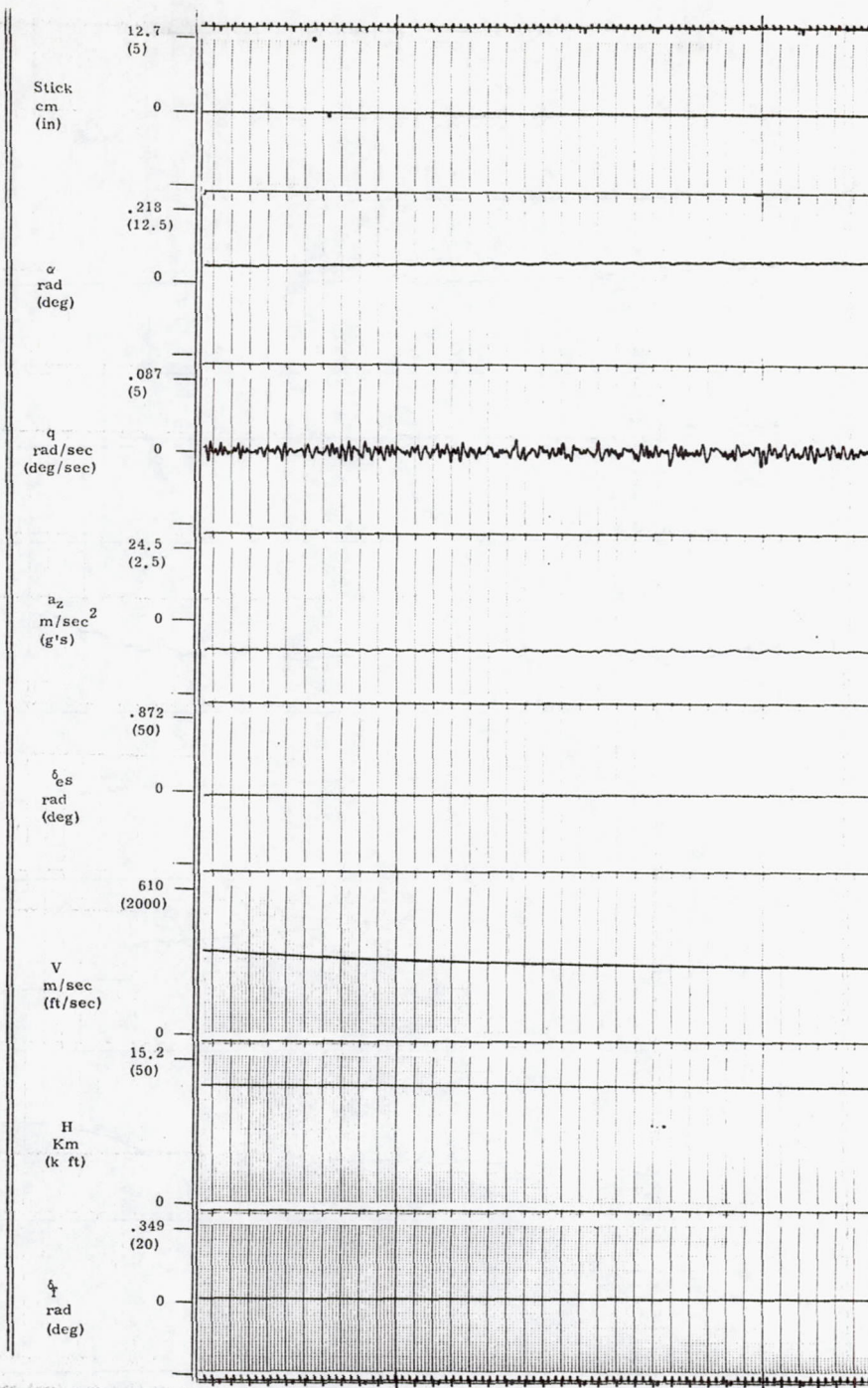


Figure 60. MLE tracking test, Transition 2.

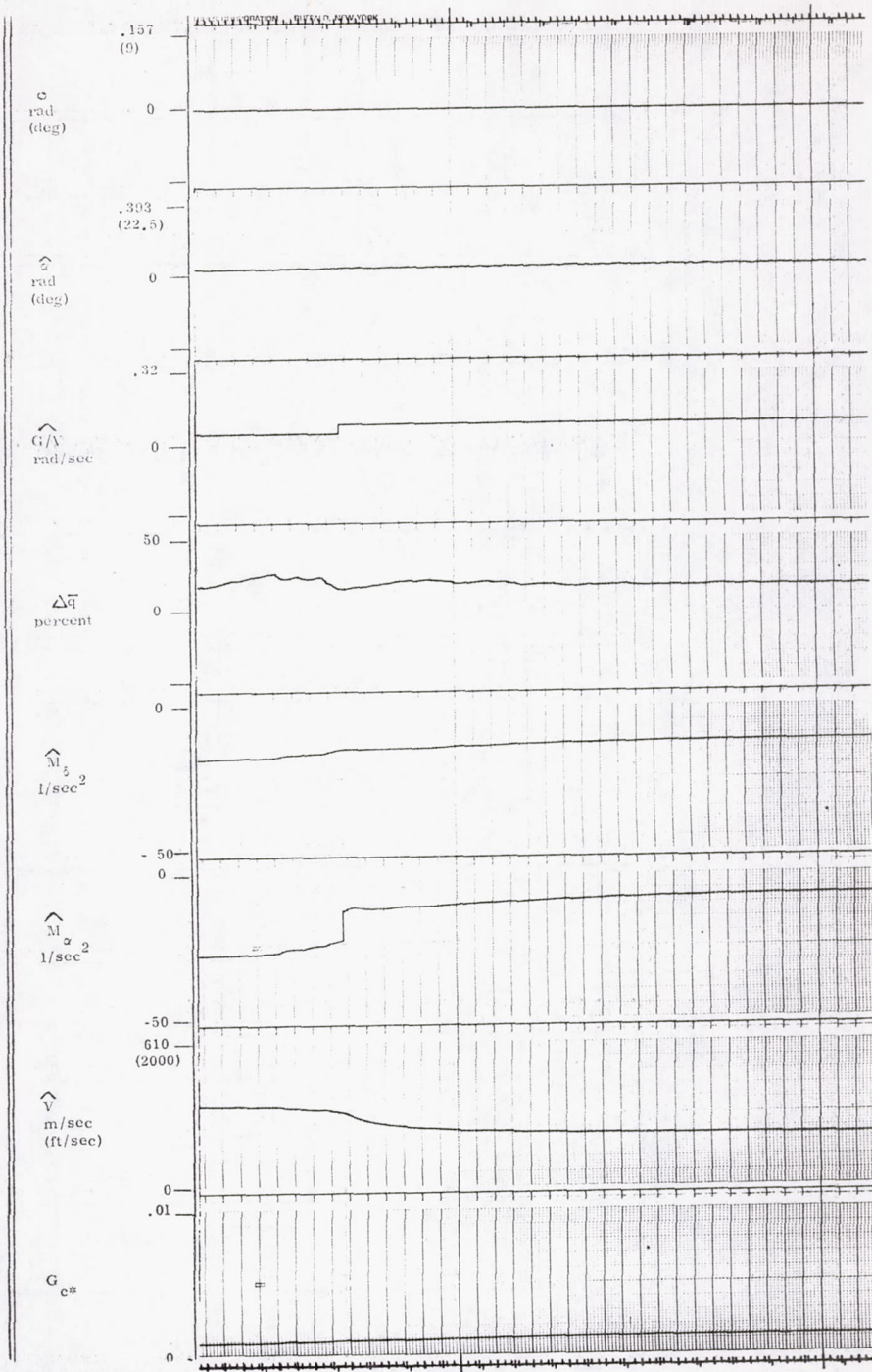


Figure 60. Concluded.

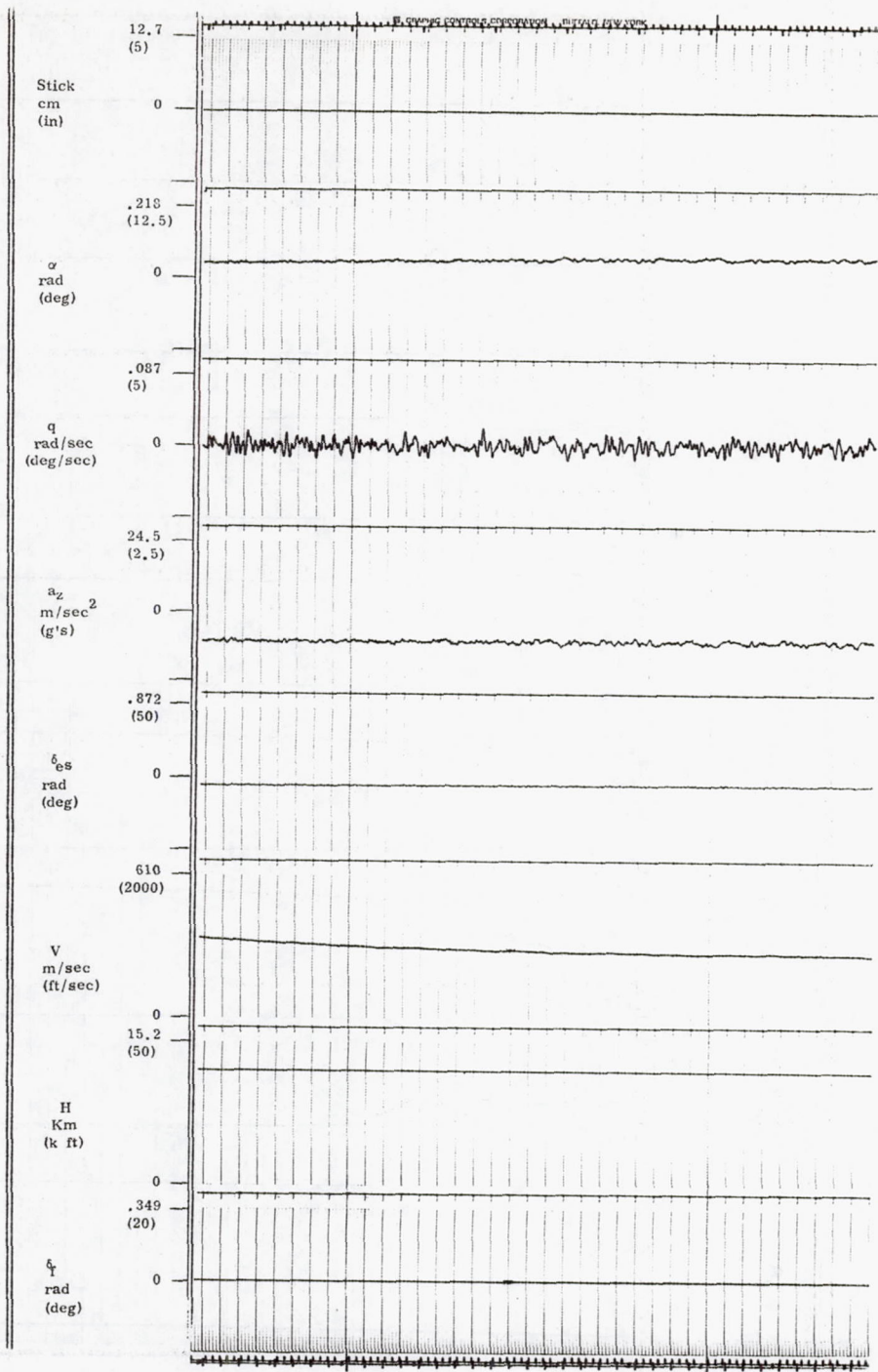


Figure 61. MLE tracking test, Transition 2 in 99 percent turbulence.

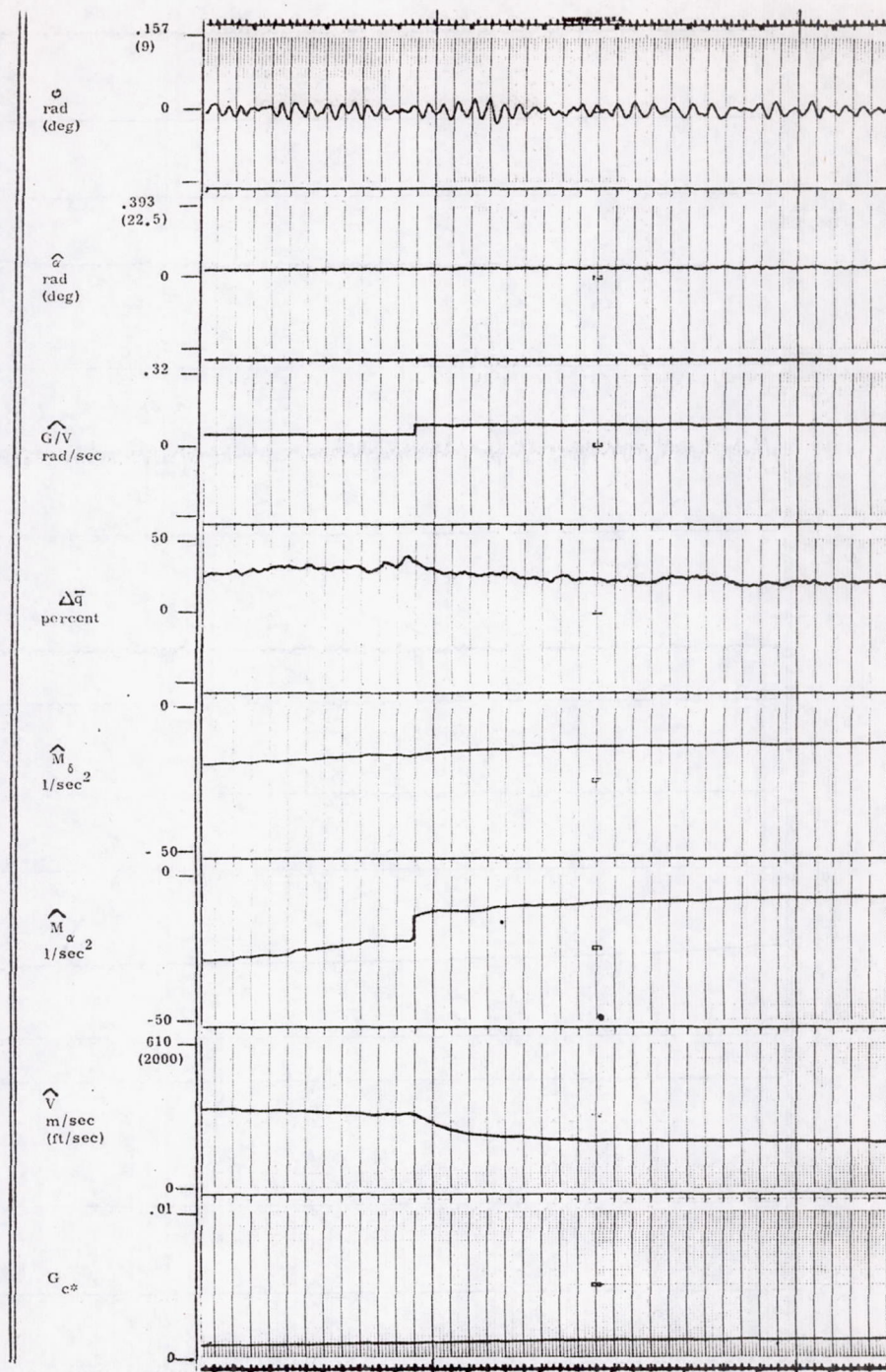


Figure 61. Concluded.

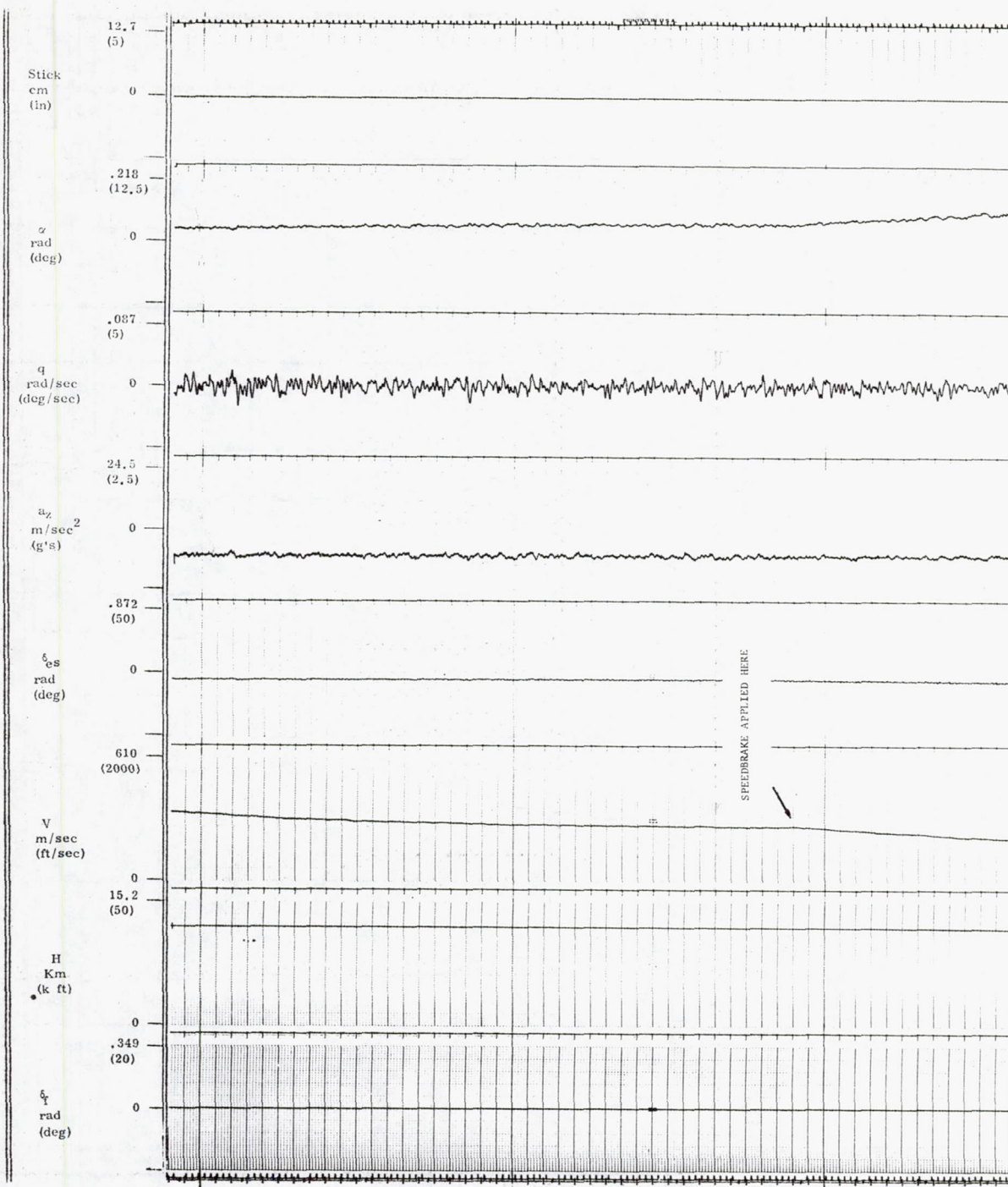


Figure 62. MLE tracking test, Transition 2 in 99 percent turbulence plus sensor noise.

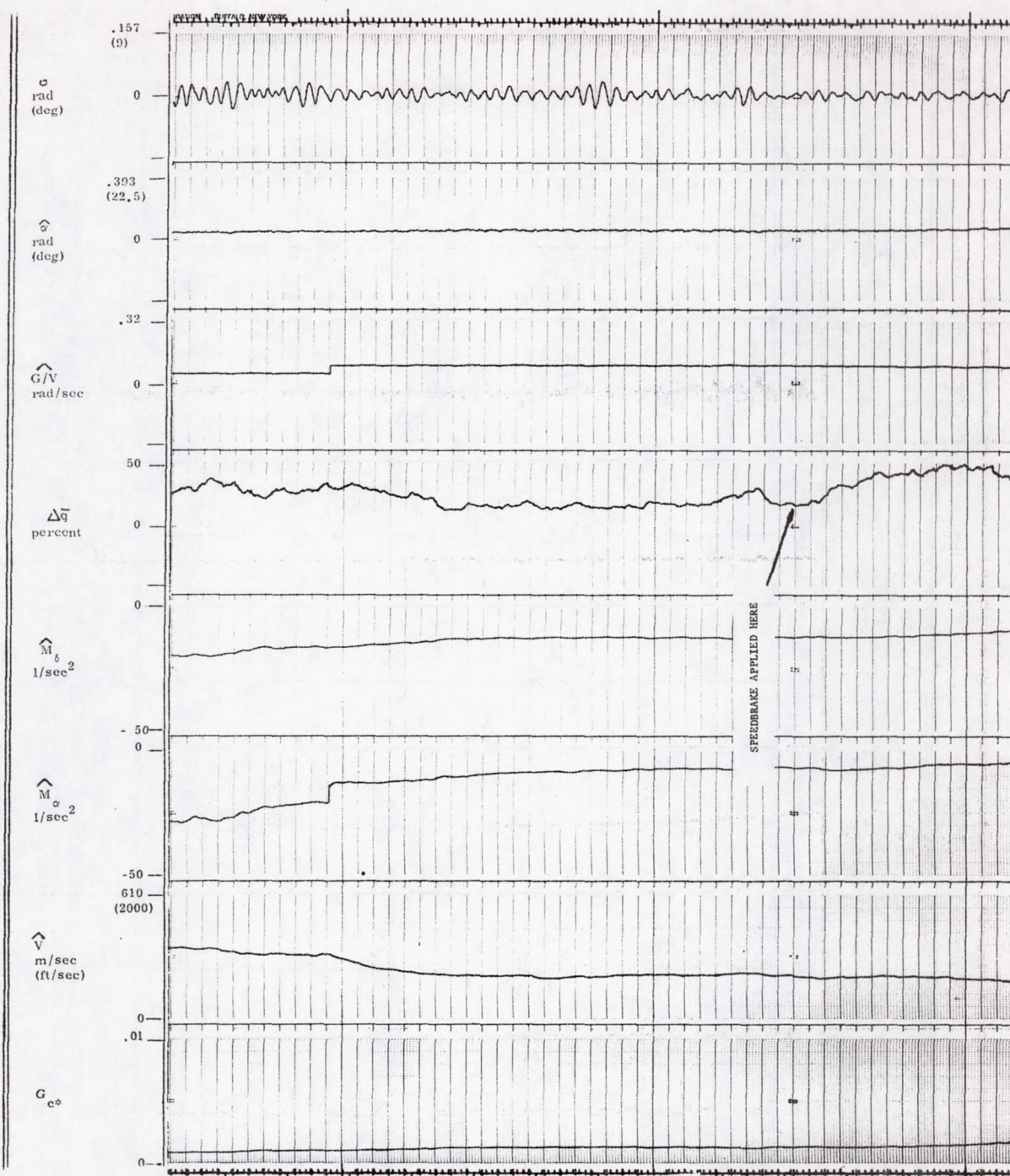


Figure 62. Concluded.

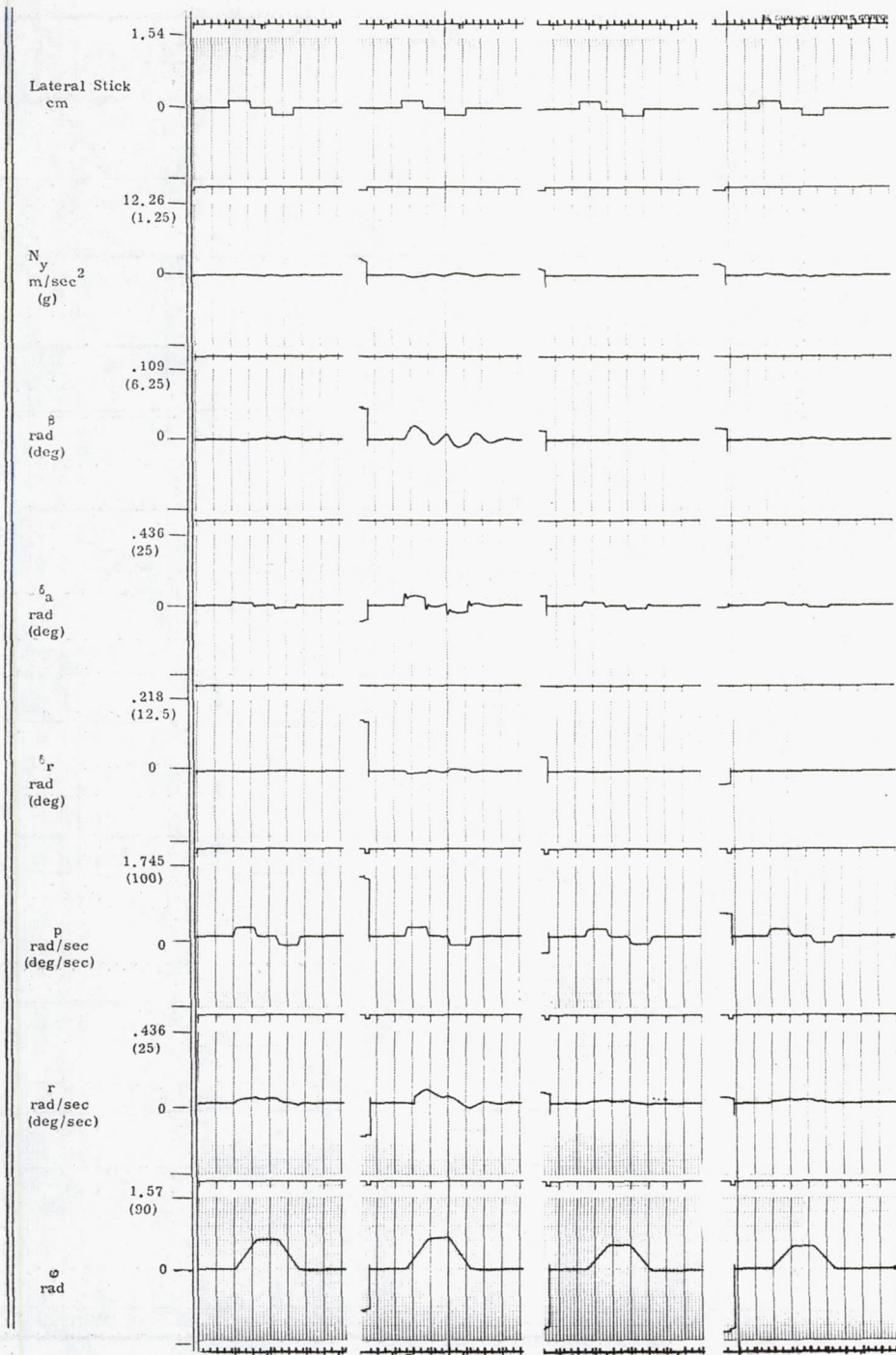


Figure 63. MLE lateral maneuvers.

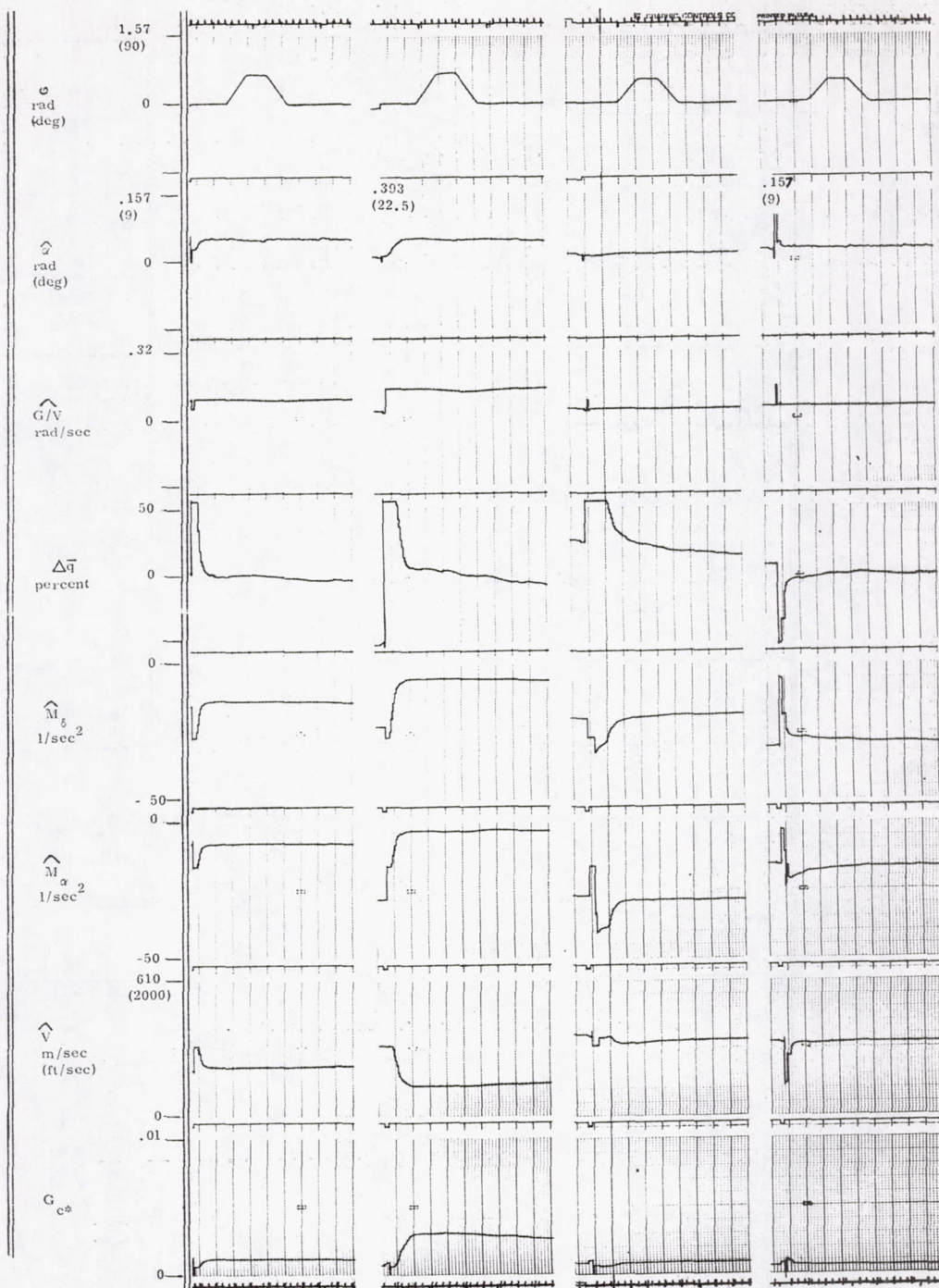


Figure 63. Concluded.

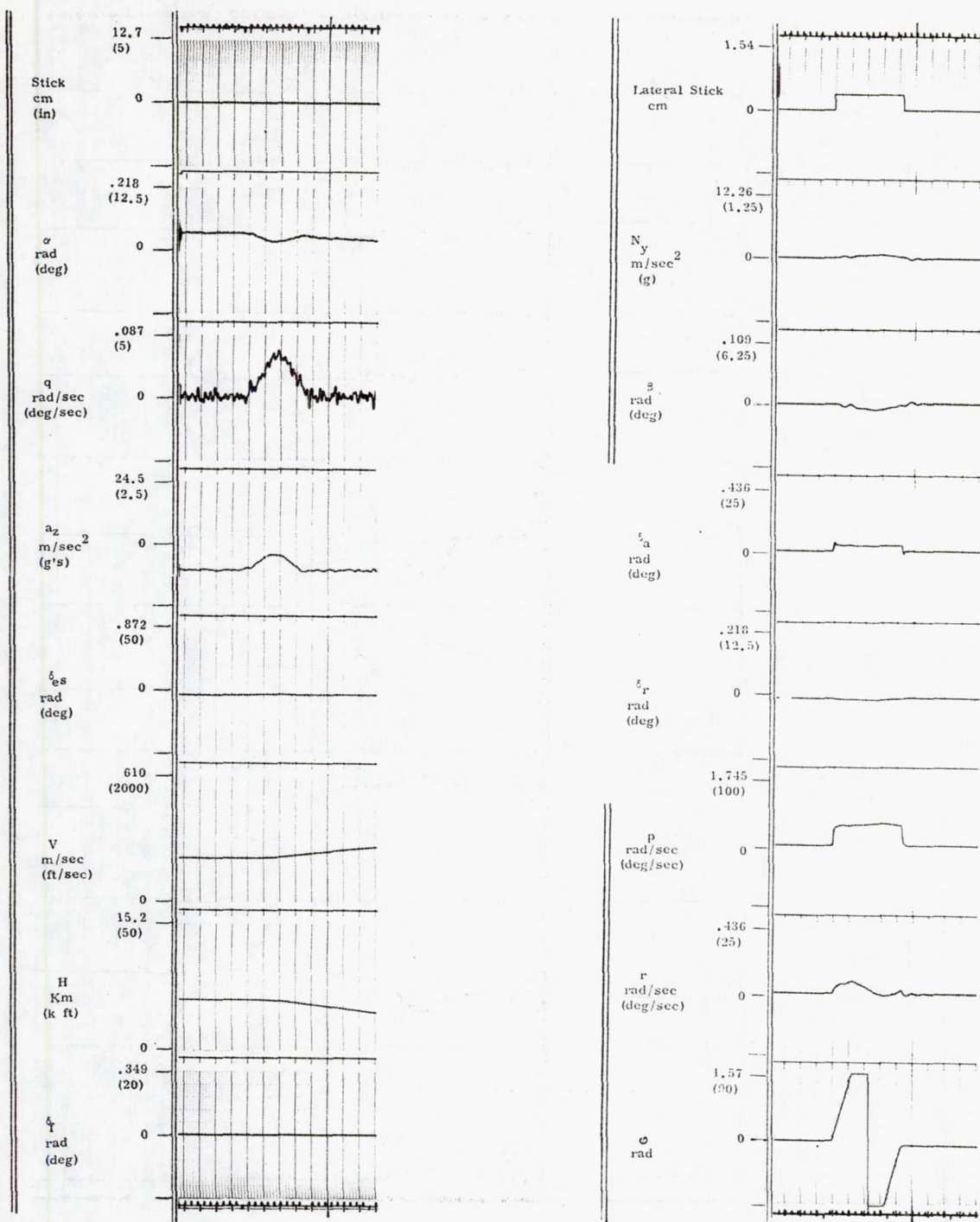


Figure 64. MLE roll over at FC1.

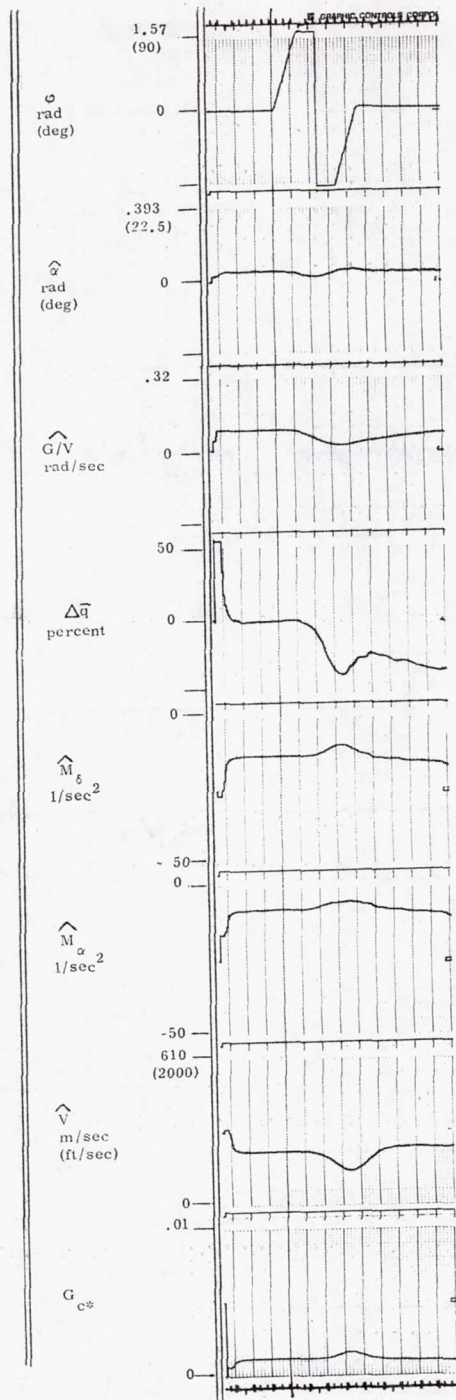


Figure 64. Concluded.

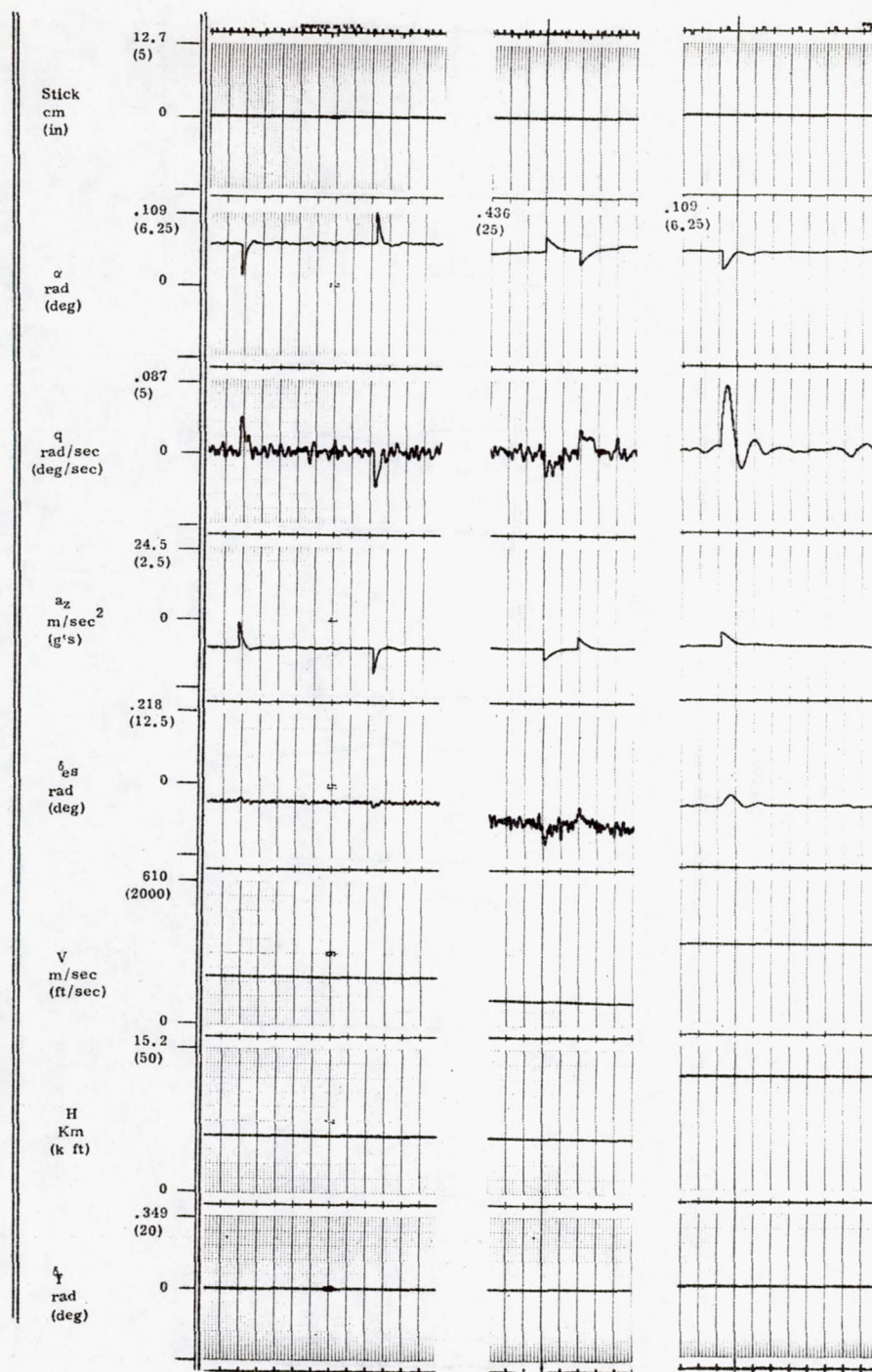


Figure 65. MLE α gusts.

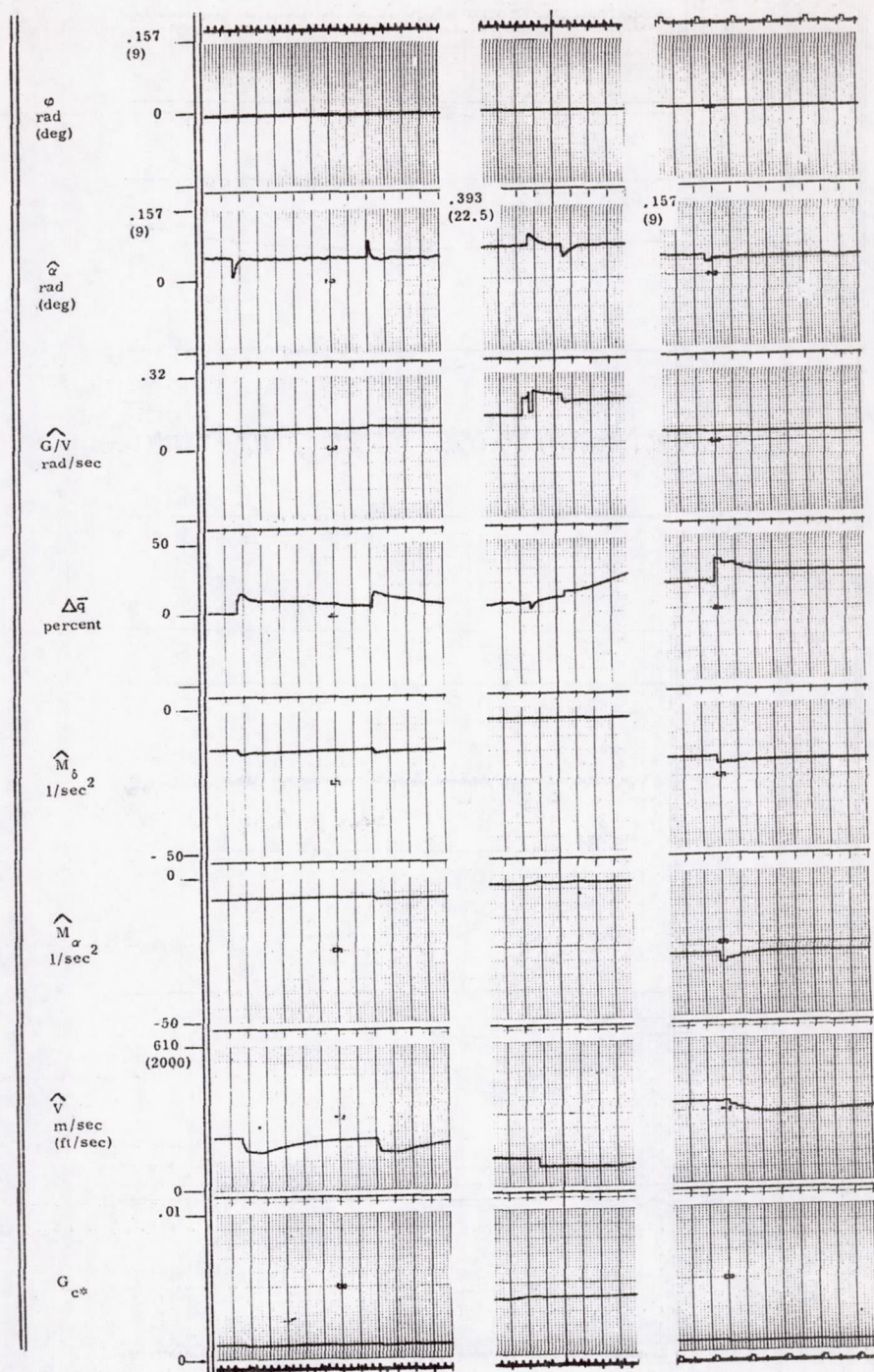


Figure 65. Concluded.

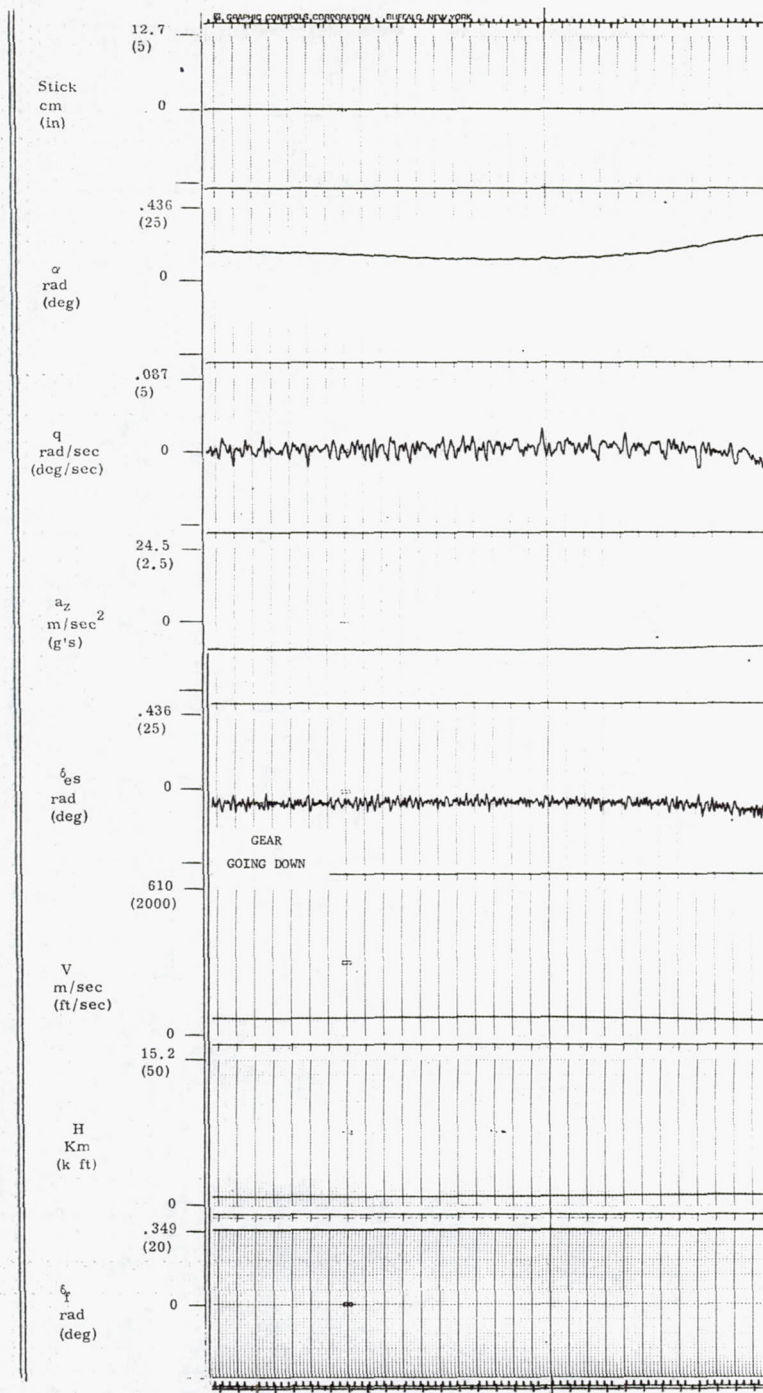


Figure 66. MLE gear-down transients.

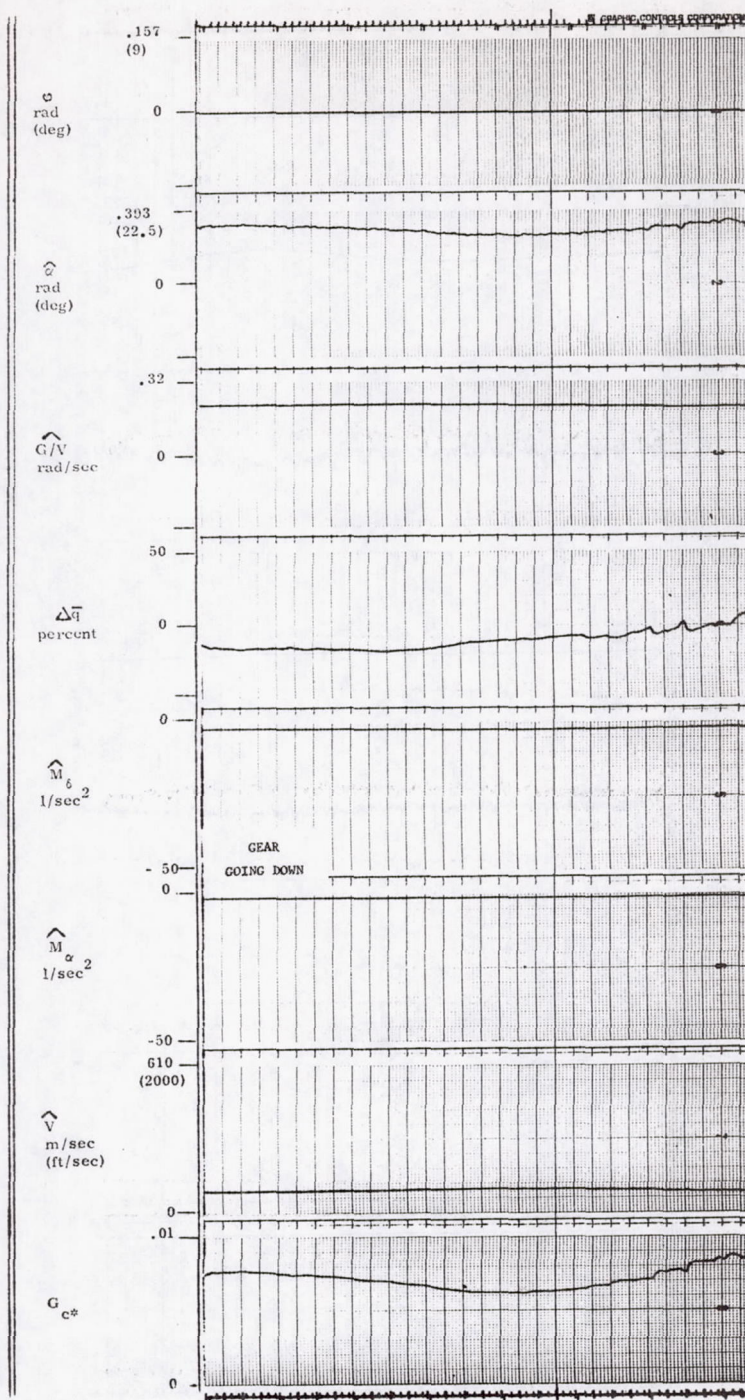


Figure 66. Concluded.

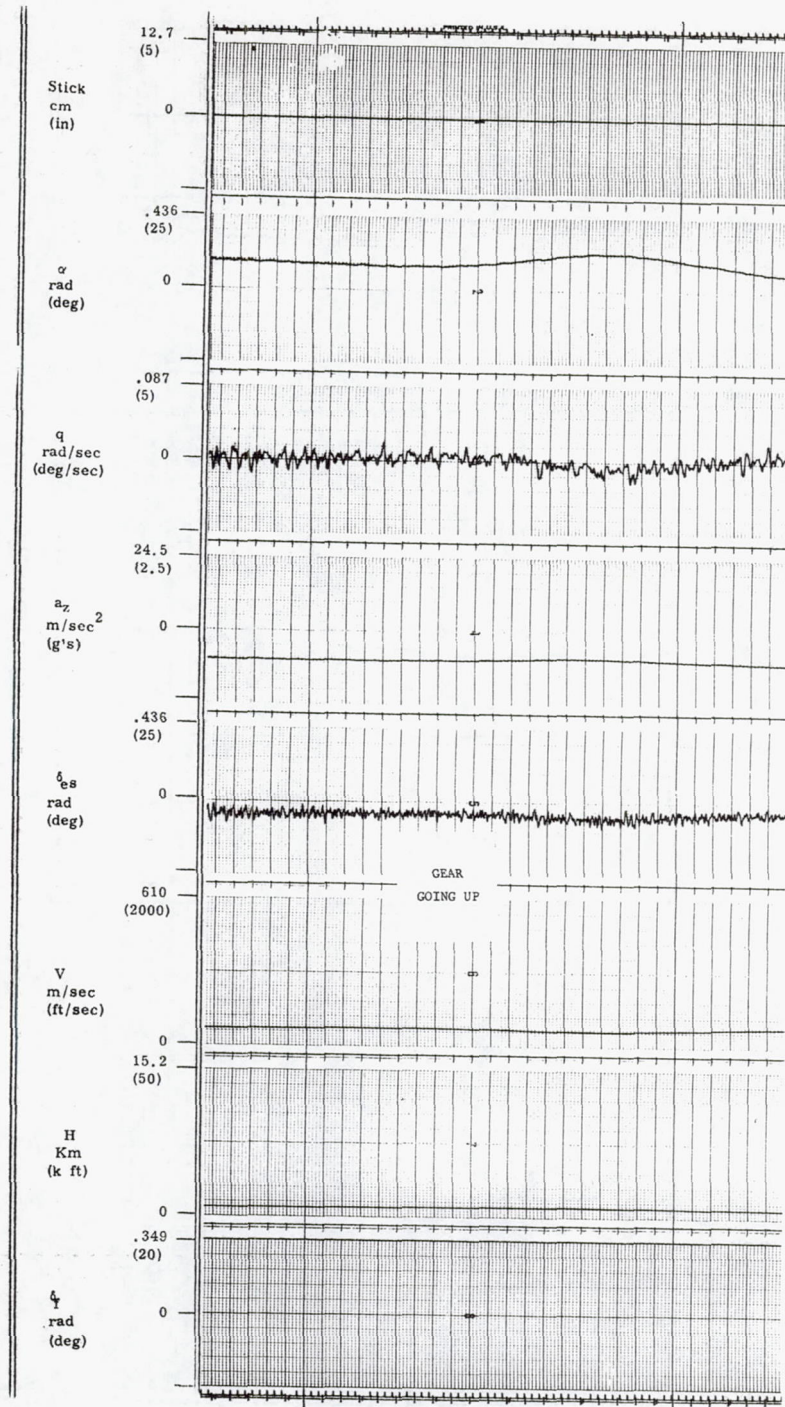


Figure 67. MLE gear-up transient.

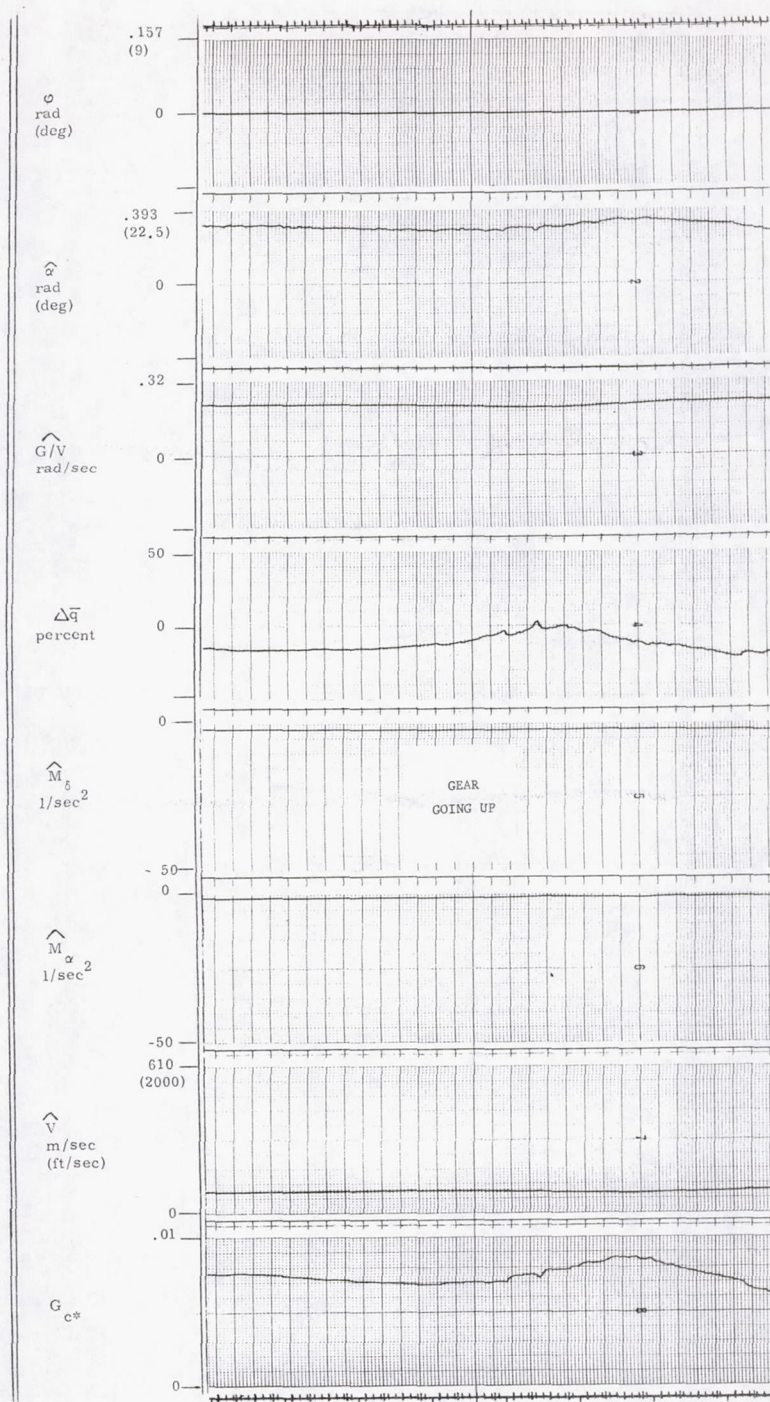


Figure 67. Concluded.

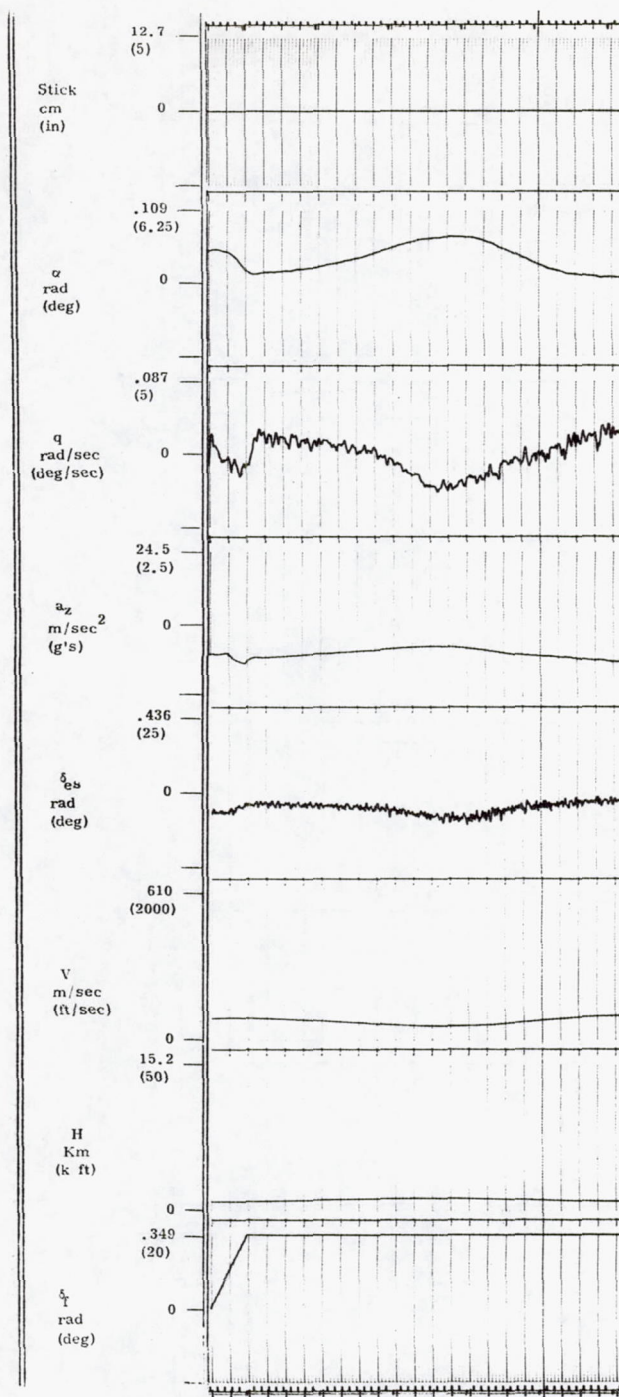


Figure 68. MLE wing-up transient.

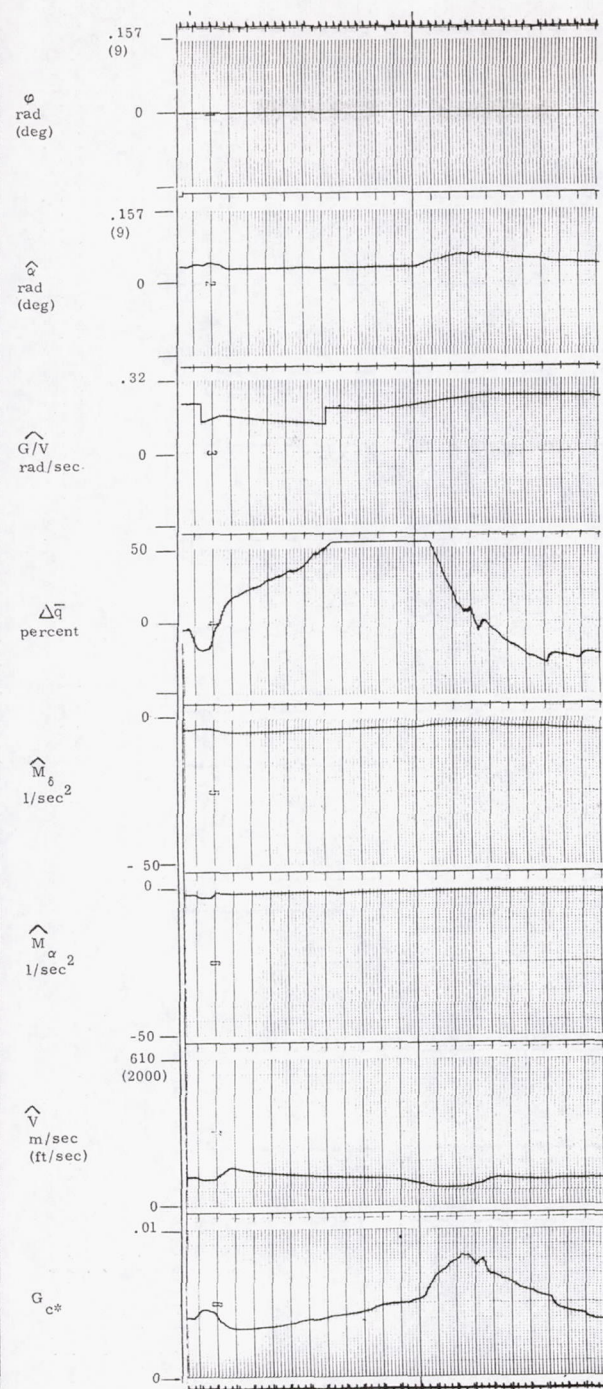


Figure 68. Concluded.

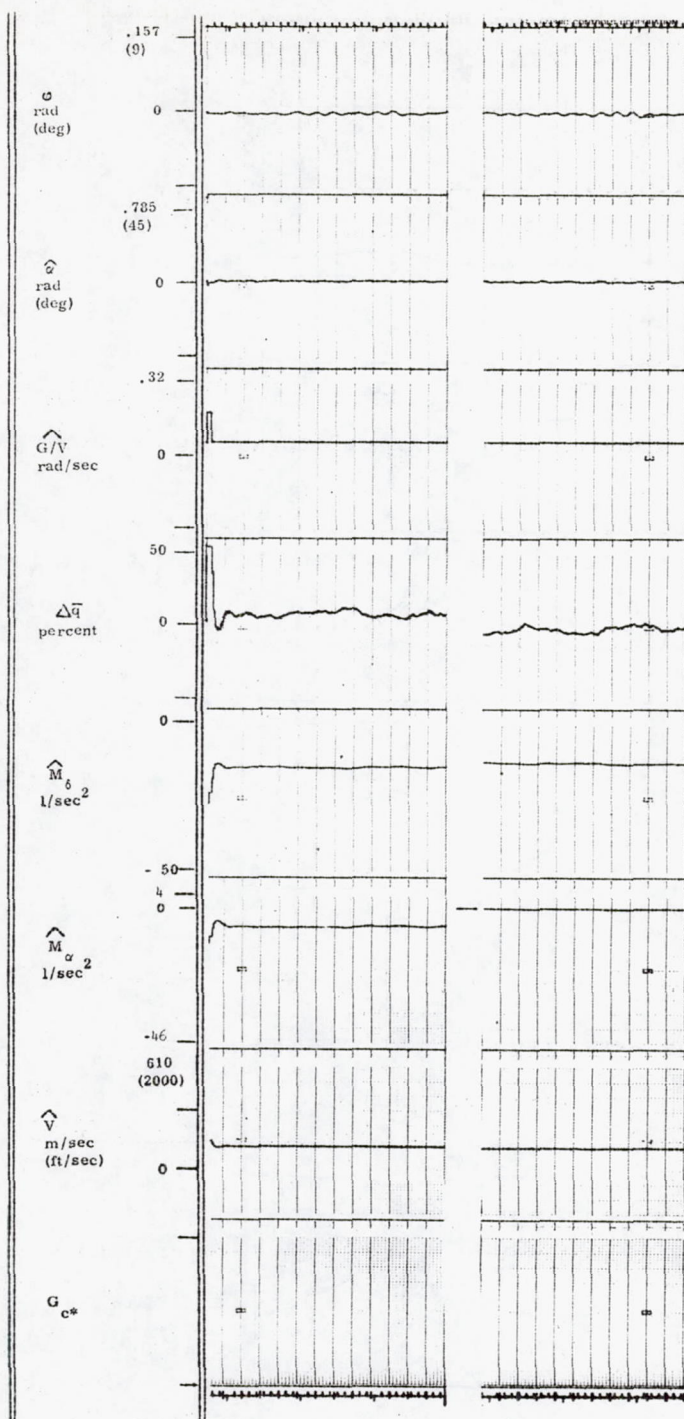


Figure 69. Relaxed-static-stability flight, FC1.

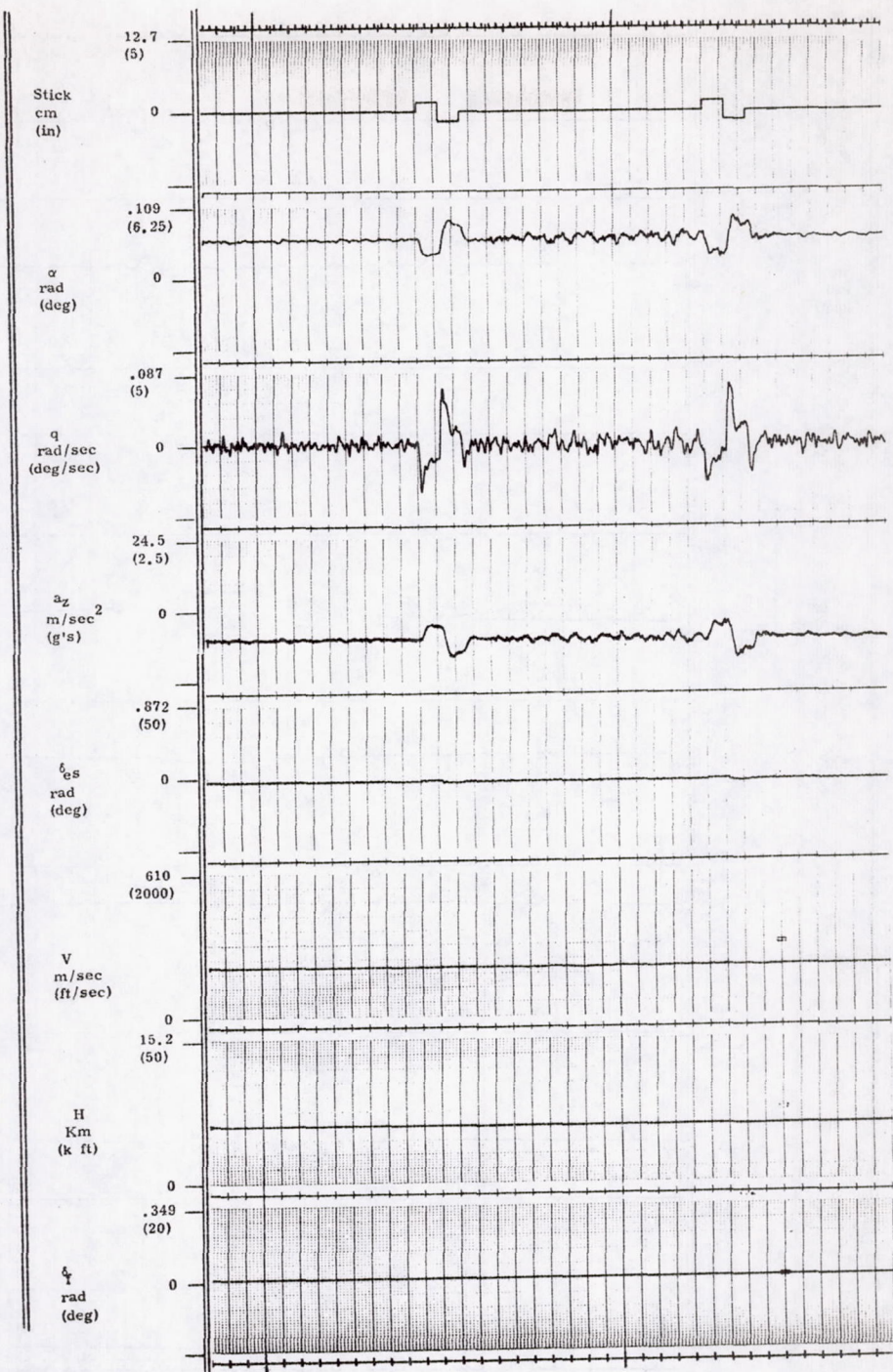


Figure 70. MLE tracking test, FC1, no sensor noise, half test signal.

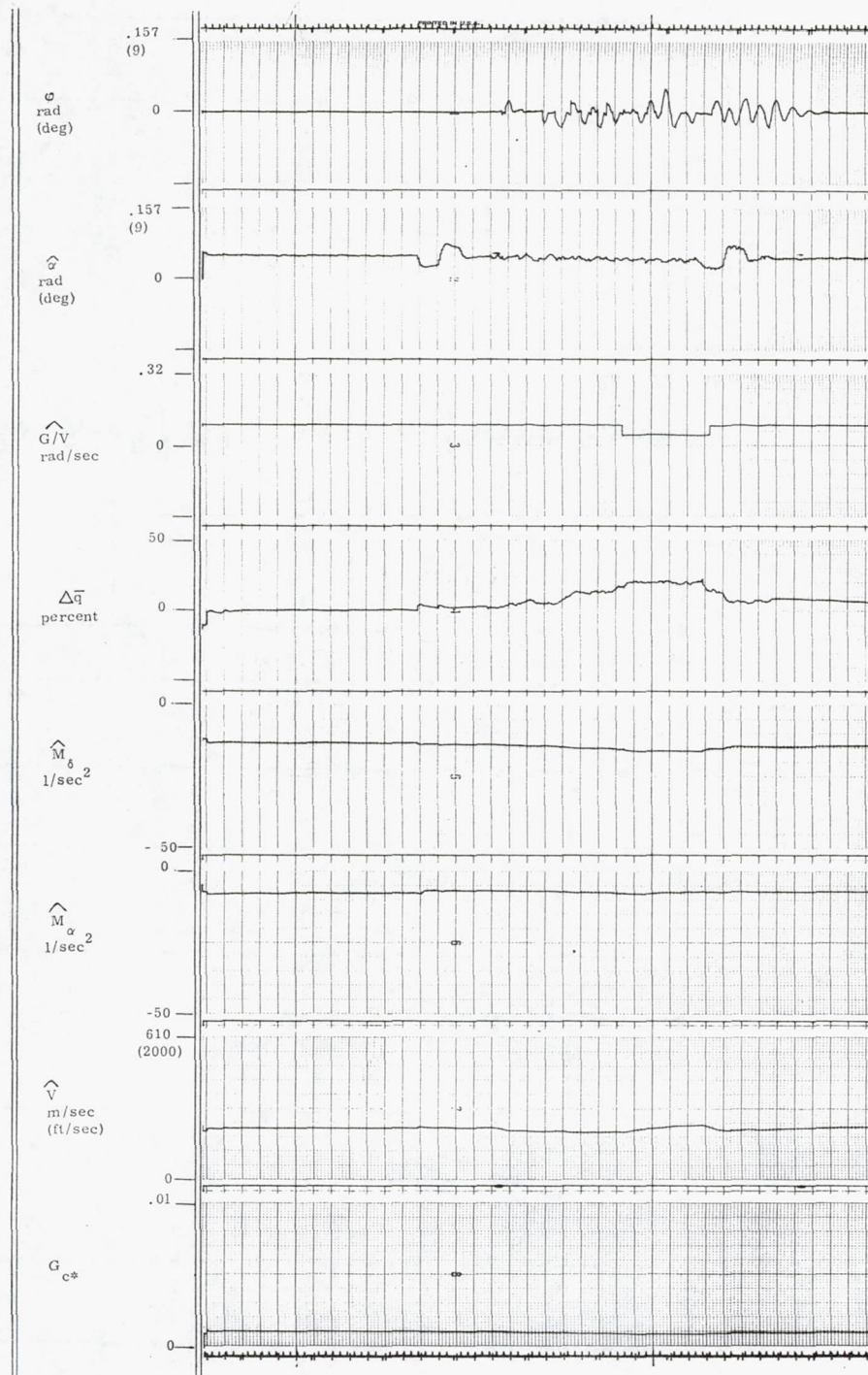


Figure 70. Concluded.

APPENDIX C

MODEL TRACKER TIME HISTORIES

APPENDIX C

MODEL TRACKER TIME HISTORIES

The time histories taken on the Langley simulator consisted of 16 channels of data (two eight-channel recorders). The channels are:

<u>Recorder 1</u>	<u>Recorder 2</u>
Pitch stick deflection	Dynamic pressure ($\bar{q}$)
Angle-of-attack (α)	$\bar{q}$ error
Pitch rate ($\dot{q}$)	Bank angle (φ)
Normal acceleration (a_3)	$\hat{f}_{11}$
Elevator position (δ_e)	$\hat{f}_{12}$
Total velocity (V)	$\hat{g}_{11}$
Altitude (H)	G_{C*} gain
Flap position (δ_f)	ET (output of nonlinear error function)

Scales for each of these parameters are indicated in MKS and conventional units.

ACCURACY AT FIXED FLIGHT CONDITIONS

Time histories for standard pitch sequences consisting of periods of quiet, turbulence, and doublet commands are presented in Figures 71 through 75 for Flight Conditions 1, 5, 8, 10 and 17. This sequence of runs was then repeated with gyro noise, accelerometer noise, and servo measurement noise simultaneously applied (Figures 76 through 79). The sequence for Flight Condition 1 was repeated with servo flow hysteresis of 0.05 degree (Figure 80).

CONVERGENCE RUNS

Figure 81 shows the response when the model parameters are initialized at FC10 but the aircraft is actually at FC5. Two separate inputs were applied; first turbulence

only and, secondly, a sequence of pilot doublets. Convergence rates are similar with time to reach 50 percent of steady state in two seconds, and 80 percent of steady state is reached in about four seconds. The gain response is slower due to the one-second lag on the $\hat{g}_{11}$ parameter.

Figure 82 shows a similar response when the model is initialized at FC5 and the aircraft is actually at FC1. Again, 80 percent of steady state is reached in about four seconds.

PARAMETER TRACKING

Two profiles corresponding to changing flight conditions were made. The first accelerated from FC5 ($\bar{q} = 105$ psf) with full power (and afterburner).

Figures 83 through 85 show the accelerating condition. For the quiet case, no adjustment in the model parameters occur and the errors grow as expected. Therefore, the quiet run was repeated using a random test signal bandlimited to have most of the energy around the short-period frequency. Use of a test signal held the tracking error to around 20 to 25 percent (Figure 83). For the case of turbulence, the tracking error in $\hat{g}_{11}$ is about 20 percent with a slightly large initial error buildup (Figure 84).

Tracking during turbulence and sensor noise is shown in Figure 85.

The second profile consisted of a deceleration from Flight Condition 10. Thrust is reduced to idle and the speed brake extended. Airspeed gradually decreases and altitude remains essentially constant. The quiet condition (Figure 86) again required a test signal. Peak errors are on the order of 50 percent. In tracking during turbulence (Figure 87), the error in g_{11} is less than 20 percent. For the turbulence and sensor noise condition, the peak errors are 20 percent (Figure 88).

LATERAL DISTURBANCE

Of interest is the effect of a roll maneuver on the pitch axis tracker. Figure 89 shows the time history of a snap 360° roll at Flight Condition 1. This is a severe test for the tracker since the sensor bandpass filters cannot eliminate the variation in the normal accelerometer reading. The peak error developed is 60 percent. Slower roll rate will have less effect. The figure shows that any activity following the roll will reduce the error. In the example, turbulence is applied following the roll and the error is reduced.

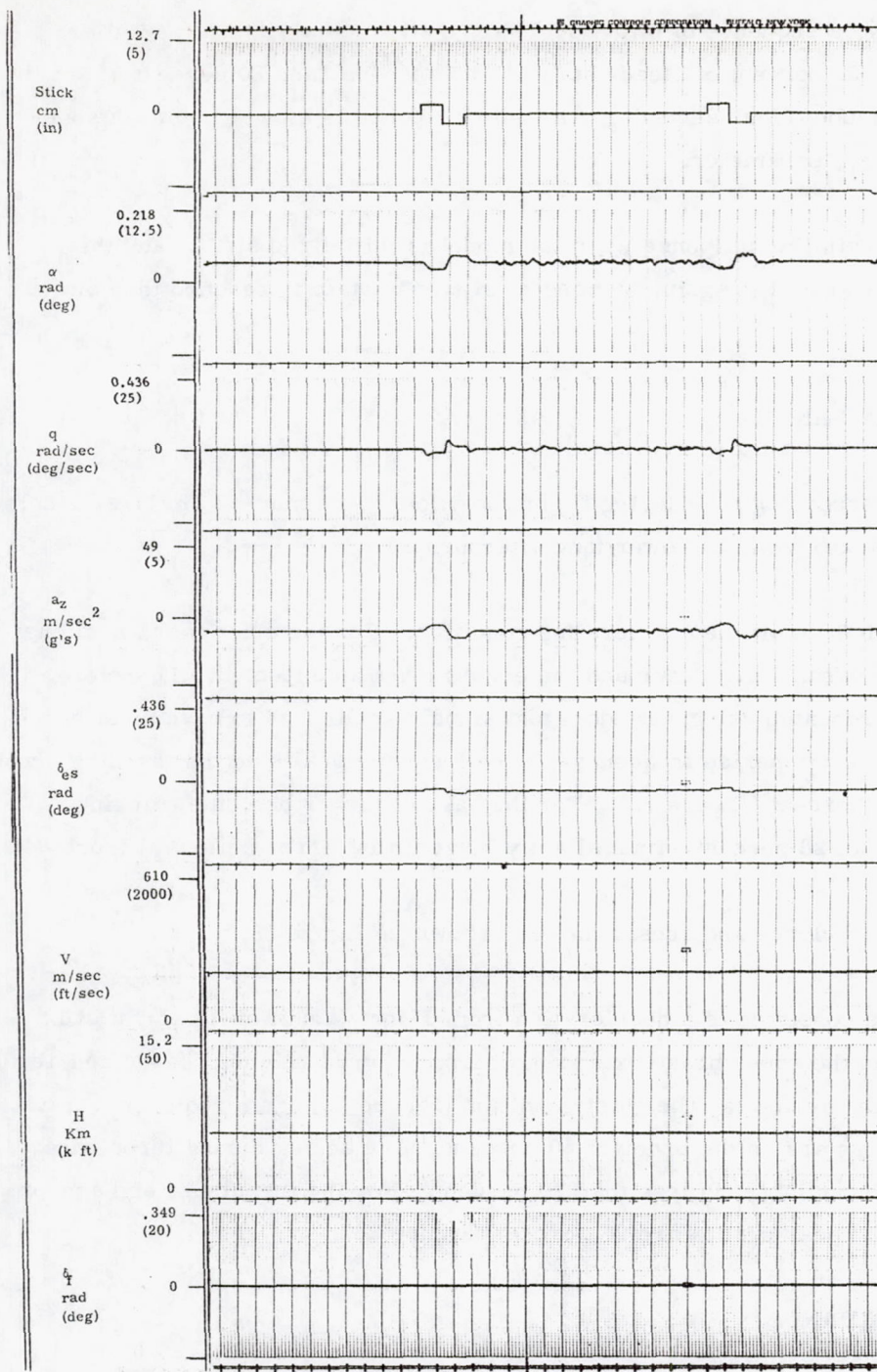


Figure 71. Model tracker at FC1.

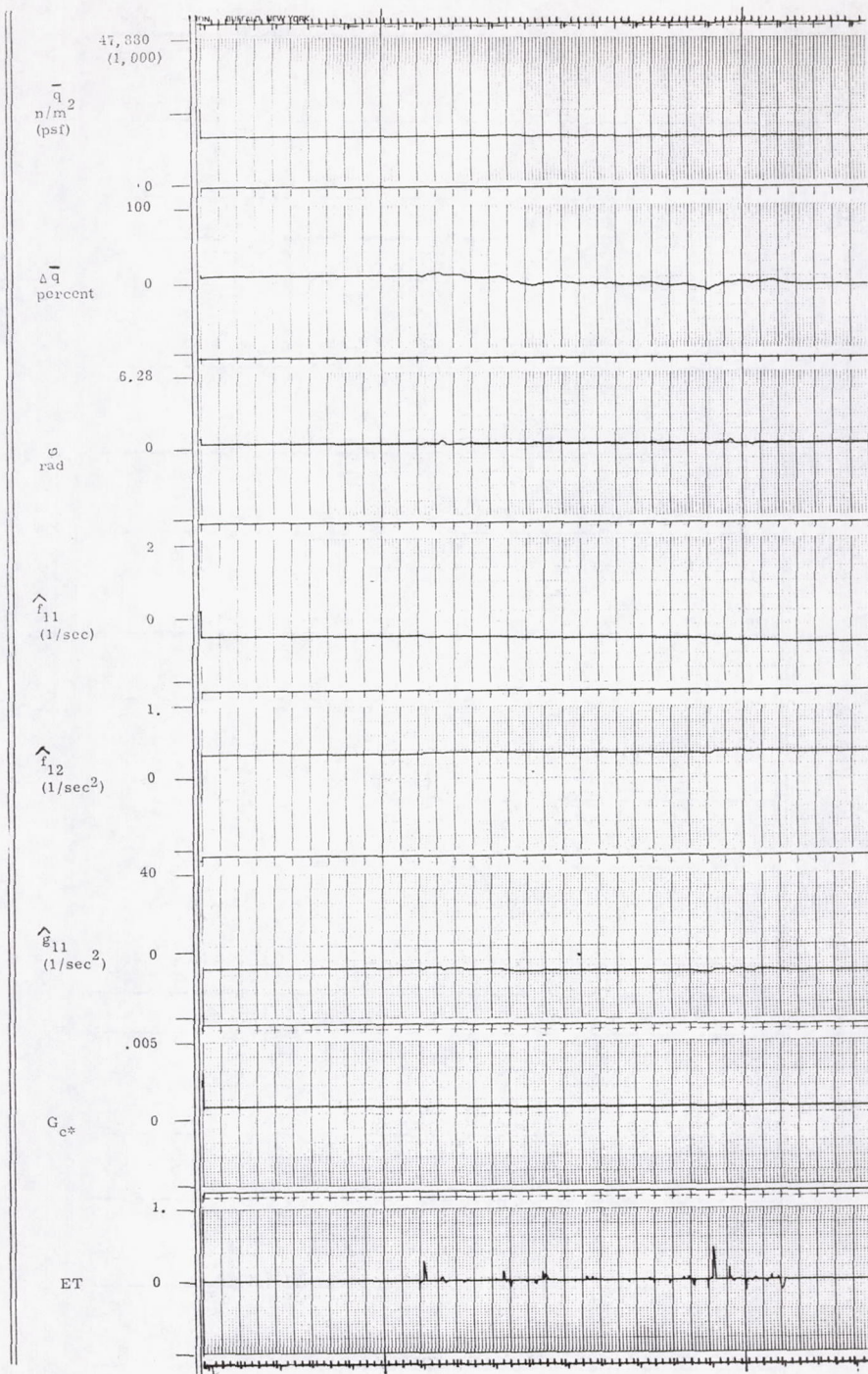


Figure 71. Concluded.

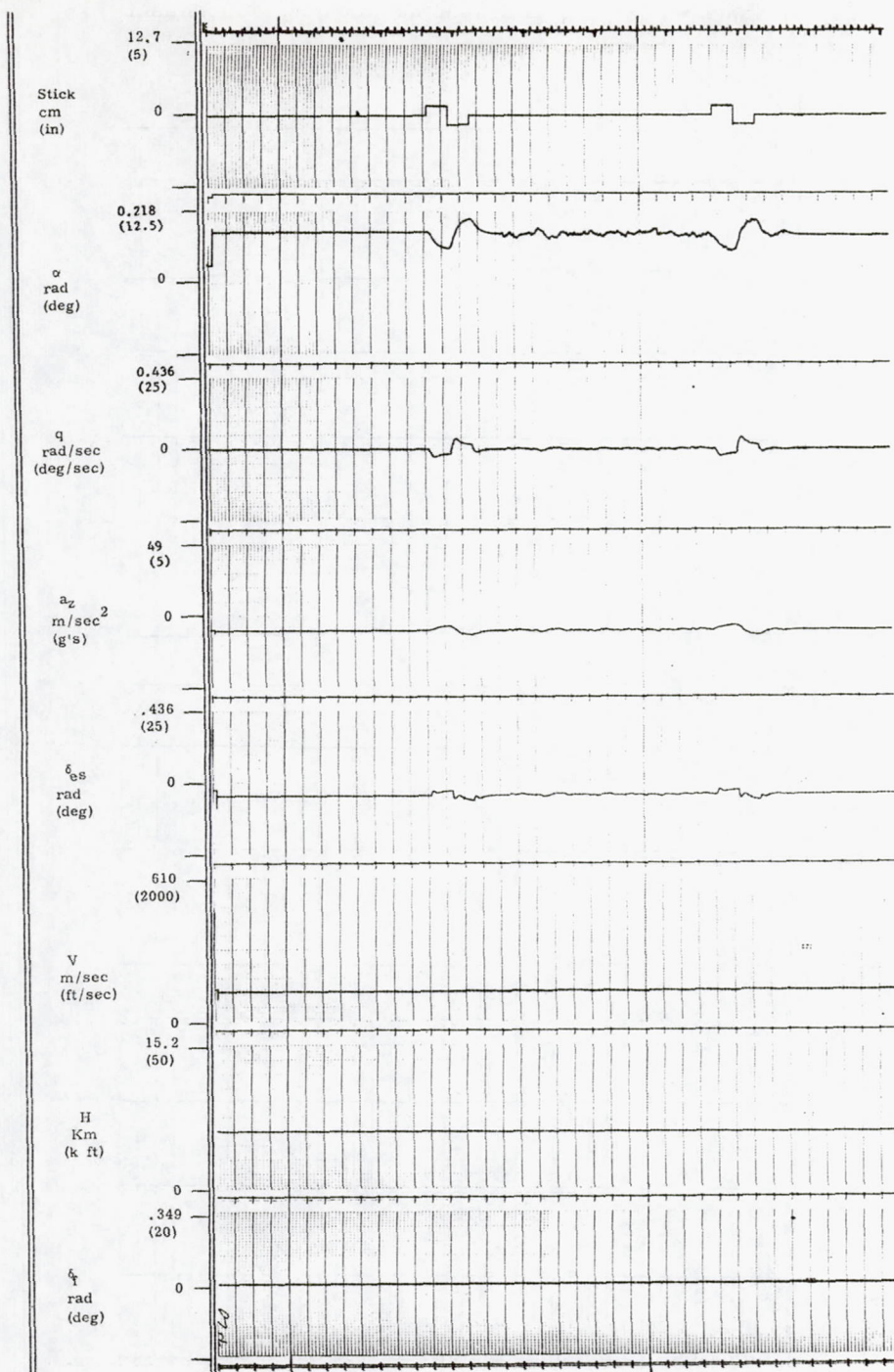


Figure 72. Model tracker at FC5.

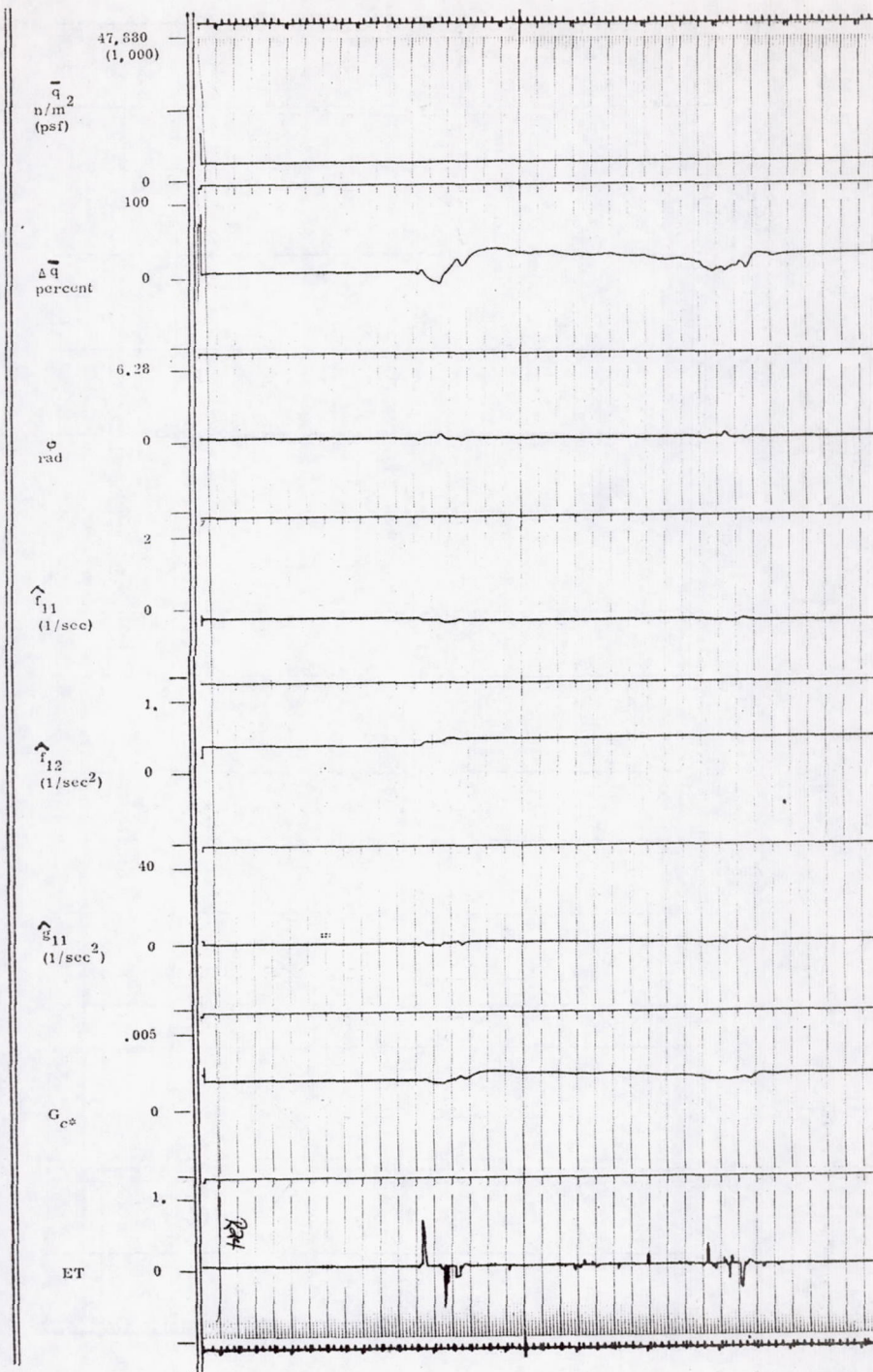


Figure 72. Concluded.

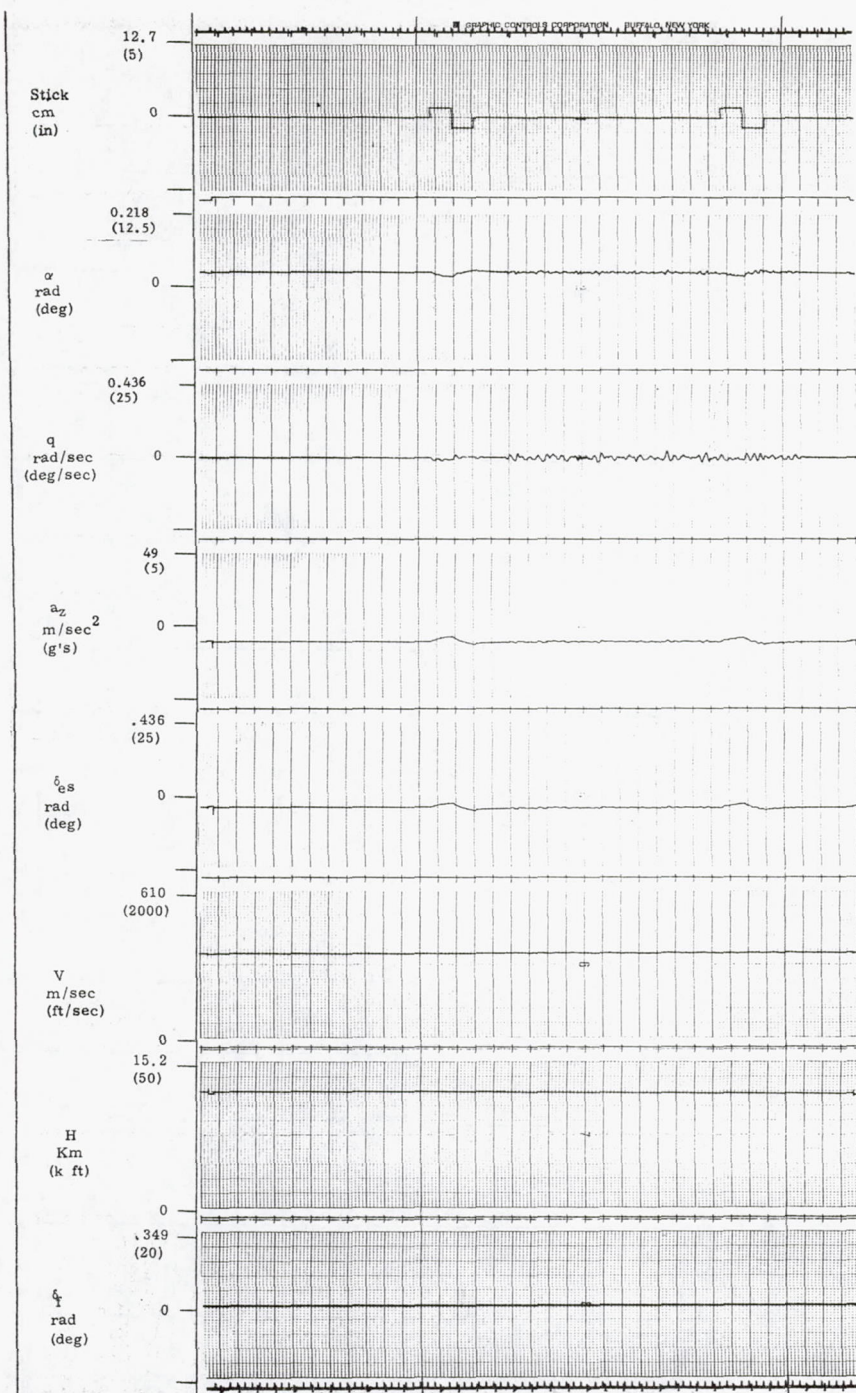


Figure 73. Model tracker at FC8.

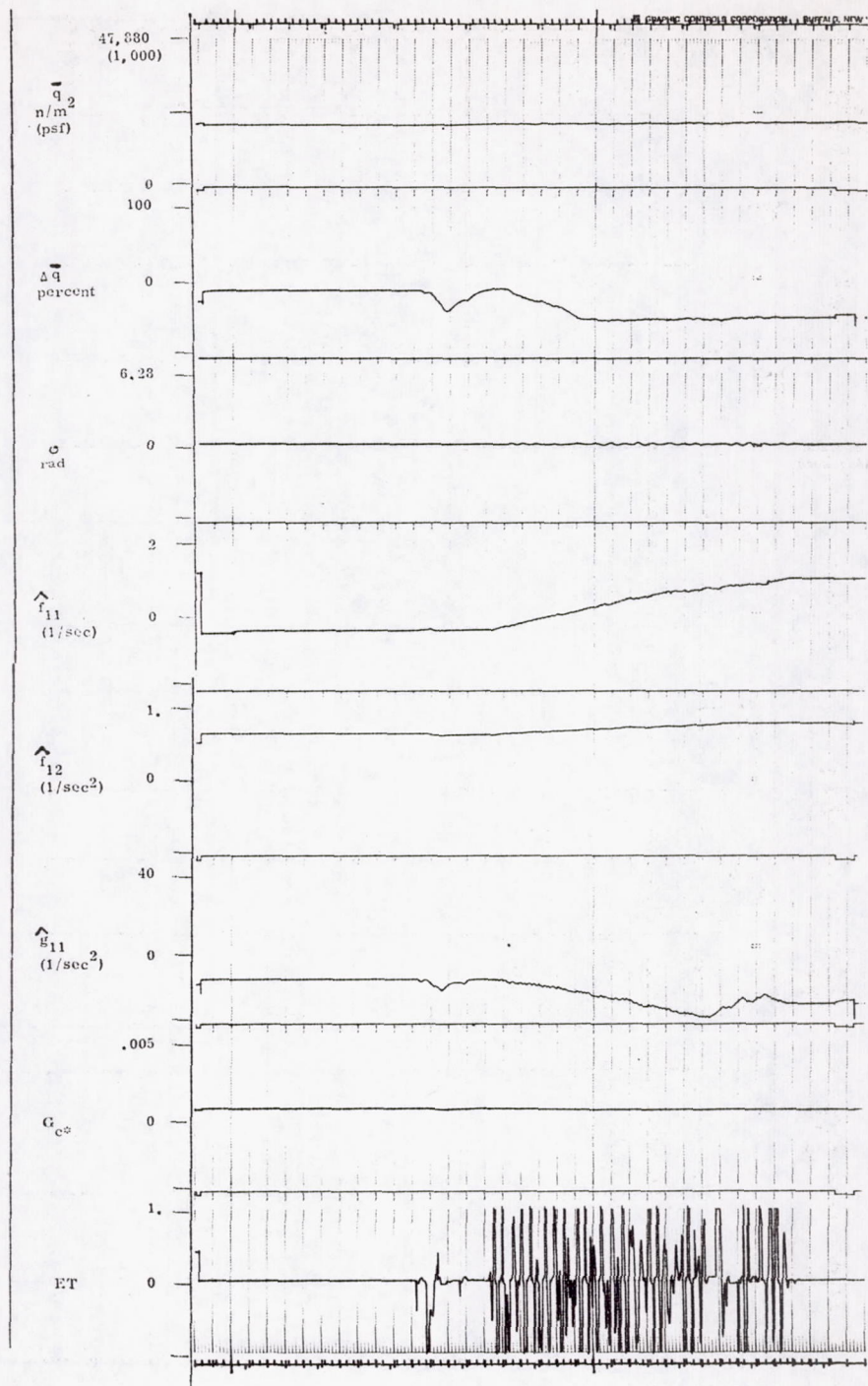


Figure 73. Concluded.

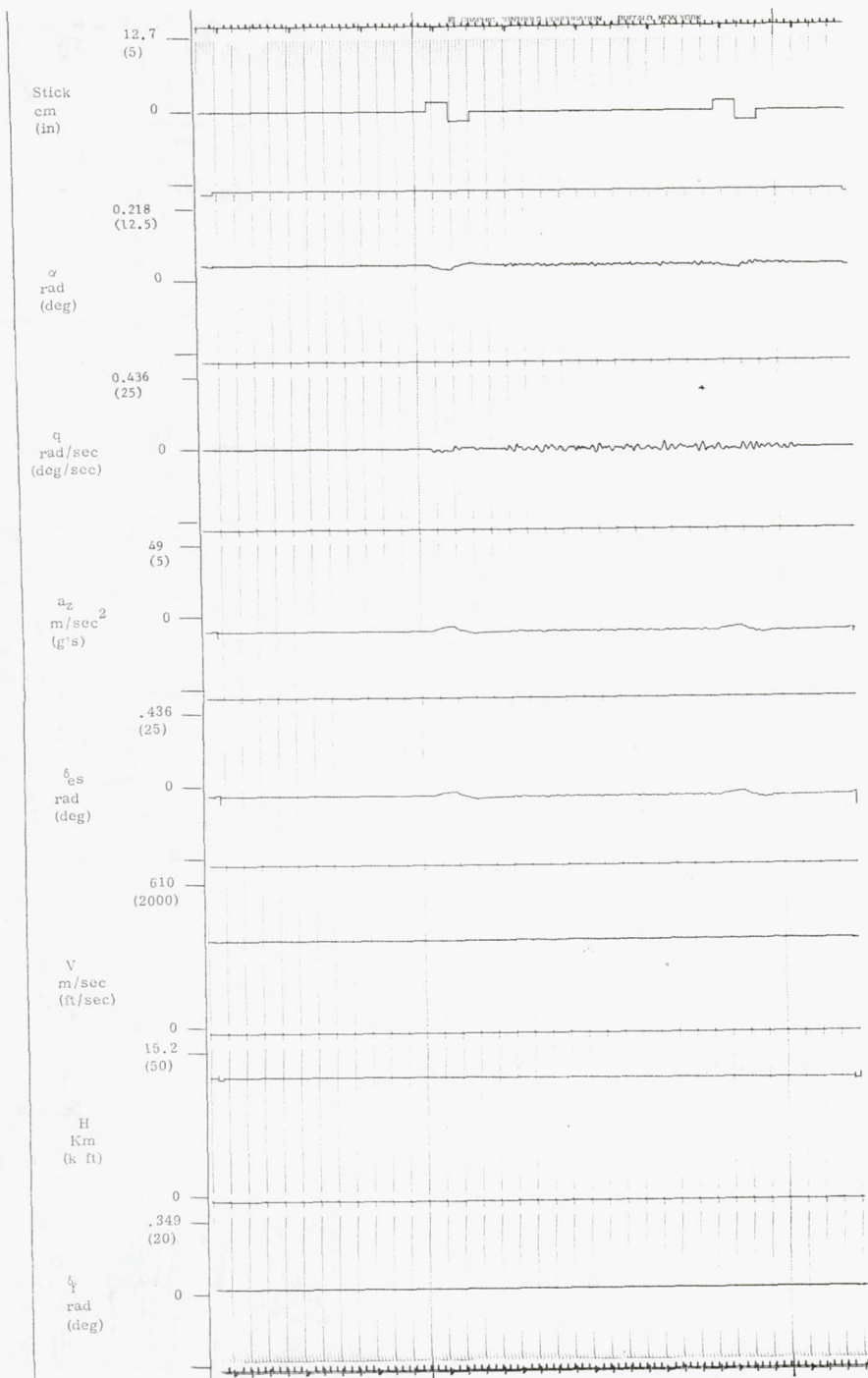


Figure 74. Model tracker at FC10.

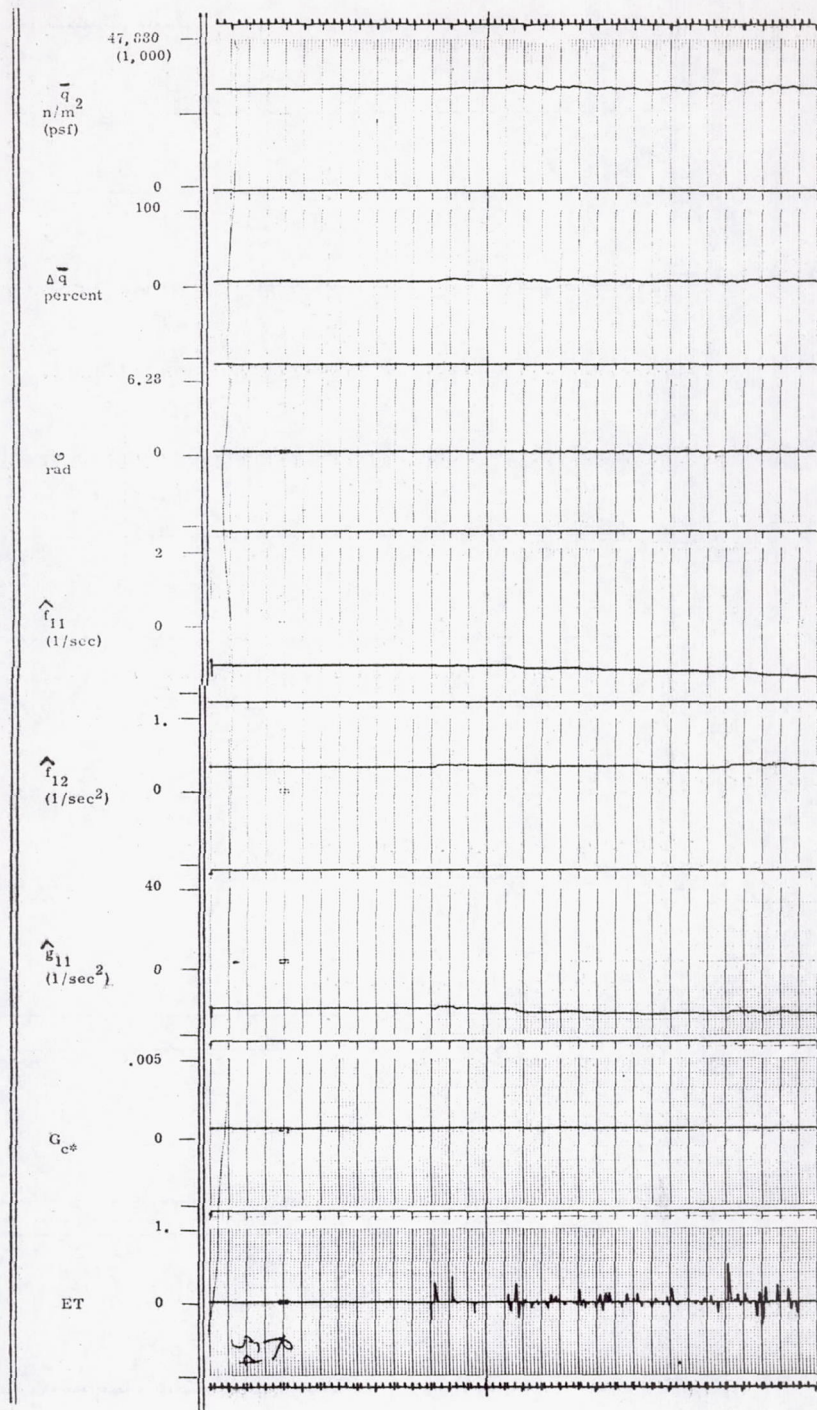


Figure 74. Concluded.

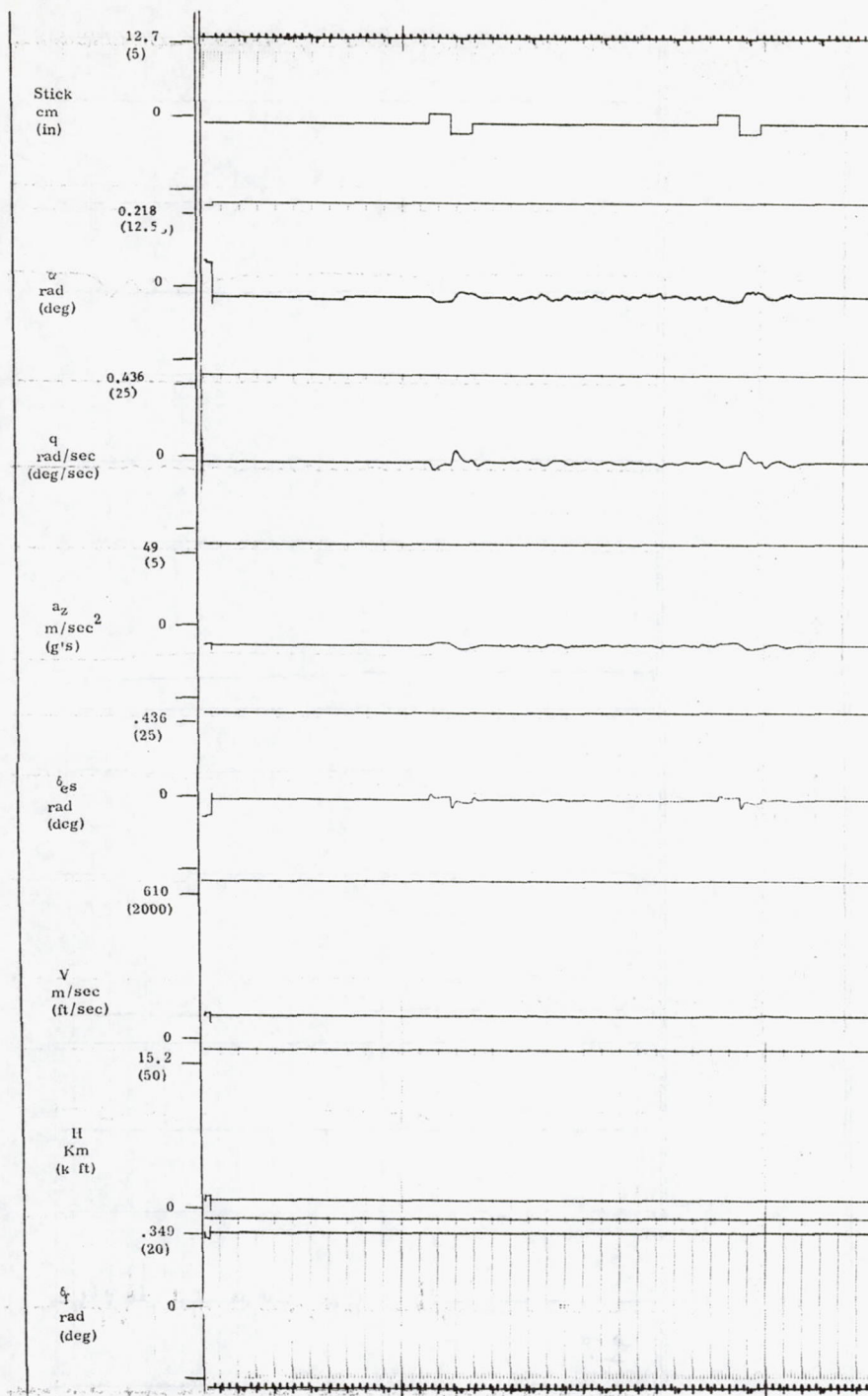


Figure 75. Model tracker at FC17.

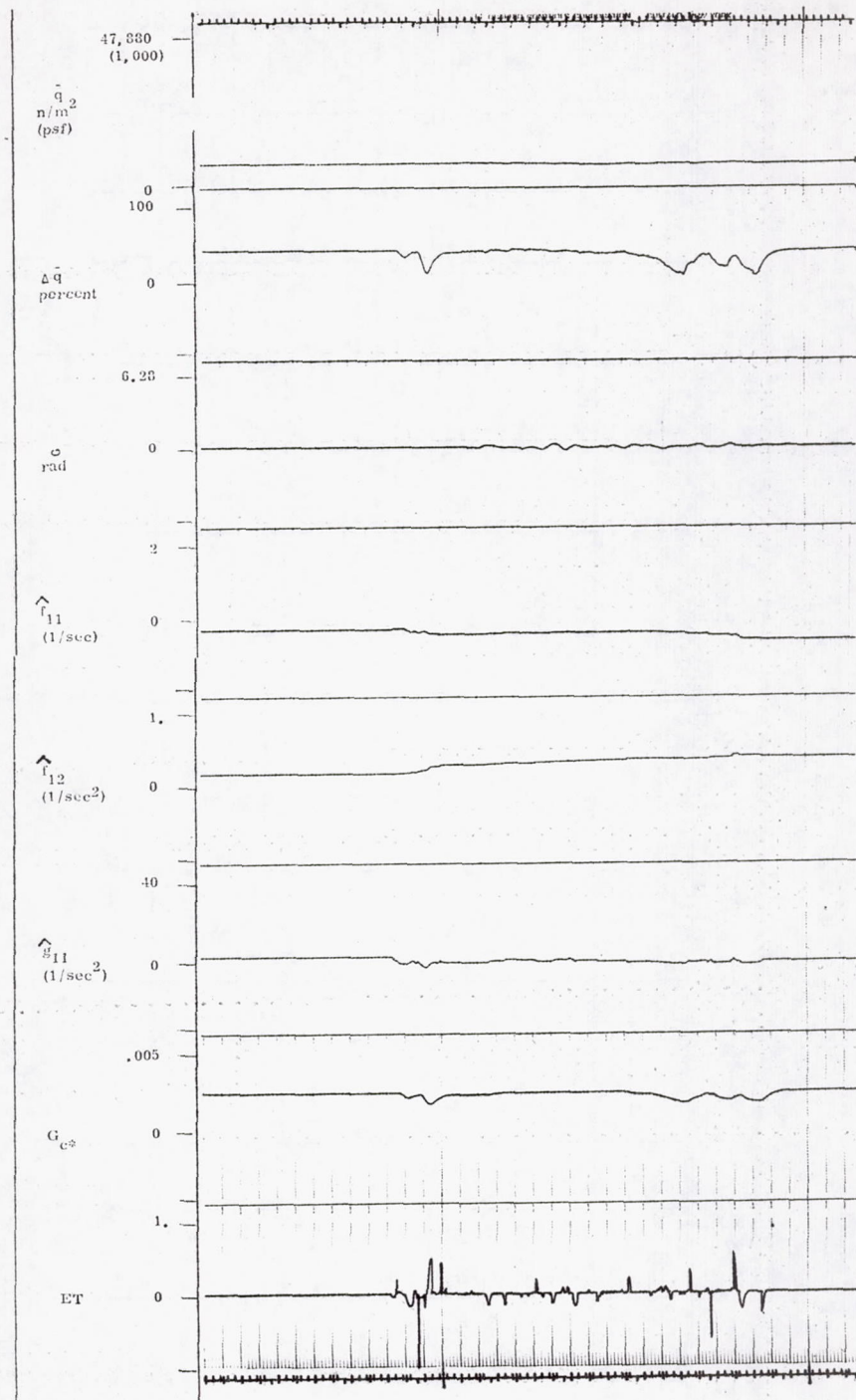


Figure 75. Concluded.

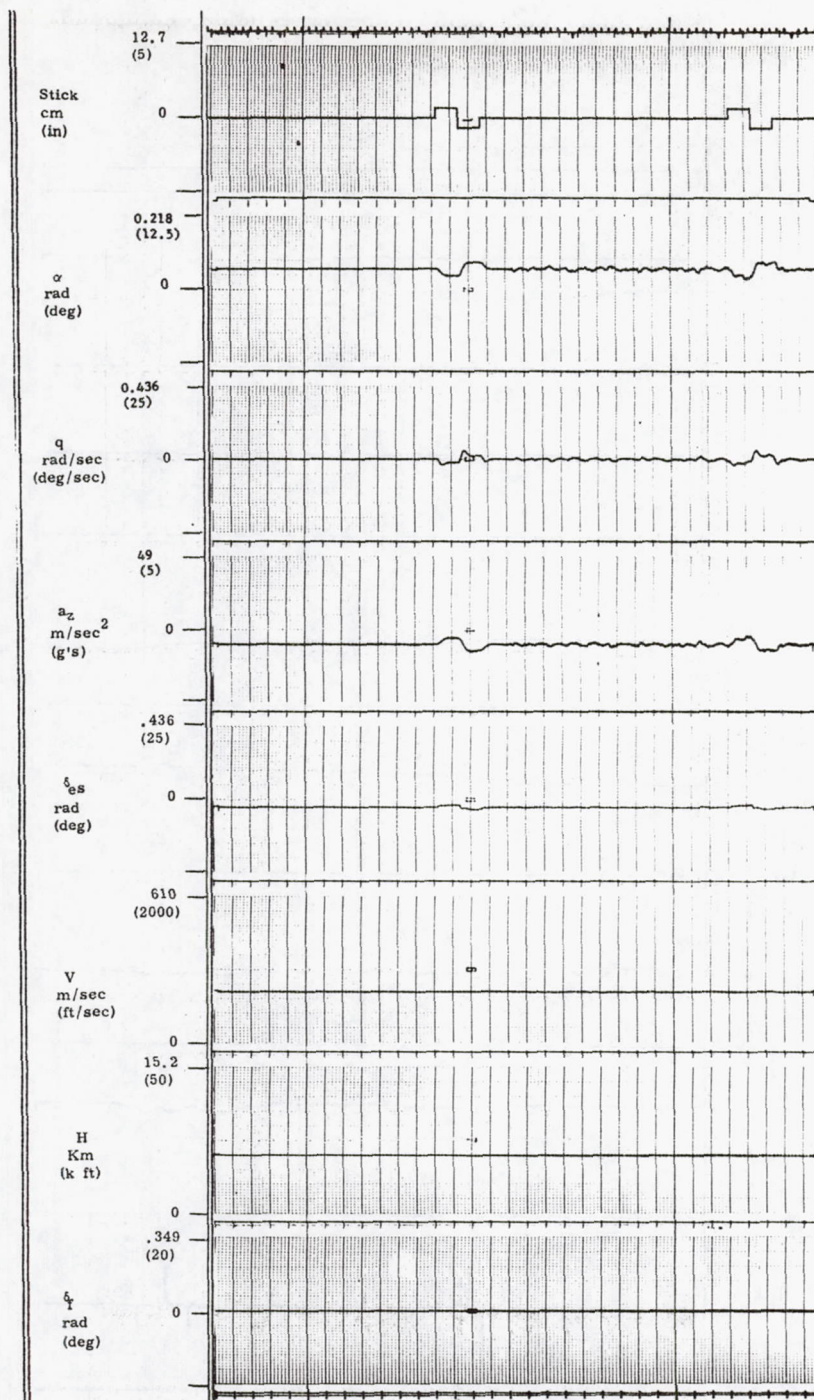


Figure 76. Model tracker at FC1 (sensor noise).

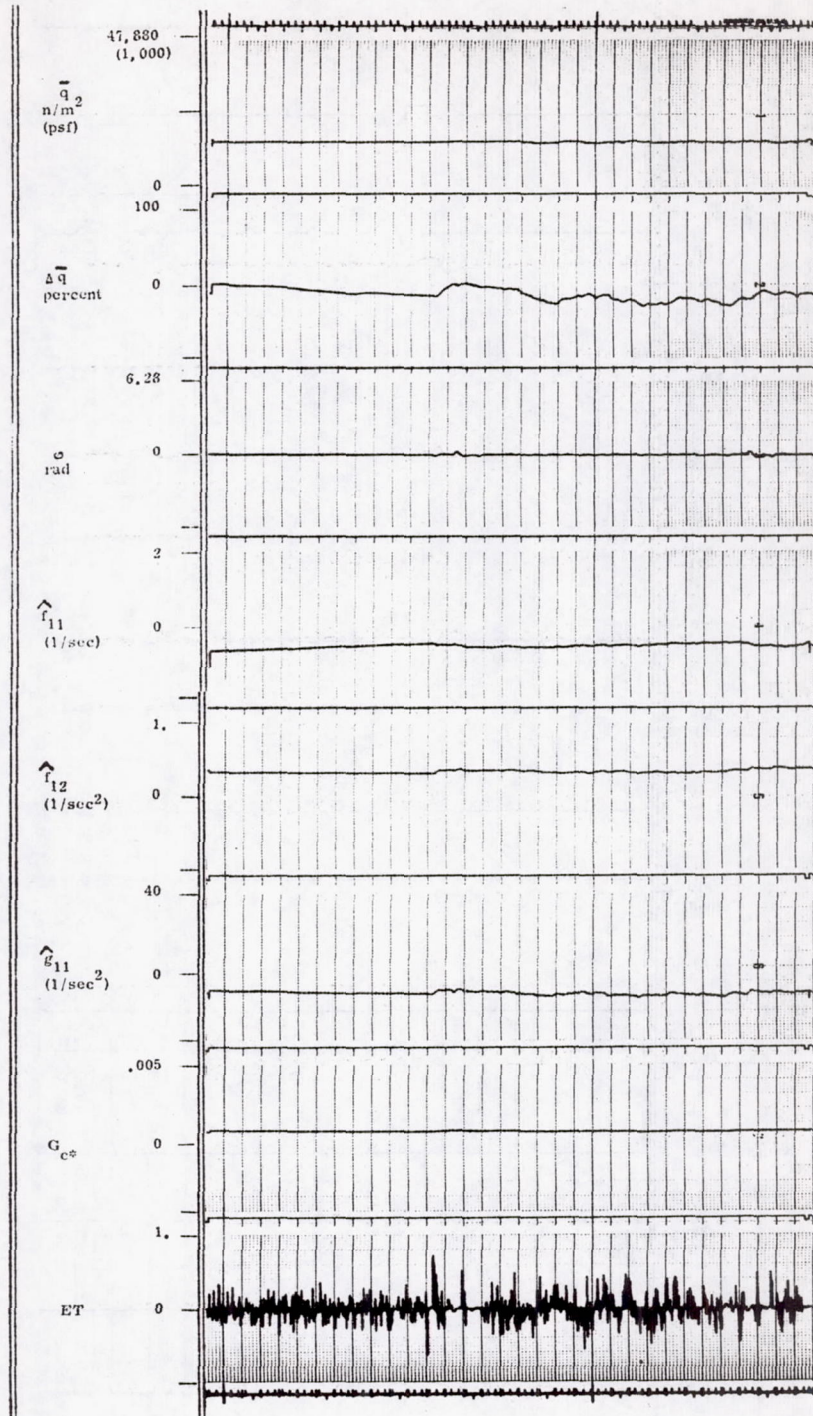


Figure 76. Concluded.

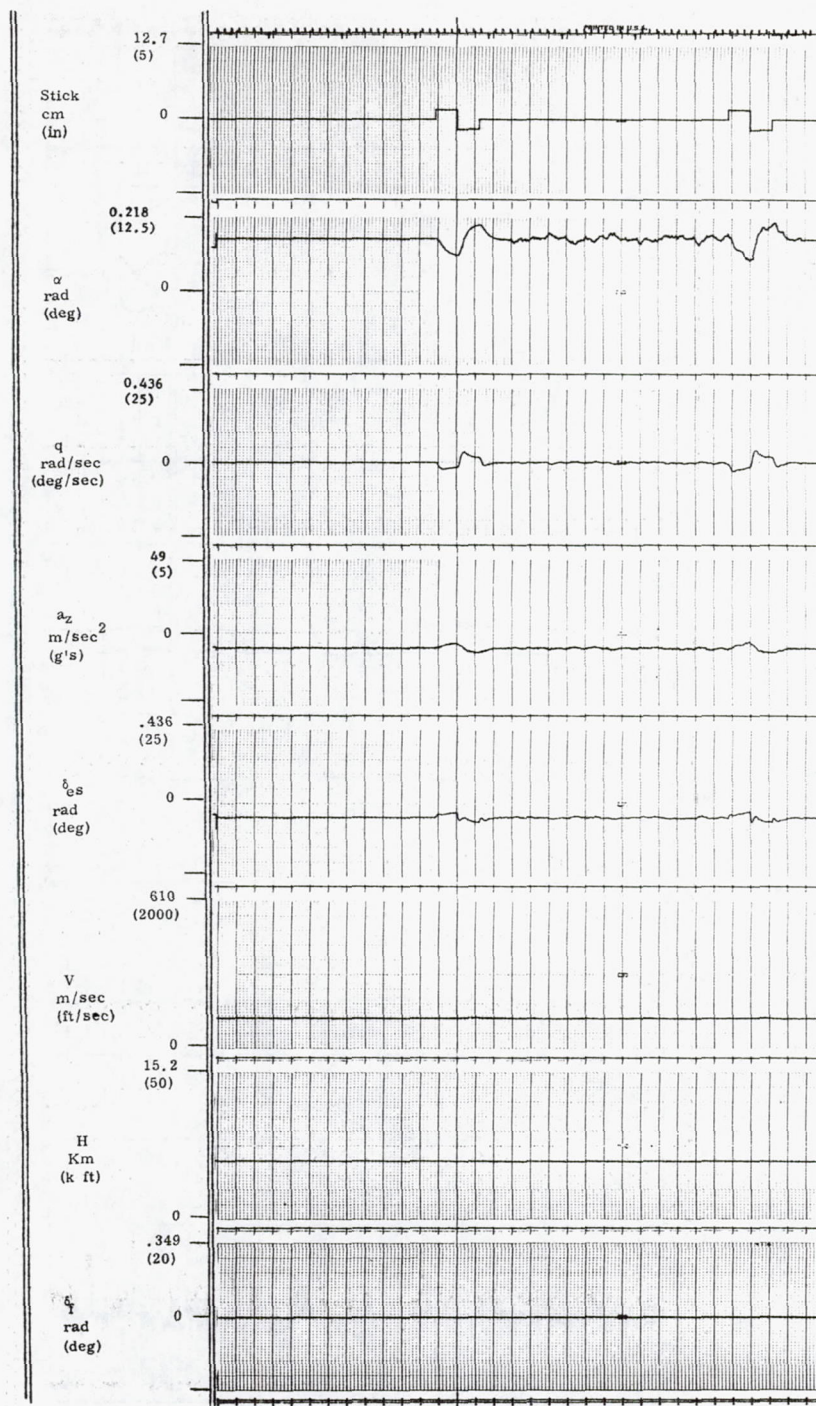


Figure 77. Model tracker at FC5 (sensor noise).

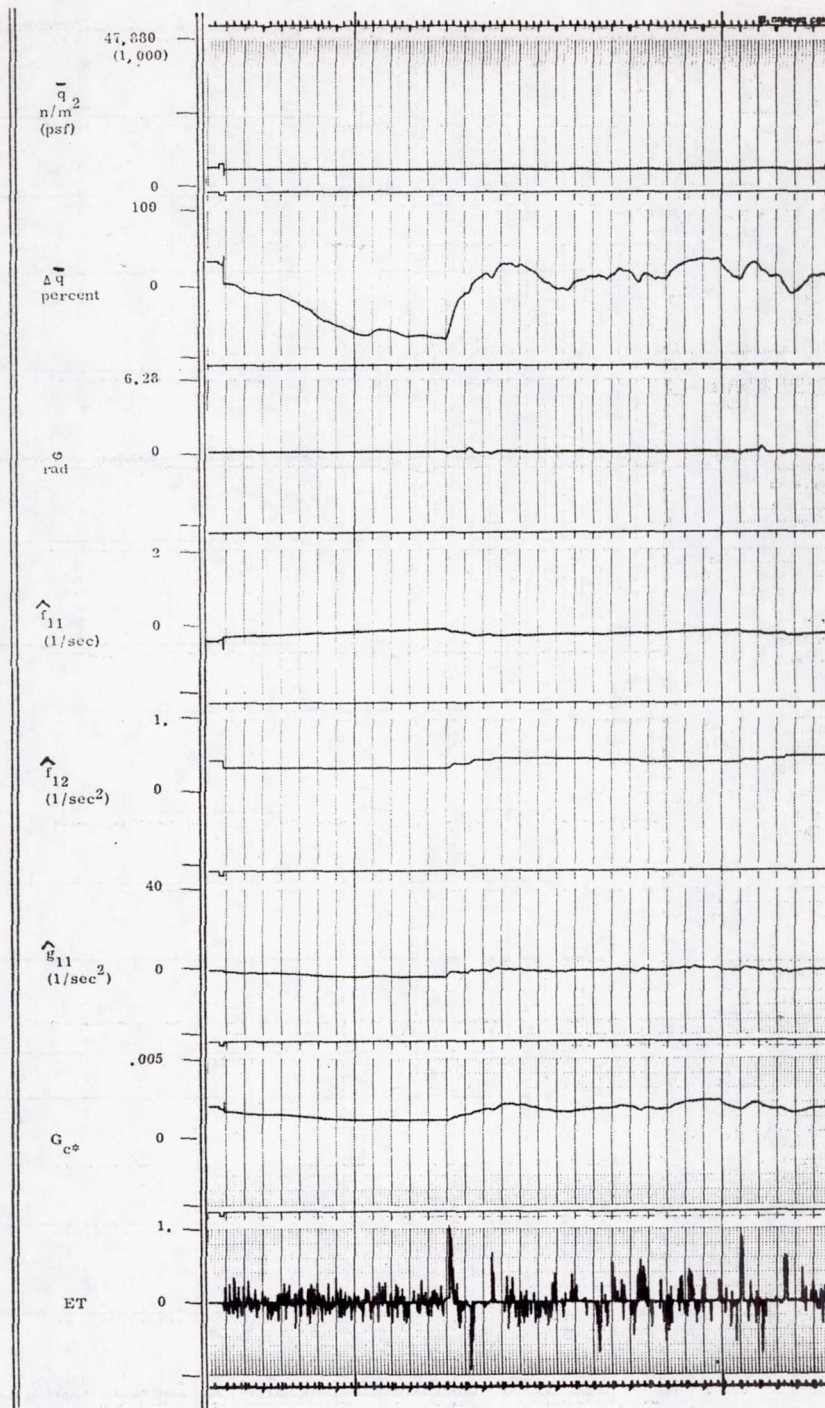


Figure 77. Concluded.

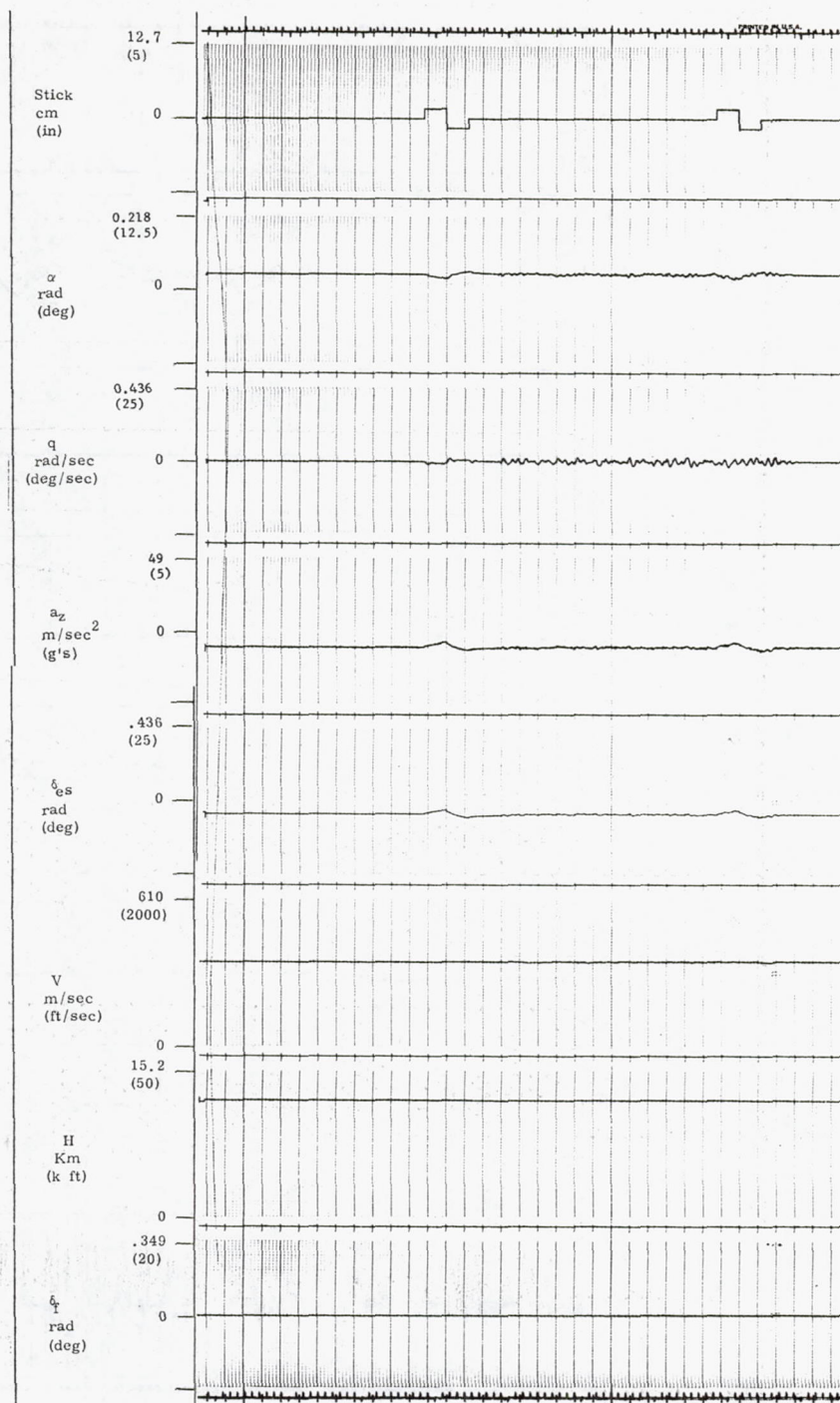


Figure 78. Model tracker at FC8 (sensor noise).

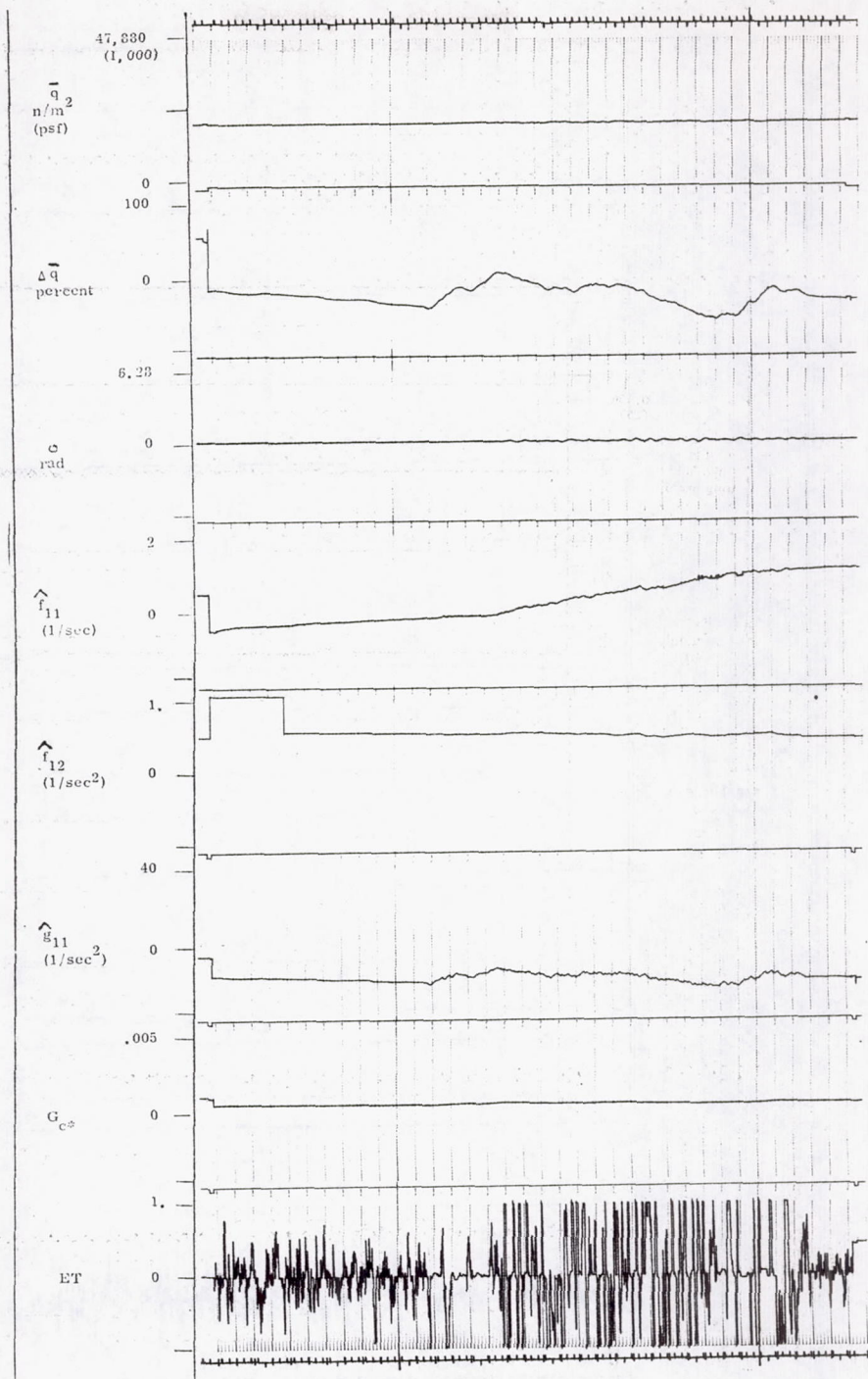


Figure 78. Concluded.

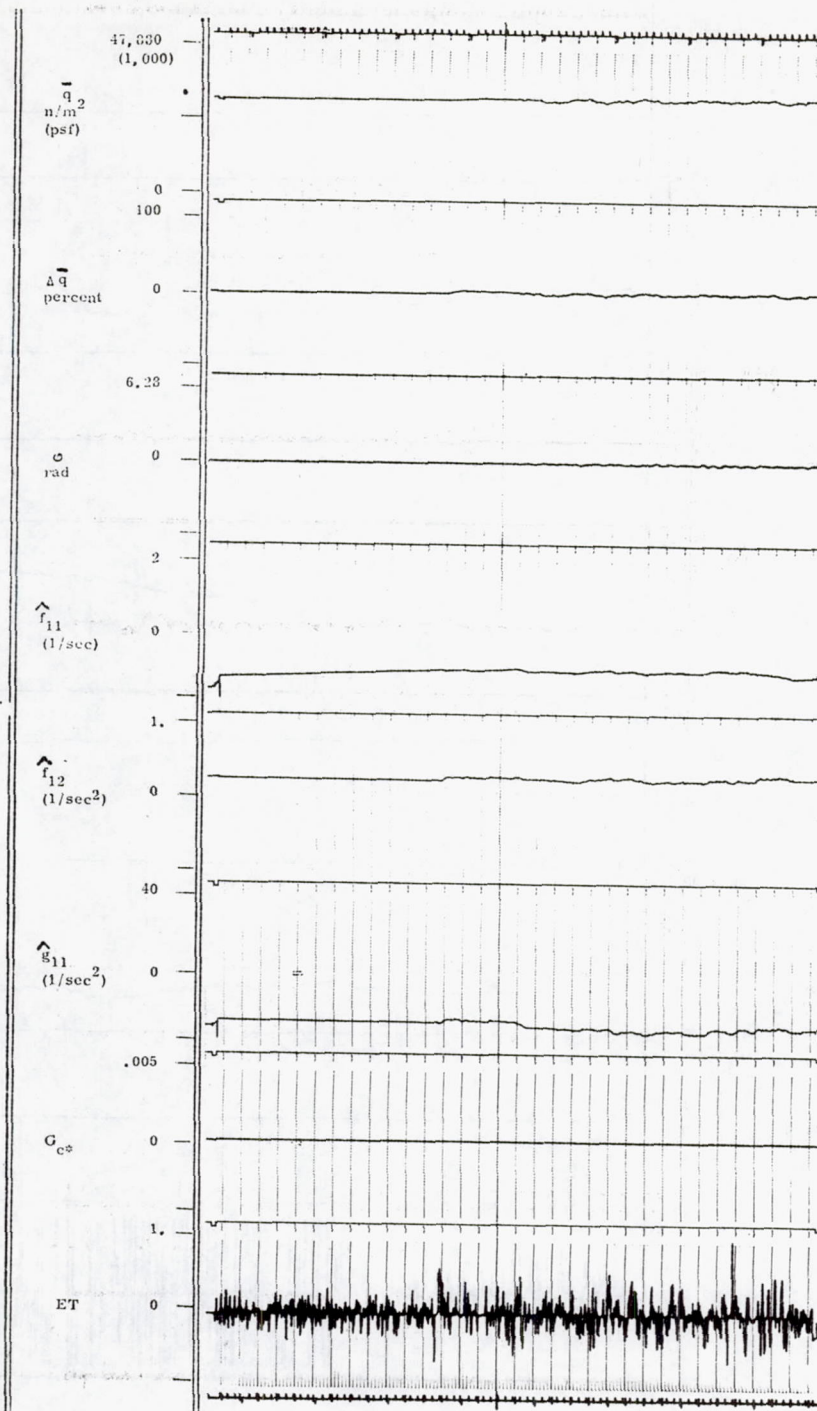


Figure 79. Model tracker at FC10 (sensor noise).

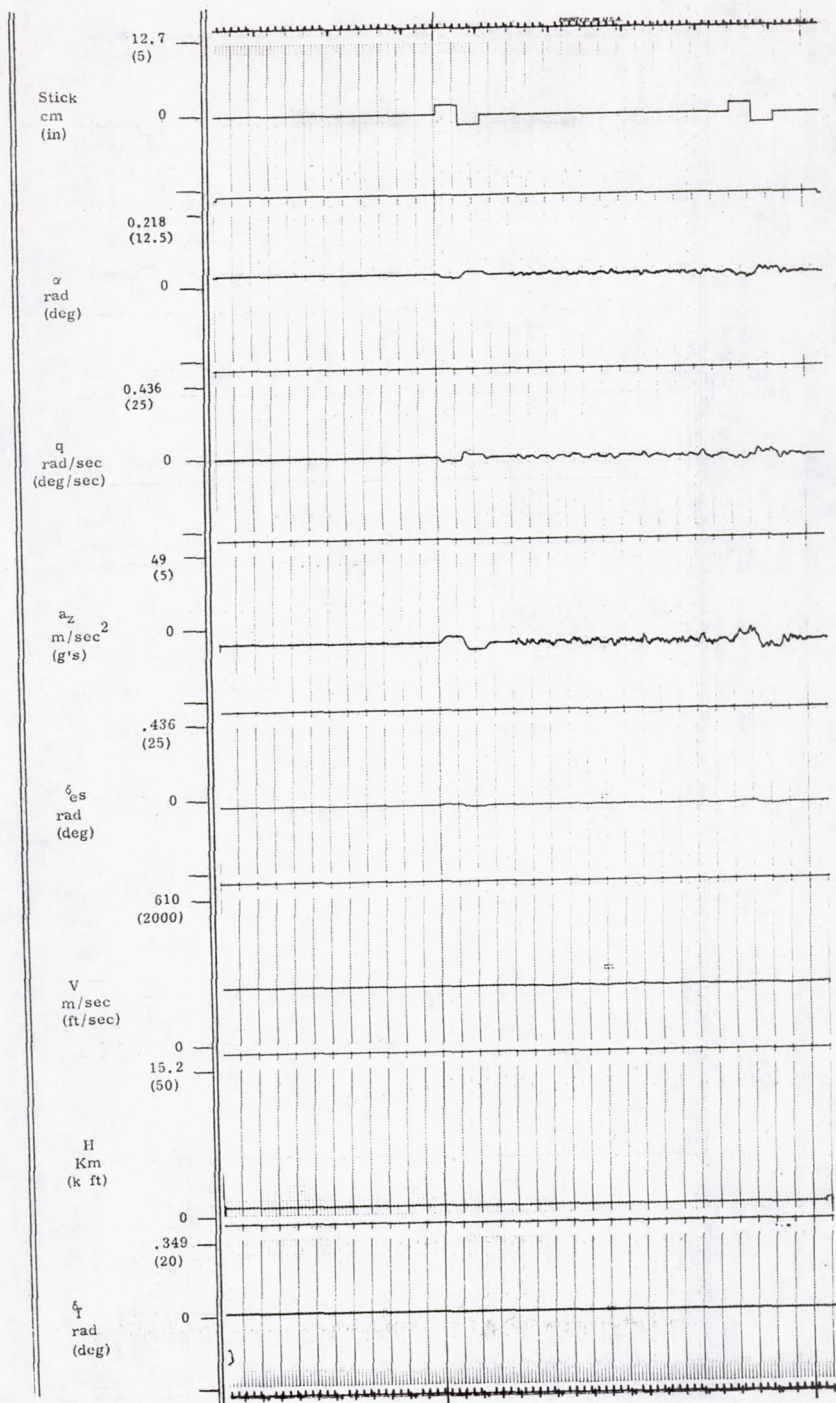


Figure 79. Concluded.

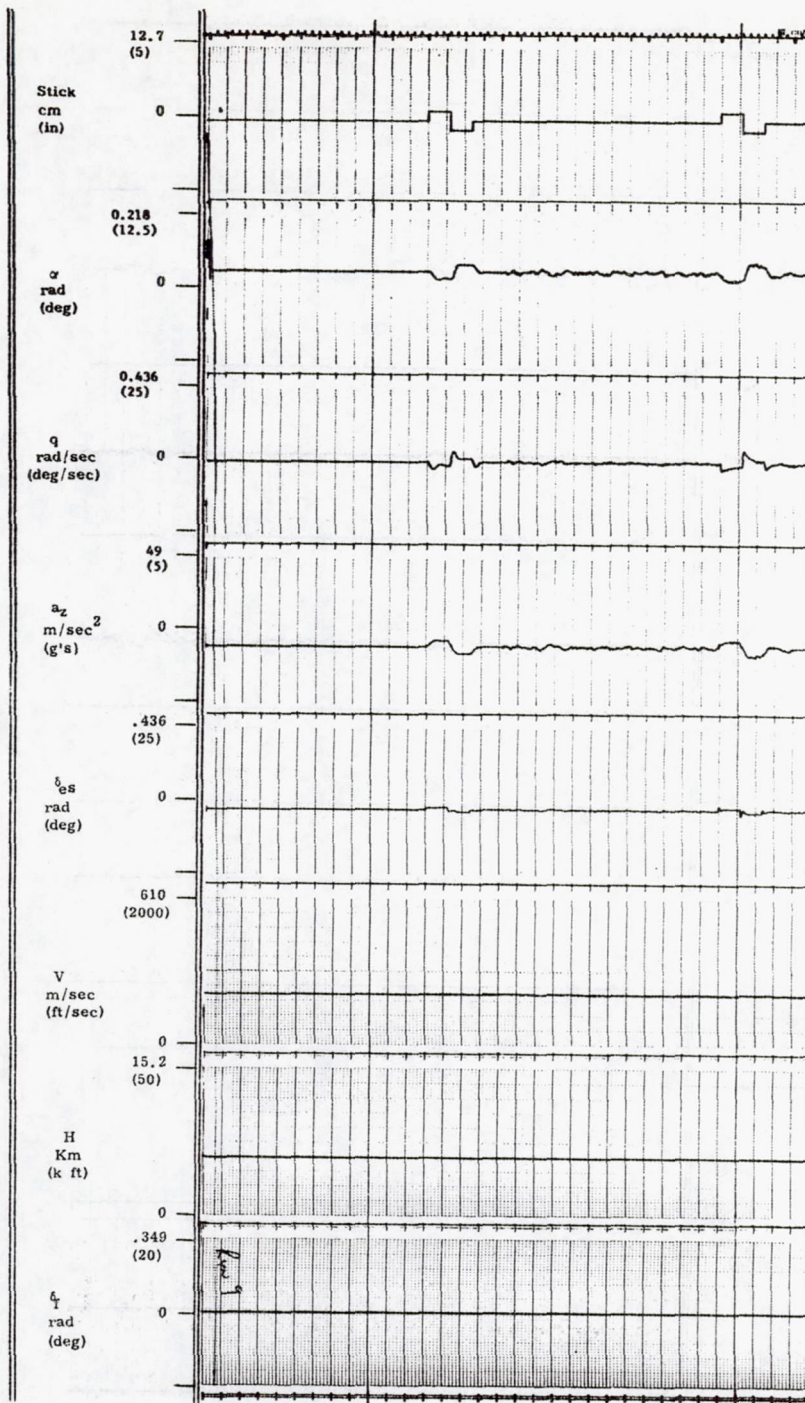
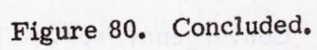


Figure 80. Model tracker at FC1 (with hysteresis).



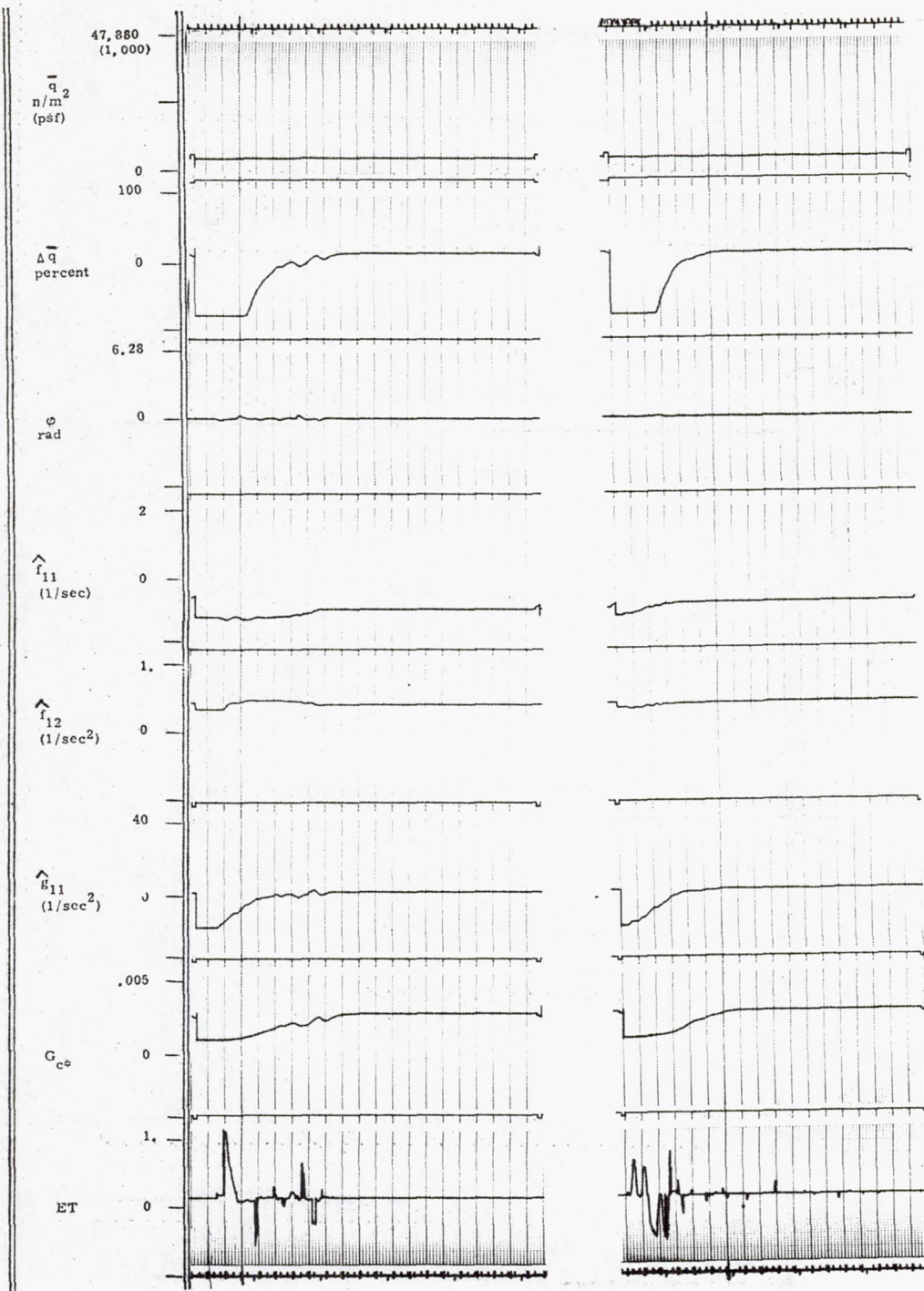


Figure 81. Convergence from FC10 to FC5.

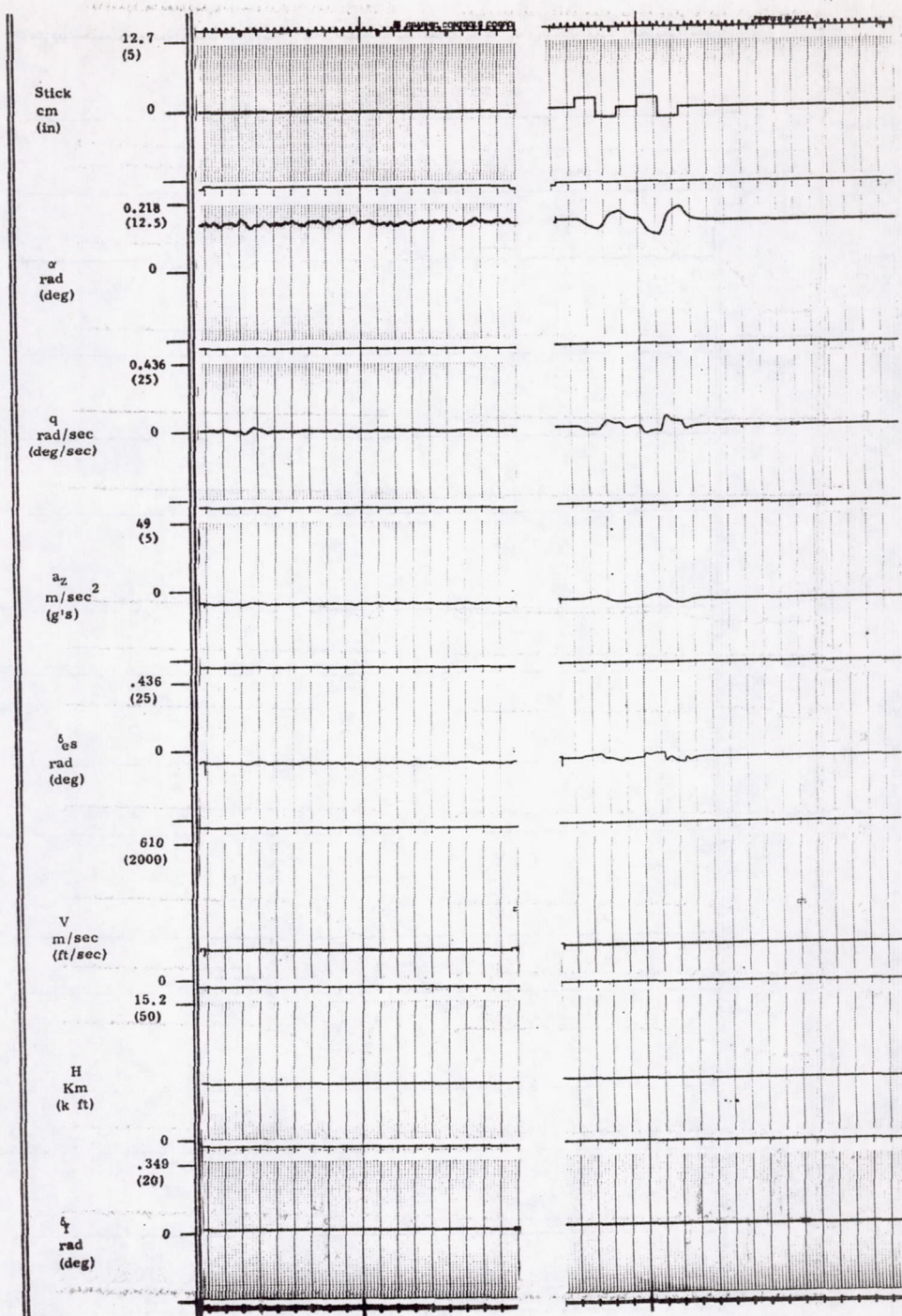


Figure 81. Concluded.

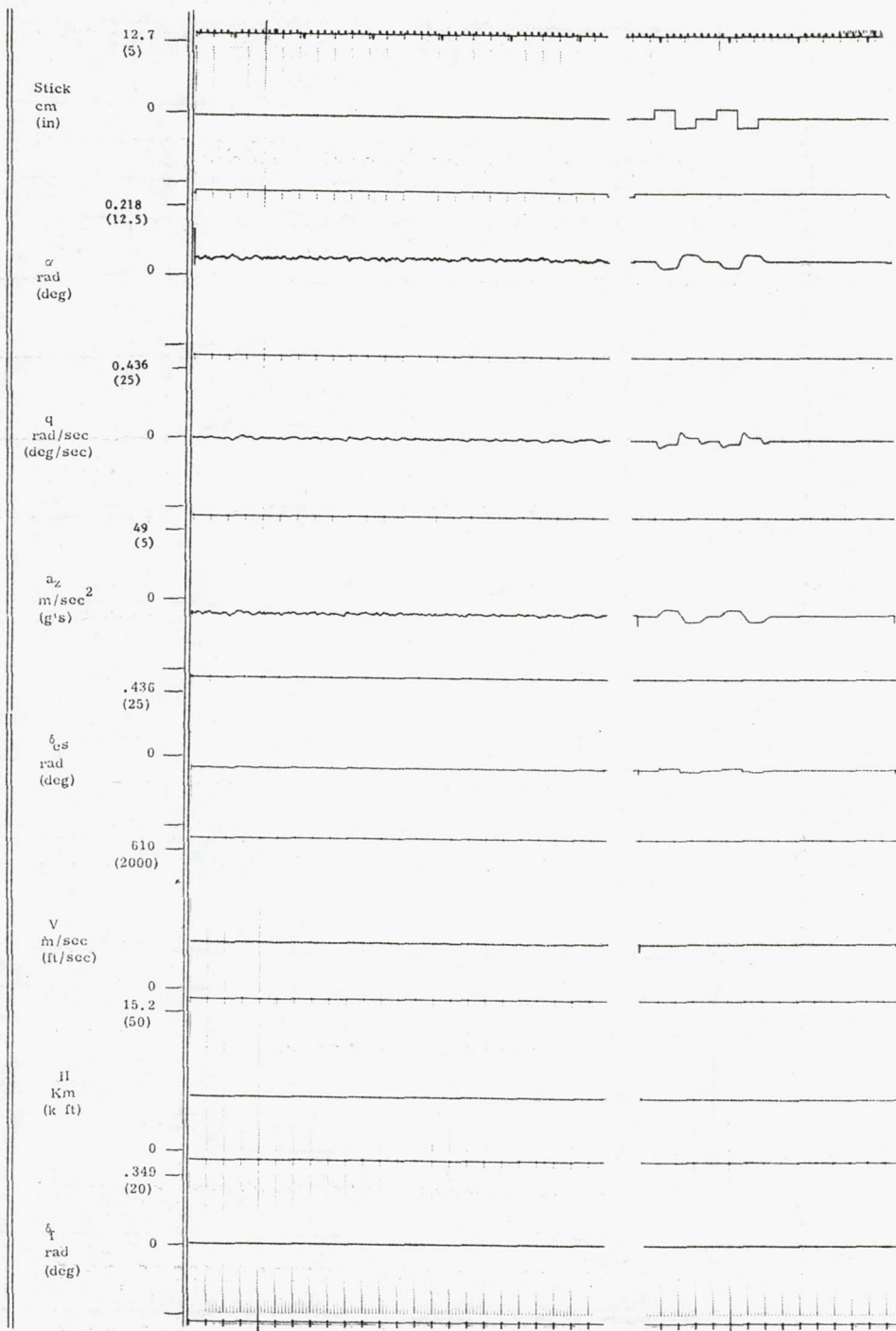


Figure 82. Convergence from FC5 to FC1.

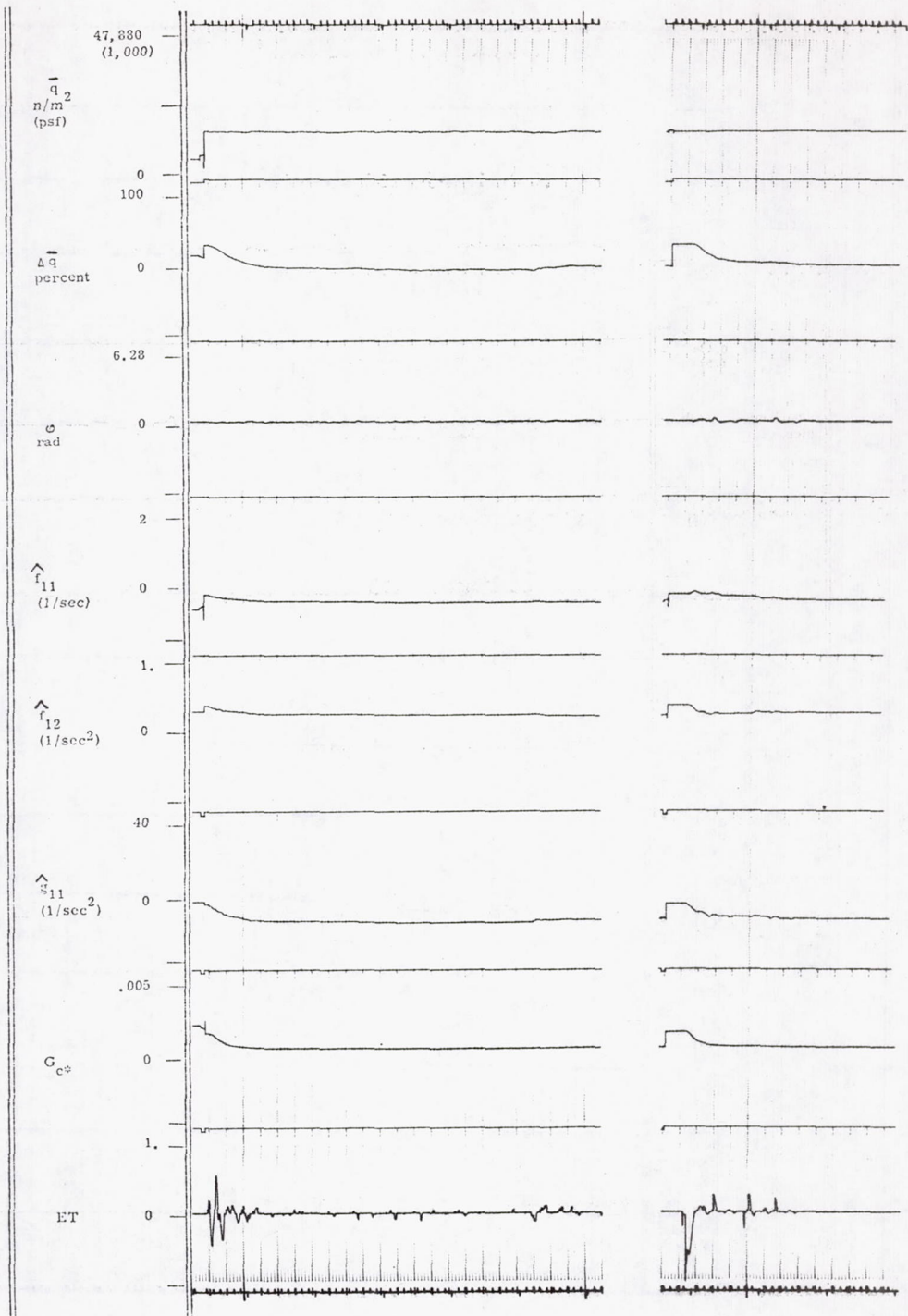


Figure 82. Concluded.

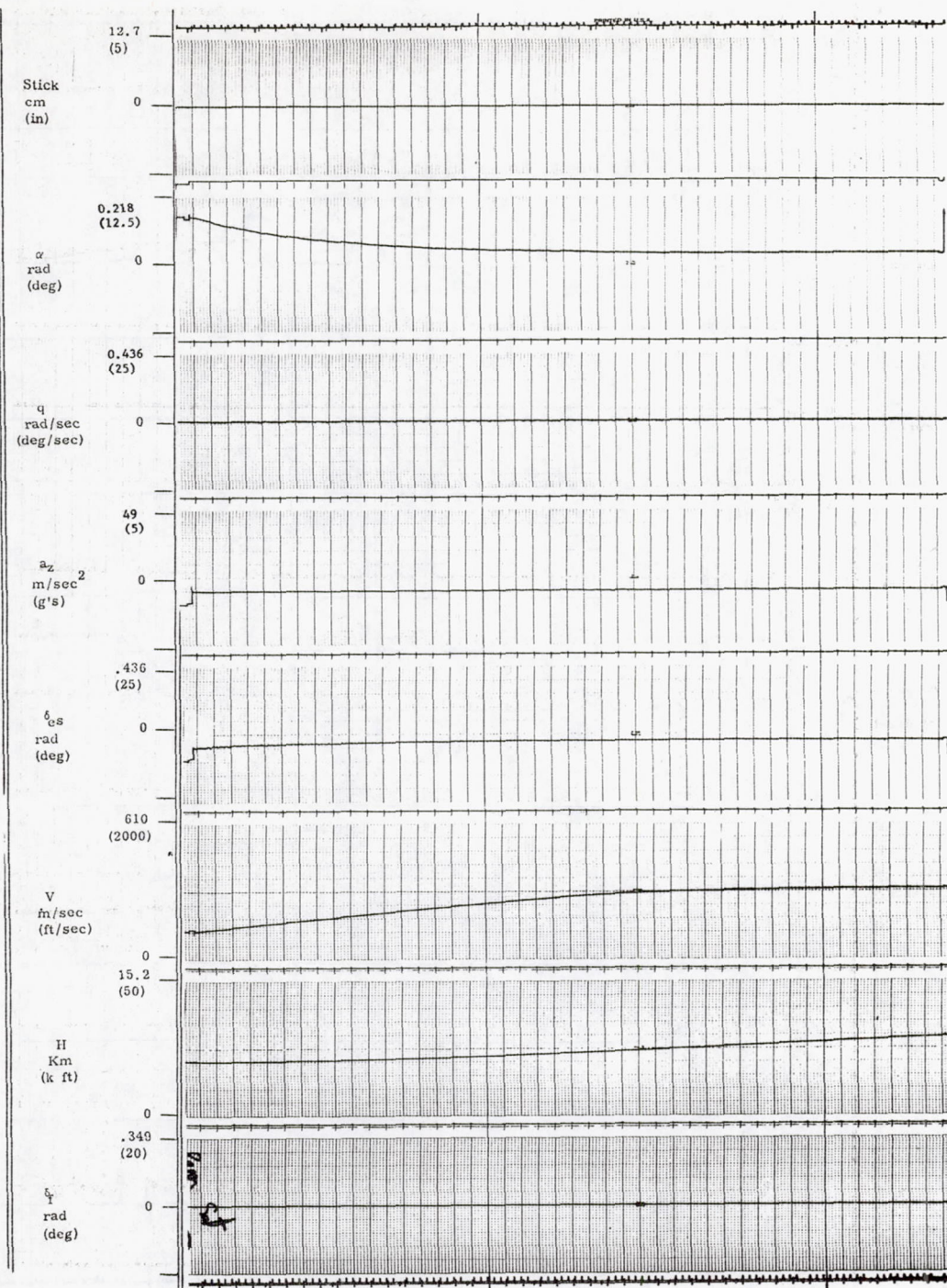


Figure 83. Model tracker during acceleration (test signal).

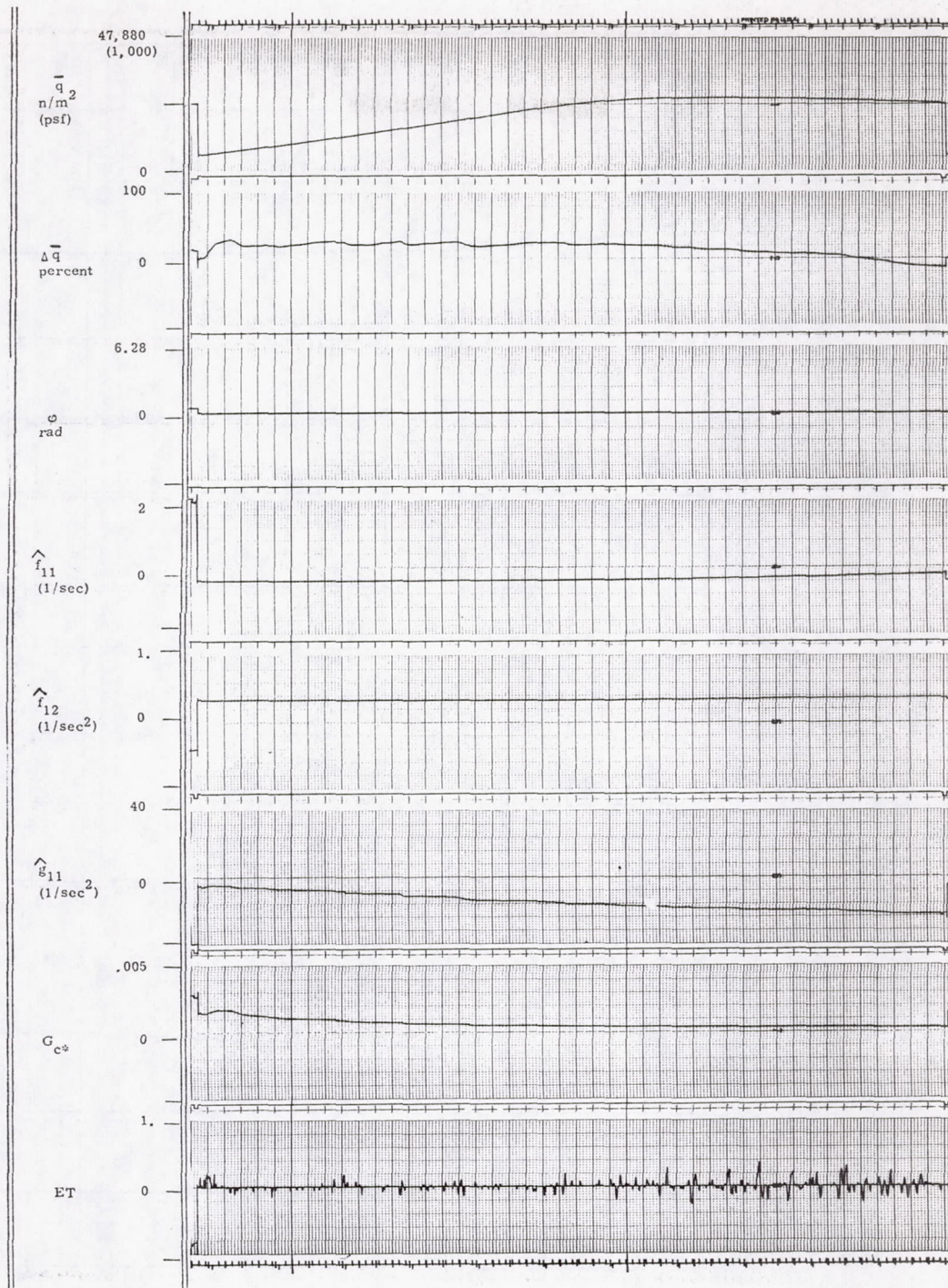


Figure 83. Concluded.

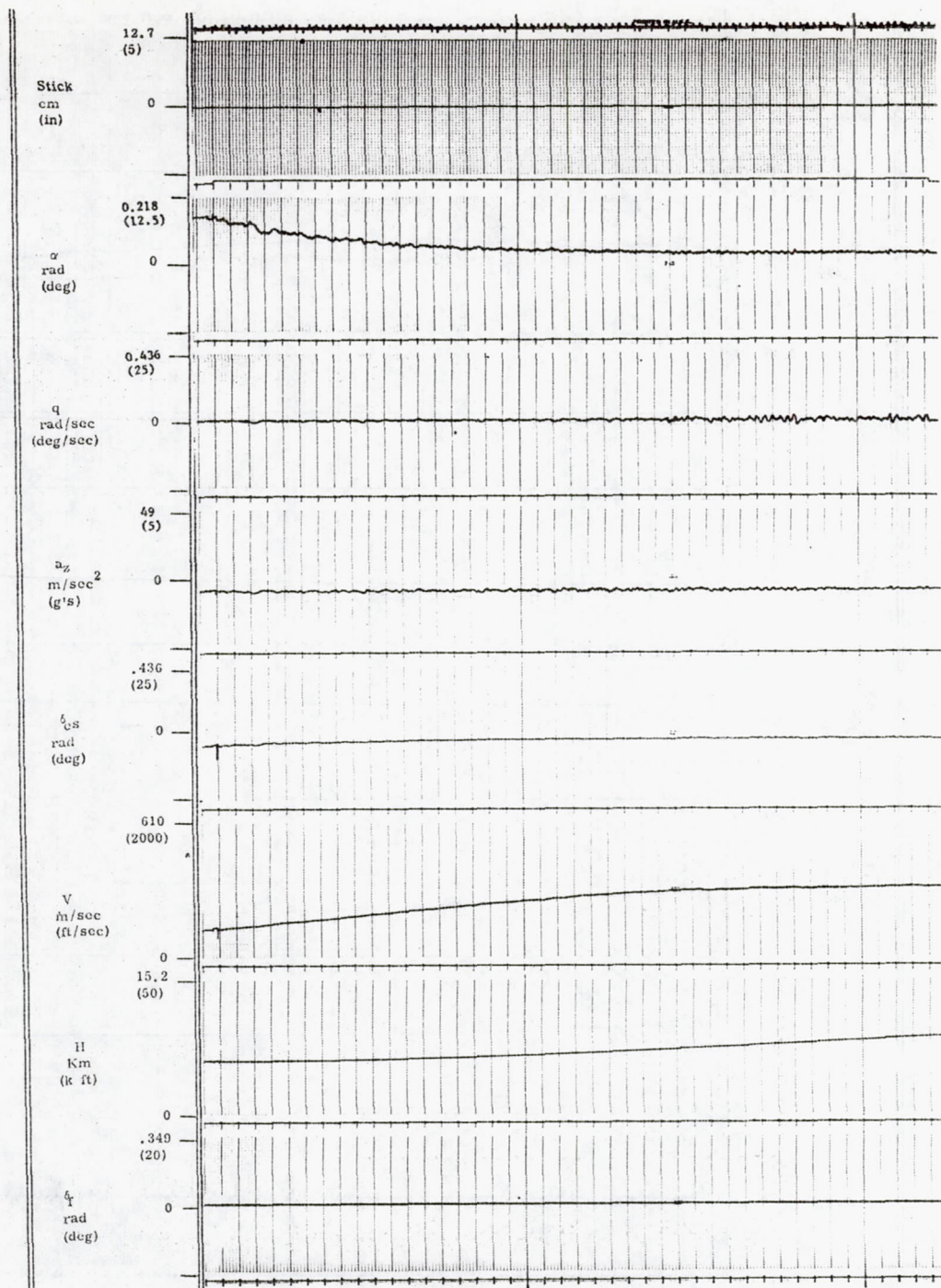


Figure 84. Model tracker during acceleration (gusts).

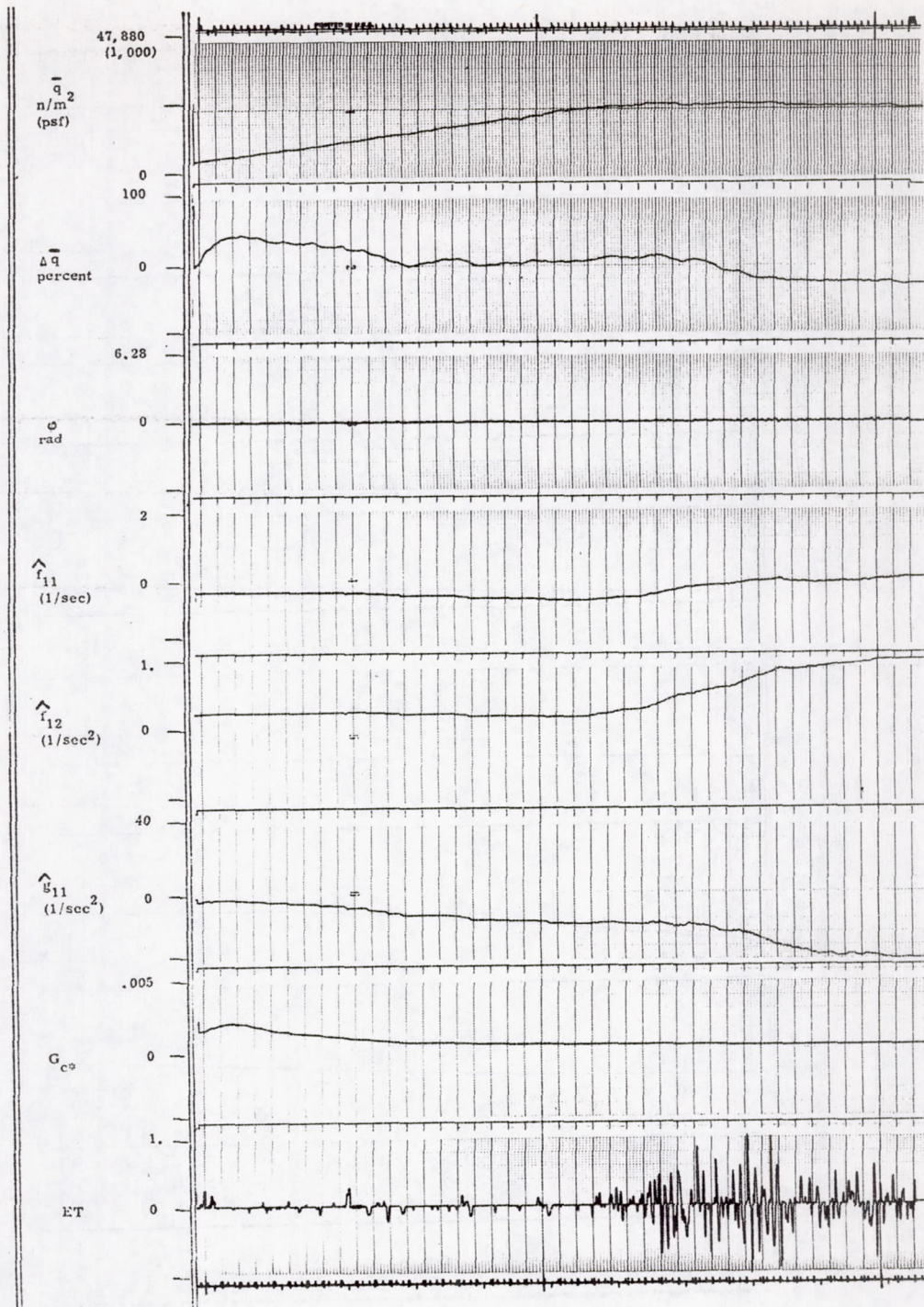


Figure 84. Concluded.

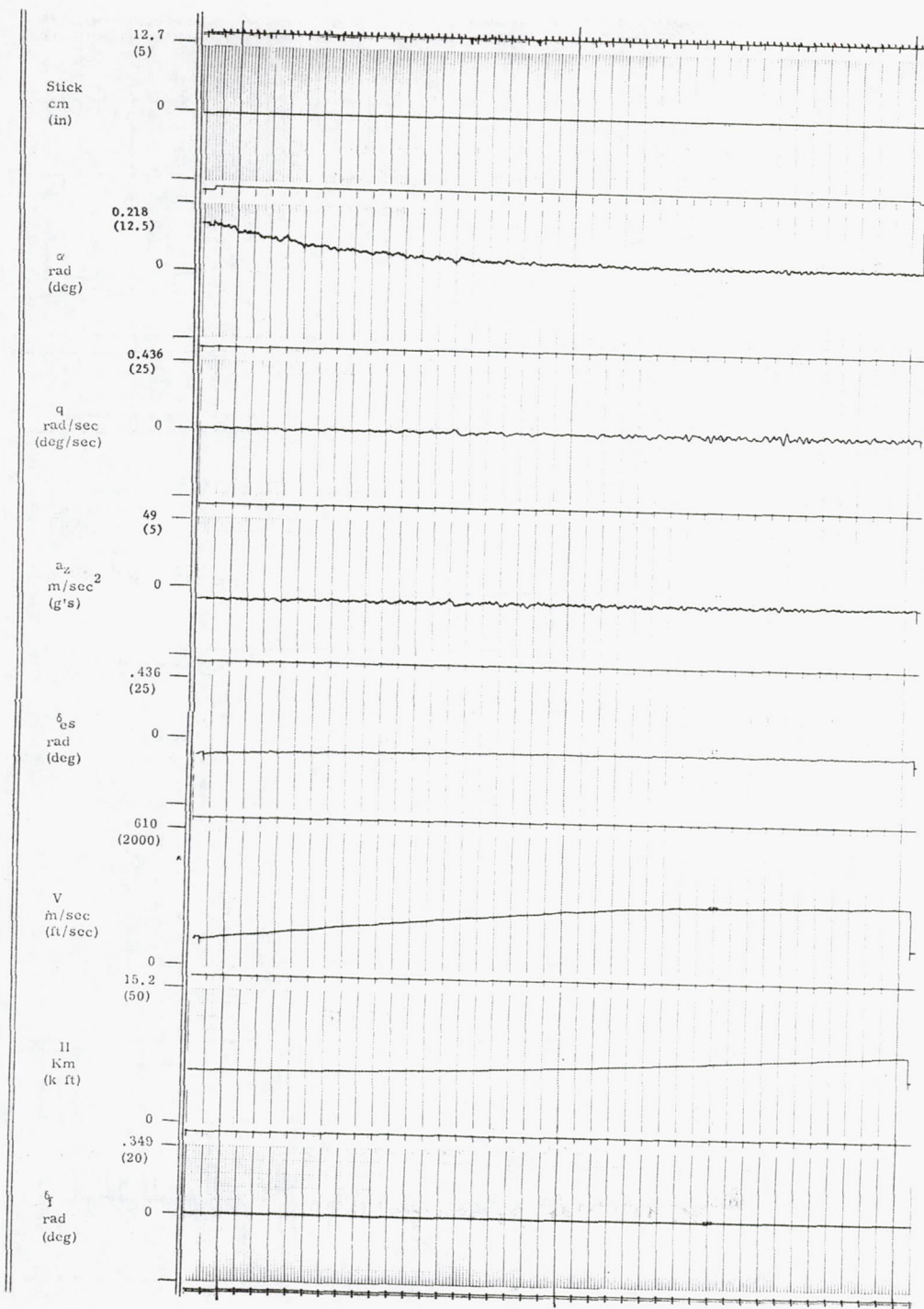


Figure 85. Model tracker during acceleration (gusts plus sensor noise).

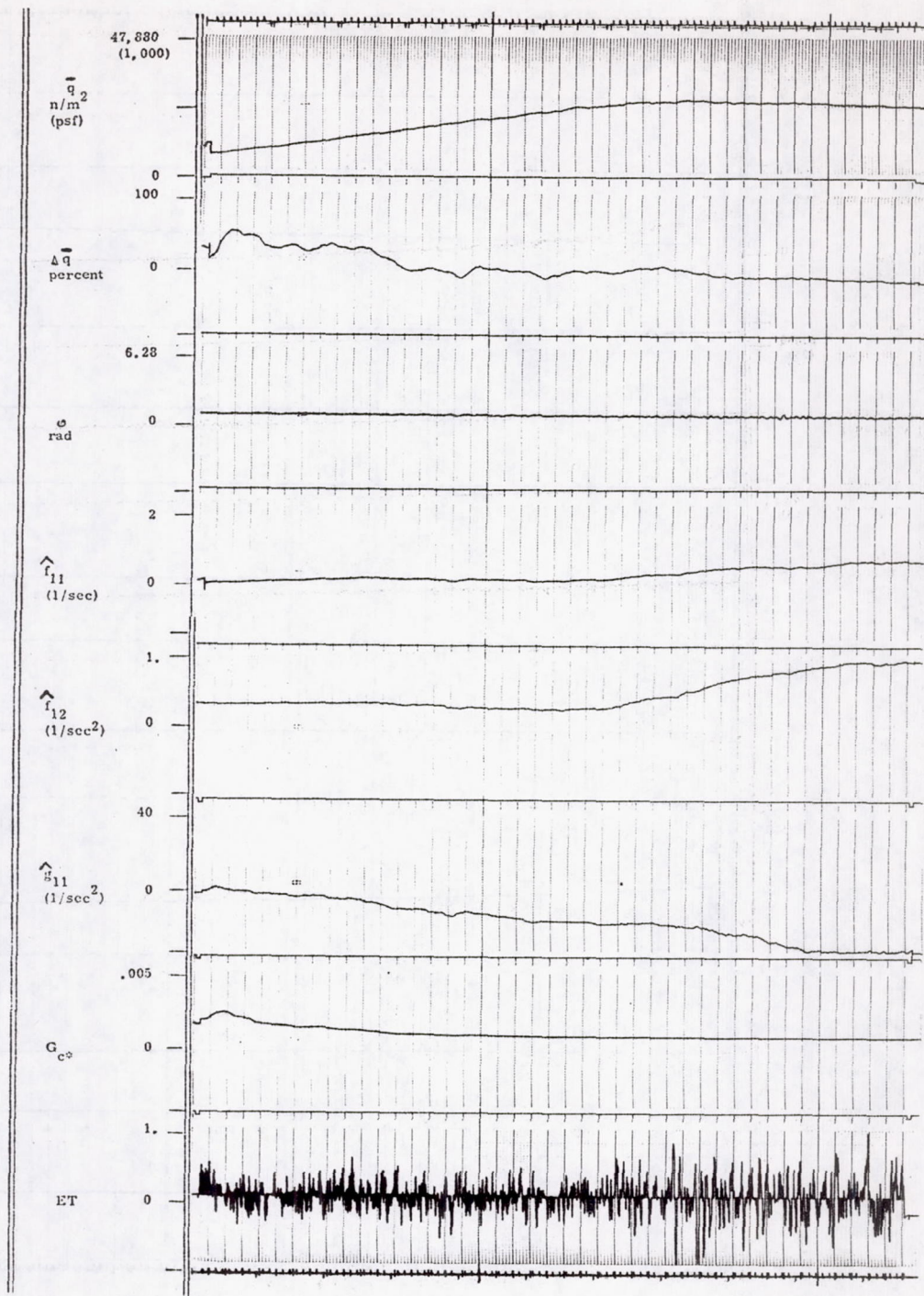


Figure 85. Concluded.

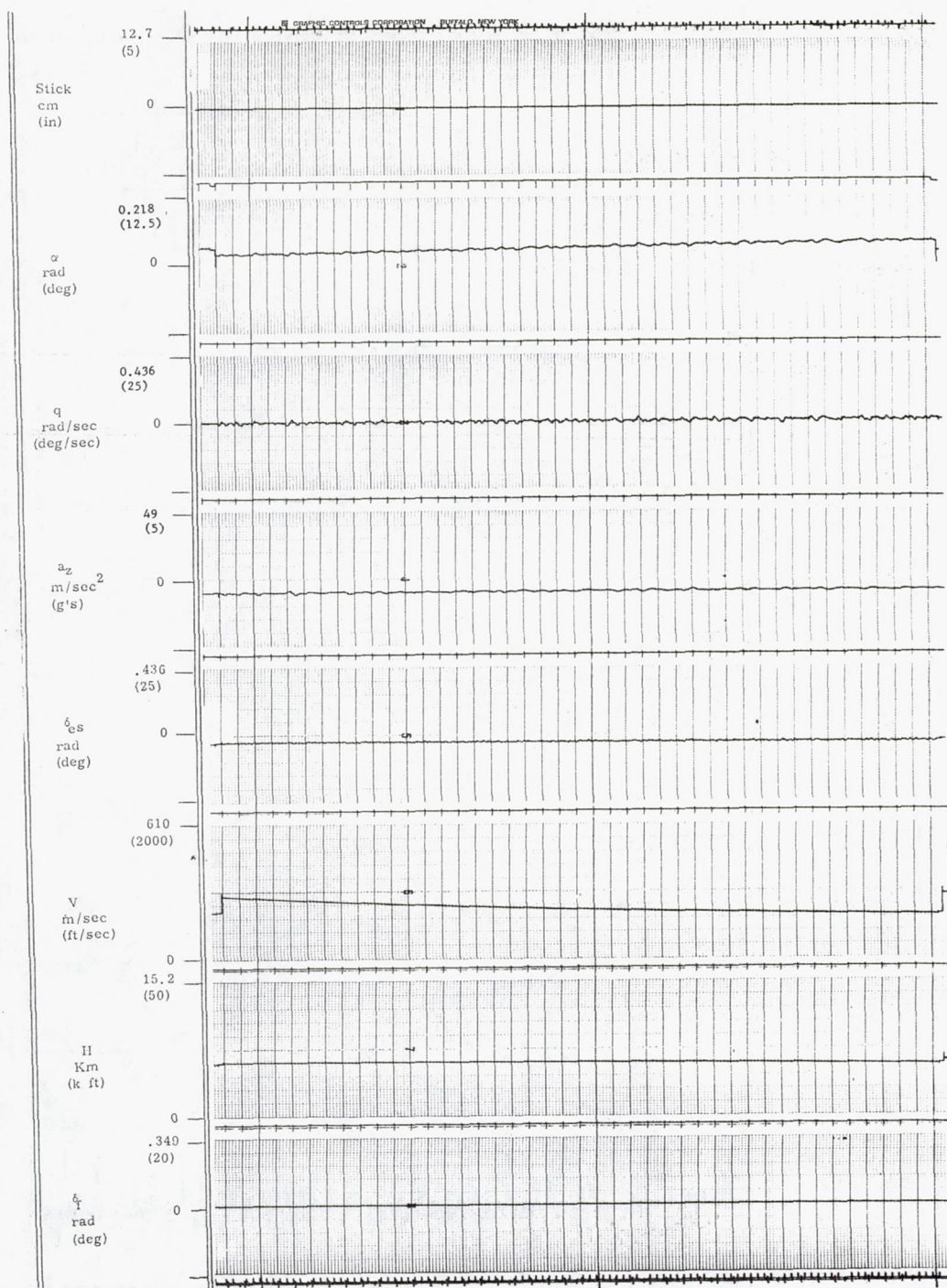


Figure 86. Model tracker in deceleration (test signal).

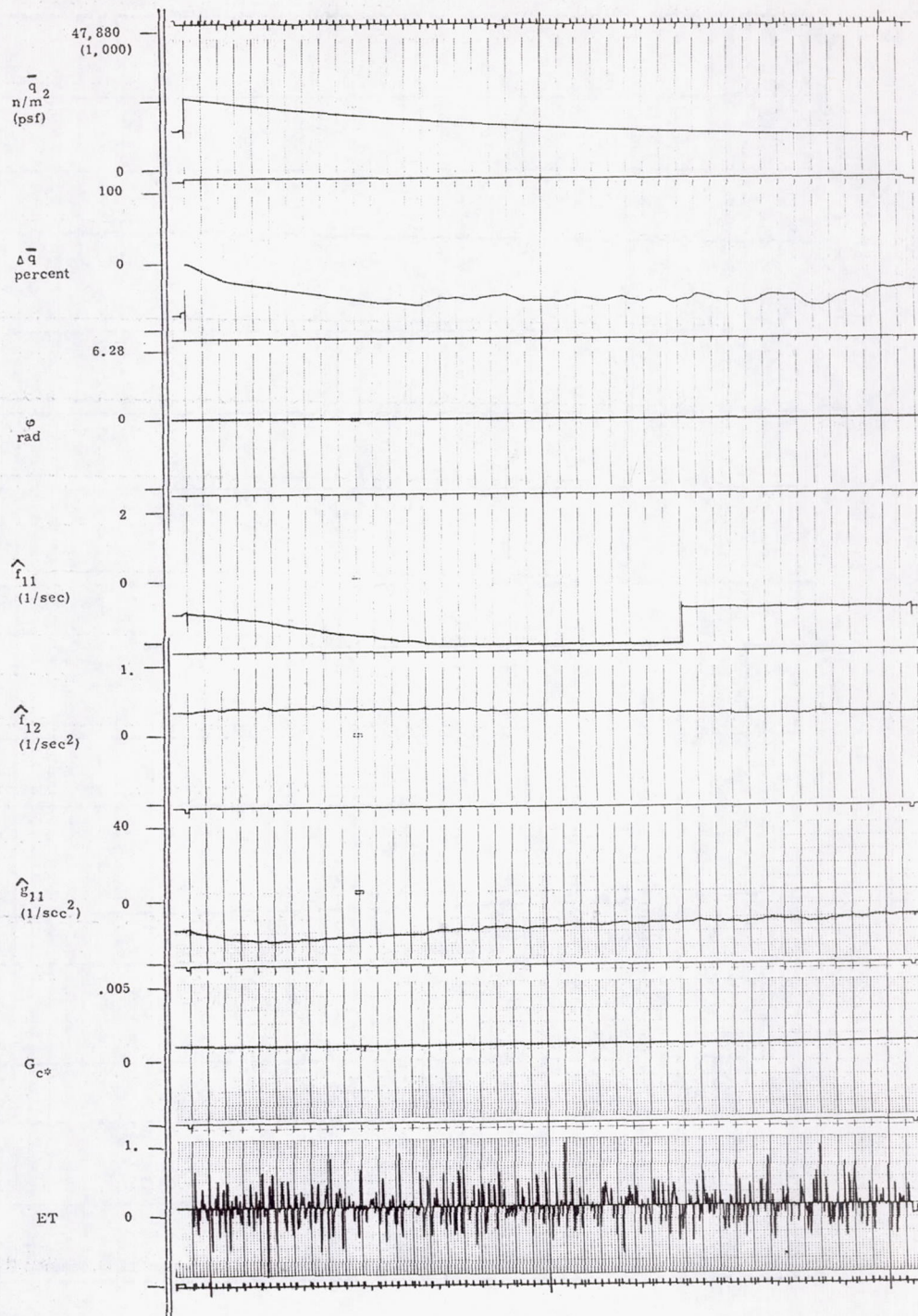


Figure 86. Concluded.

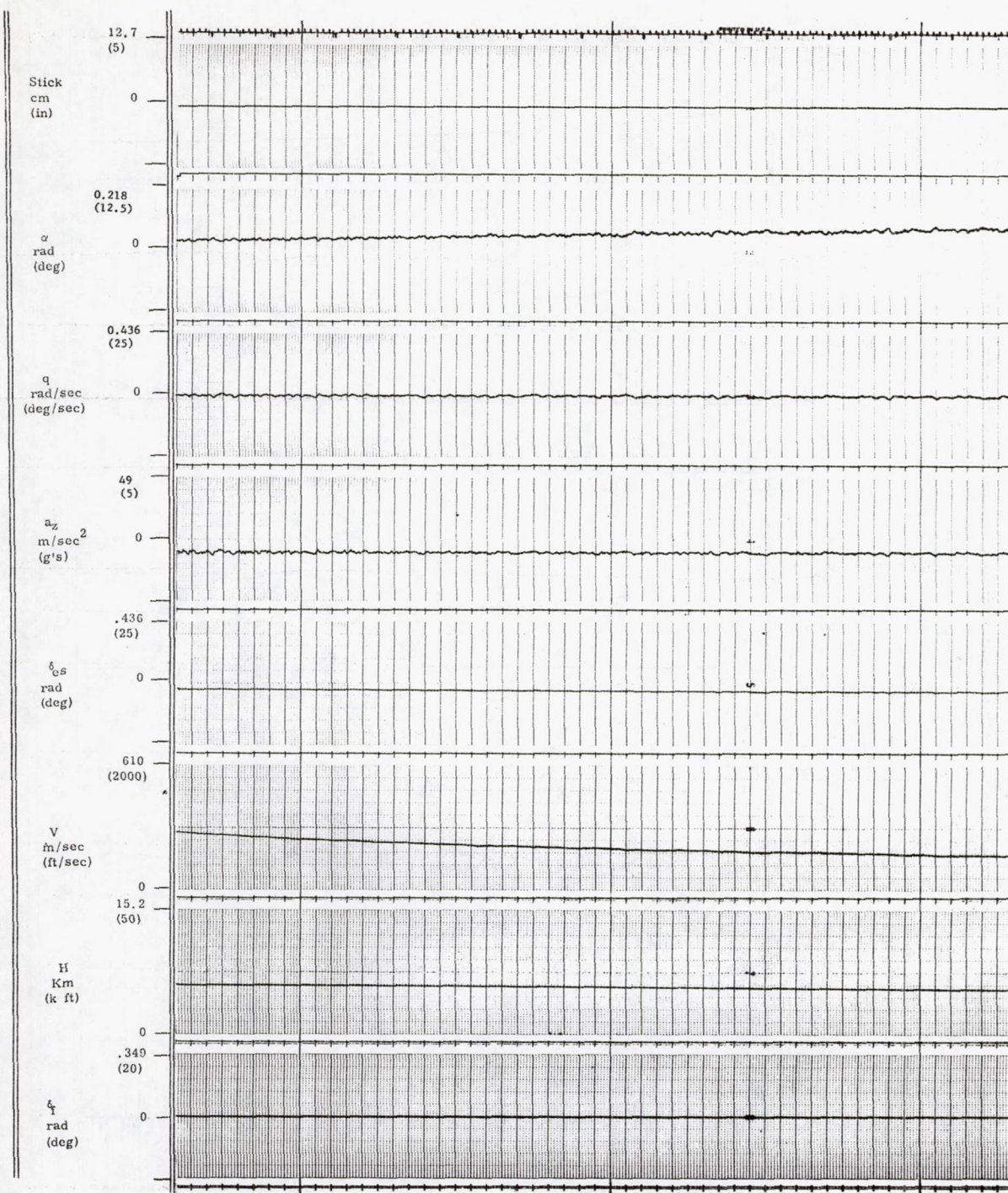


Figure 87. Model tracker in deceleration (gusts).

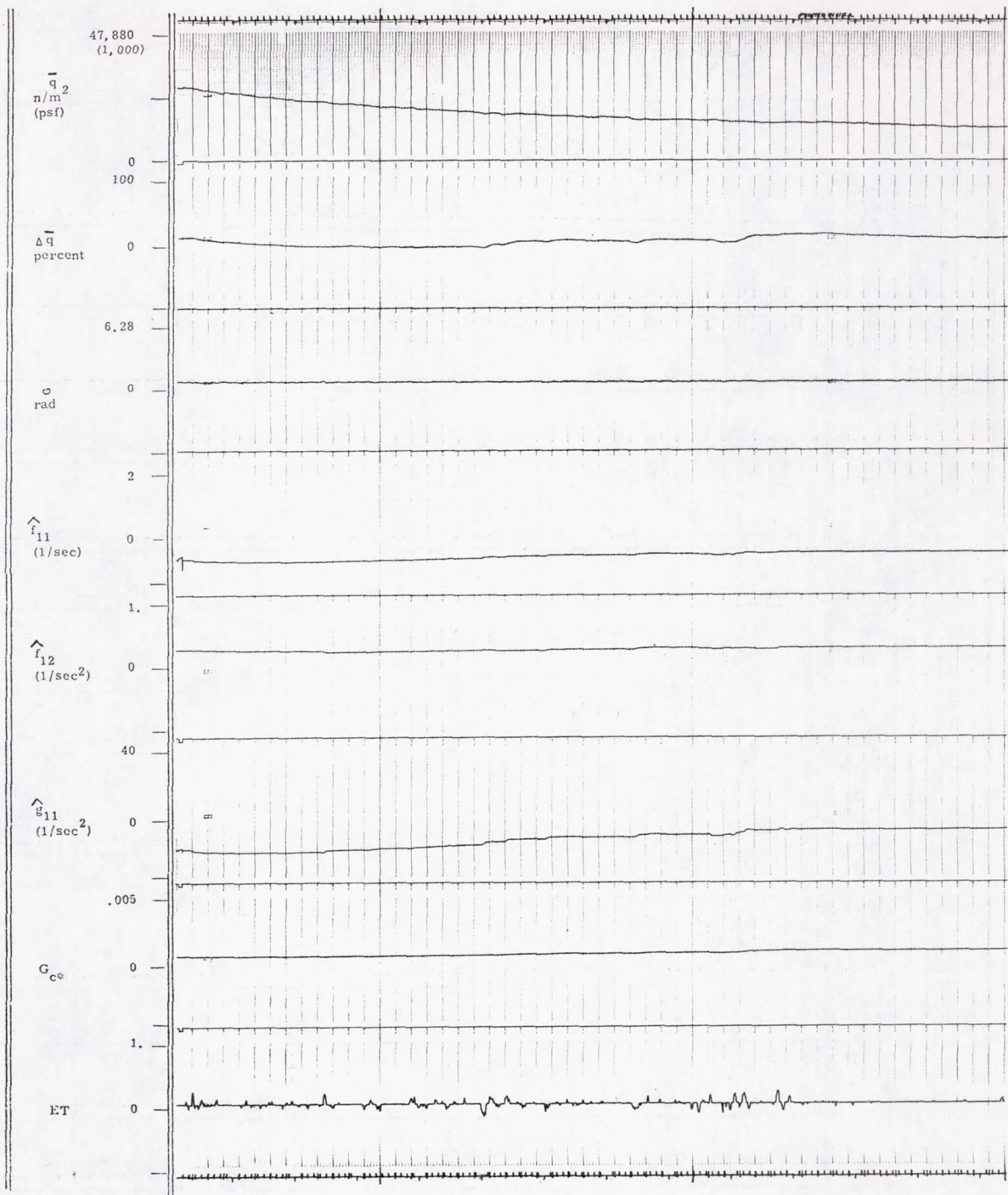


Figure 87. Concluded.

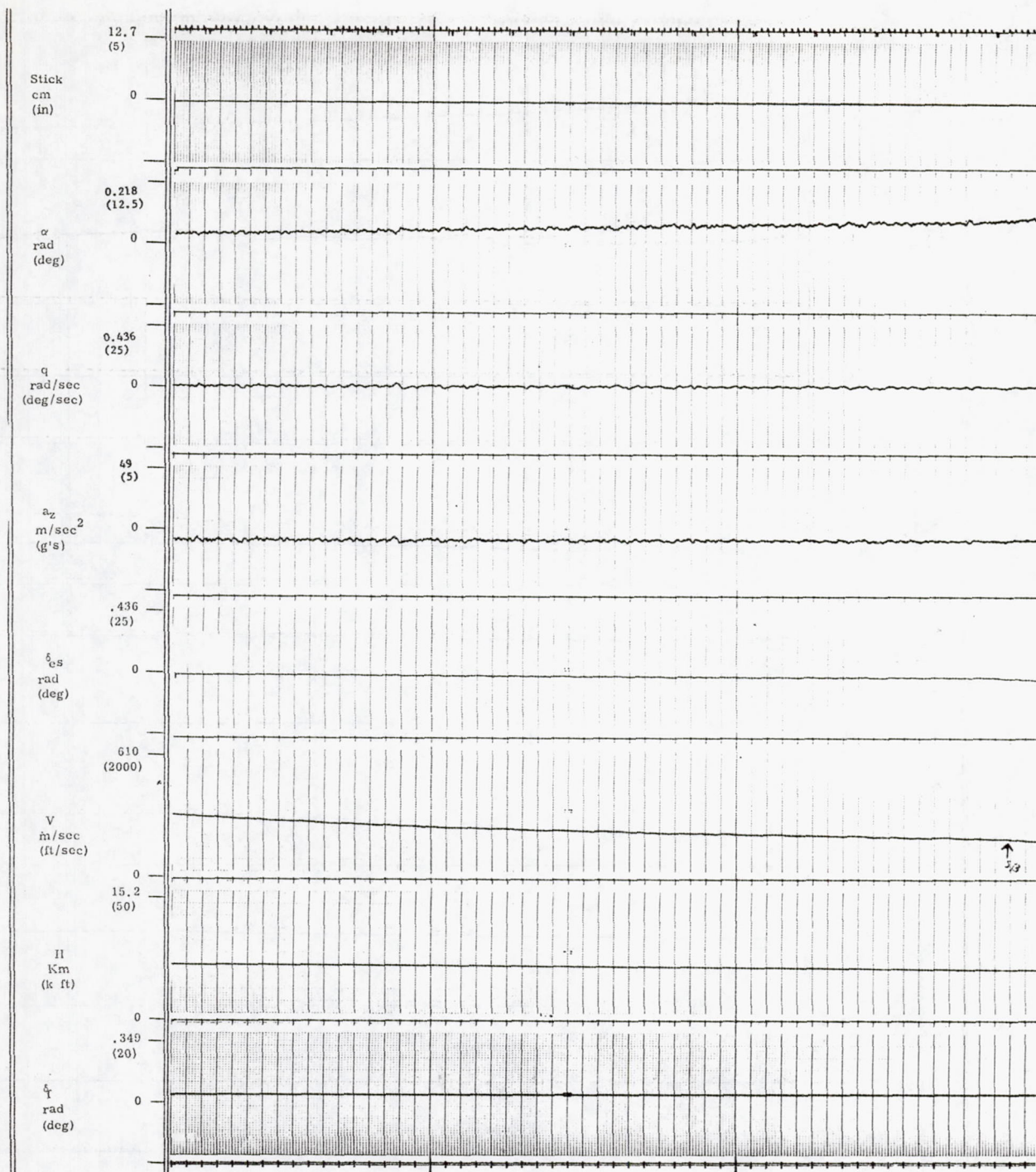


Figure 88. Model tracker in deceleration
(gusts plus sensor noise).

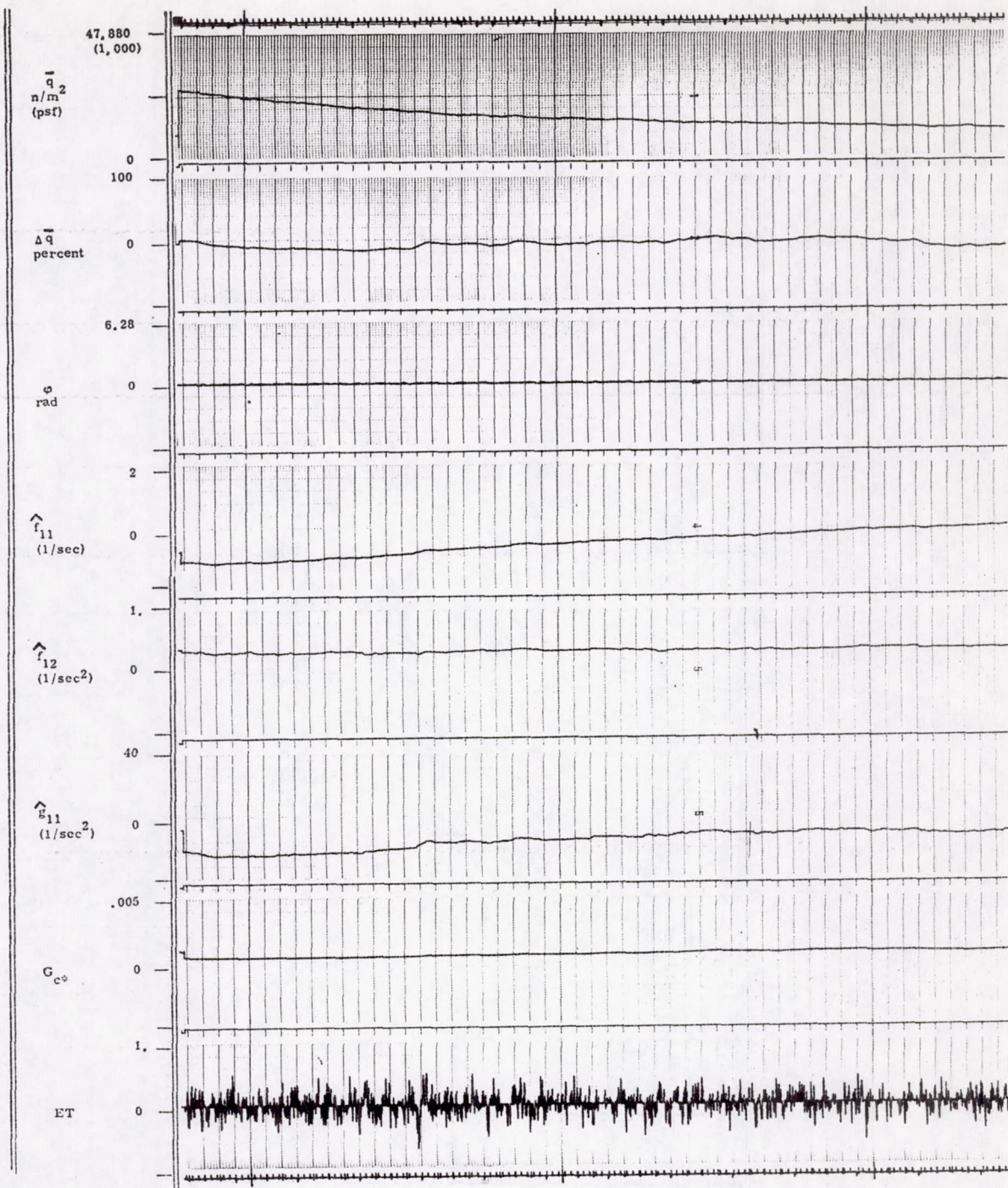


Figure 88. Concluded.

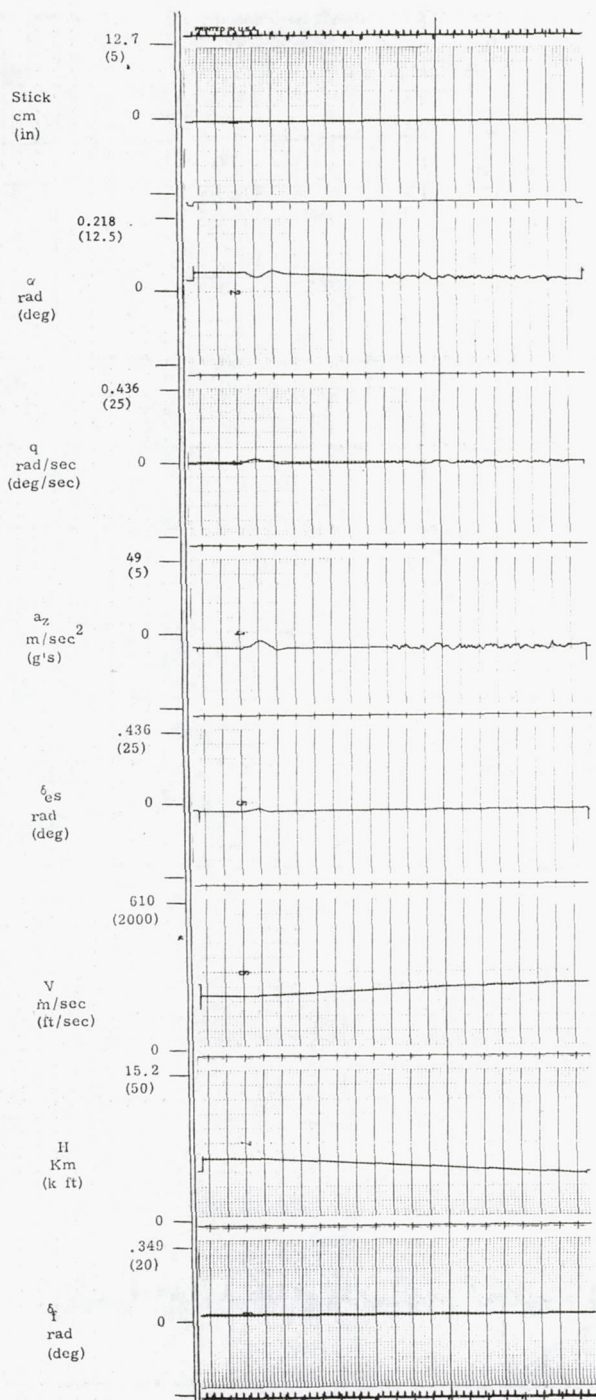


Figure 89. Model tracker response to snap roll (FC1).

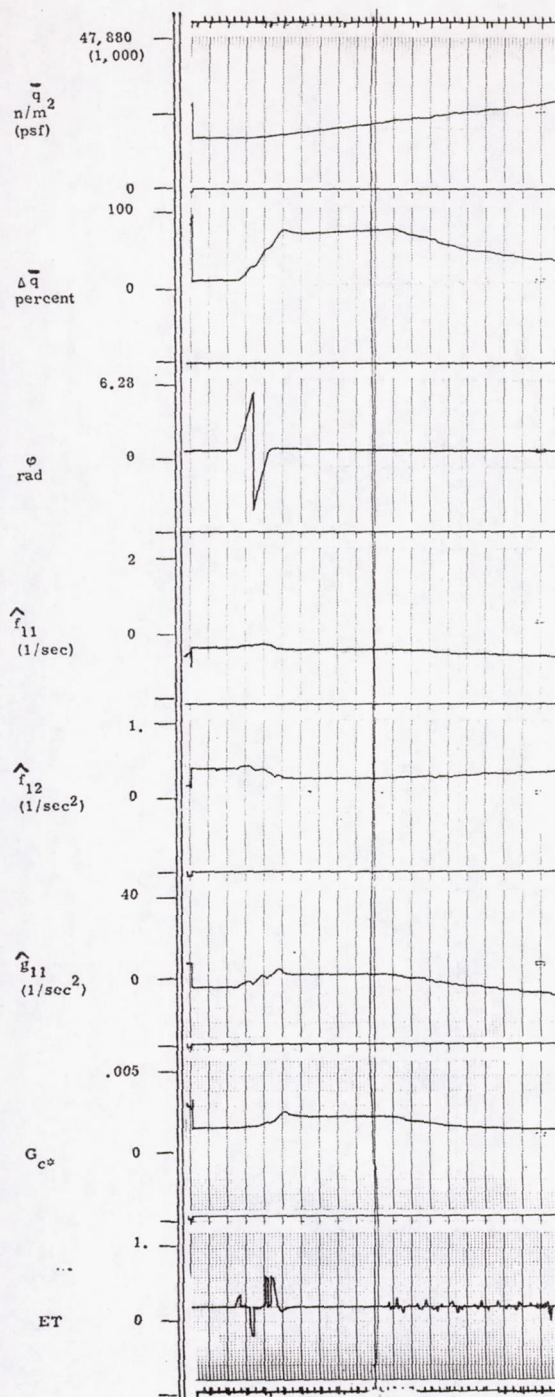


Figure 89. Concluded.

APPENDIX D

LIMIT-CYCLE GAIN-CHANGER PITCH AXIS TIME HISTORIES

APPENDIX D

LIMIT-CYCLE GAIN-CHANGER PITCH AXIS TIME HISTORIES

Time histories taken on the Langley simulator consisted of 16 channels of data (two eight-channel recorders). The channels are:

<u>Recorder 1</u>	<u>Recorder 2</u>
Pitch stick deflection	Dynamic pressure ($\bar{q}$)
Angle-of-attack (α)	G_{C*} gain
Pitch rate (q)	Elevator servo position (δ_{e_s})
Normal acceleration (a_3)	Aileron servo position (δ_{a_s})
Elevator position (δ_e)	G_{LAT} gain
Total velocity (V)	Bank angle (φ)
Altitude (H)	Roll rate (r)
Flap position (δ_f)	Sidelsip (β)

Scales for each trace are indicated in MKS and conventional units.

ACCURACY AT FIXED FLIGHT CONDITIONS

Standard pitch sequences consisting of periods of quiet, turbulence, and doublet commands are presented in Figure 90 through 94 for Flight Conditions 1, 5, 8, 10 and 17 (power approach). This sequence of runs was then repeated with gyro noise, accelerometer noise, and servo measurement noise simultaneously applied (Figure 95 through 98).

Finally, the sequence at Flight Condition 1 was repeated with servo hysteresis of 0.05 degrees. (Refer to Figure 99.)

CONVERGENCE PROPERTIES

The concept of "converging" to a gain value does not apply to the limit-cycle concept in the same way it does to the explicit identifiers. In normal operation, the gain changer will not be at a gain higher than critical. If the system starts at a value below critical, the gain increase is largely determined by the integral gain. Figures 100 and 101 show the responses at FC5 with the system initialized at FC10. In turbulence it takes on the order of ten seconds to reach "steady state" (which is six db below critical). (Refer to Figure 100). For pilot commands, the gain is reduced slightly.

ADAPTATION

In the case of increasing and decreasing dynamic pressure, the gain changer tracked the critical gain (quiet case) or tracked the "turbulence" gain value. Figure 102 shows a full power acceleration starting from FC 5. Note the constant limit-cycle amplitude. Figures 103 and 104 show the same run with turbulence and then with turbulence and sensor noise.

Figures 105 through 107 show the responses starting from FC 10 and decelerating by reducing thrust to idle. Again the limit-cycle amplitude remains constant (Figure 105). Adaptation during turbulence is shown in Figure 106. The same adaptation is repeated in Figure 107 with turbulence and sensor noise applied.

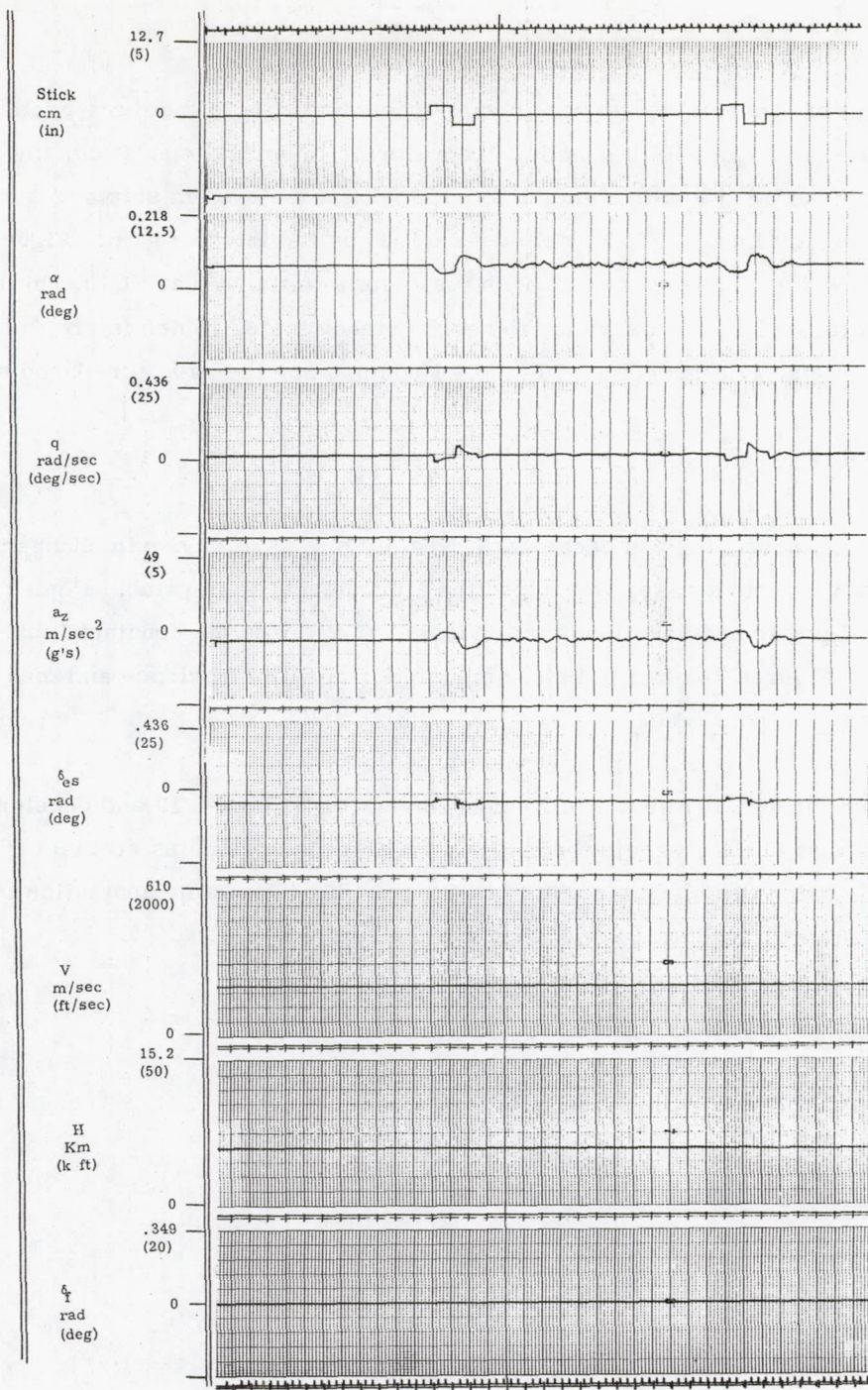


Figure 90. Limit-cycle design at FC1.

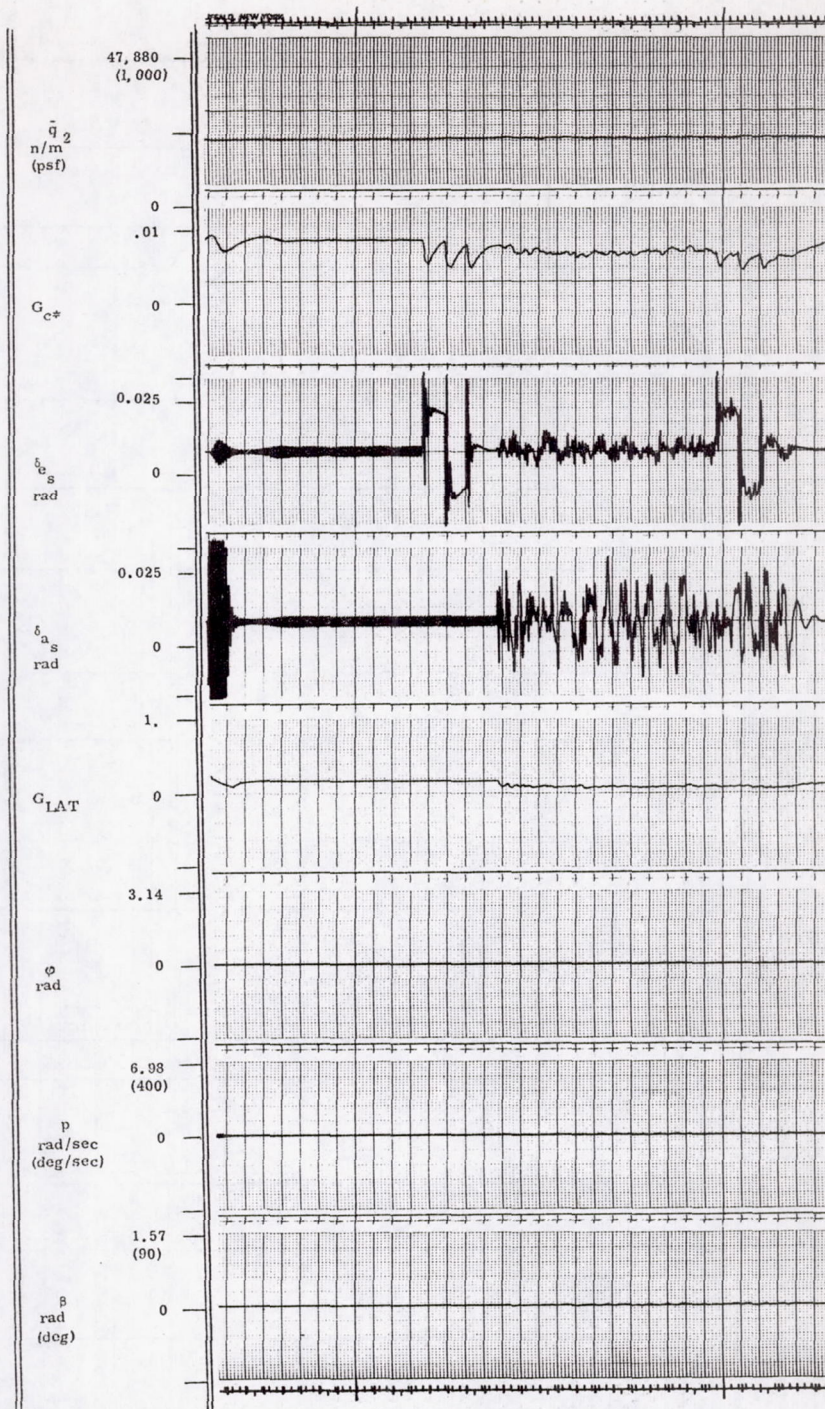


Figure 90. Concluded.

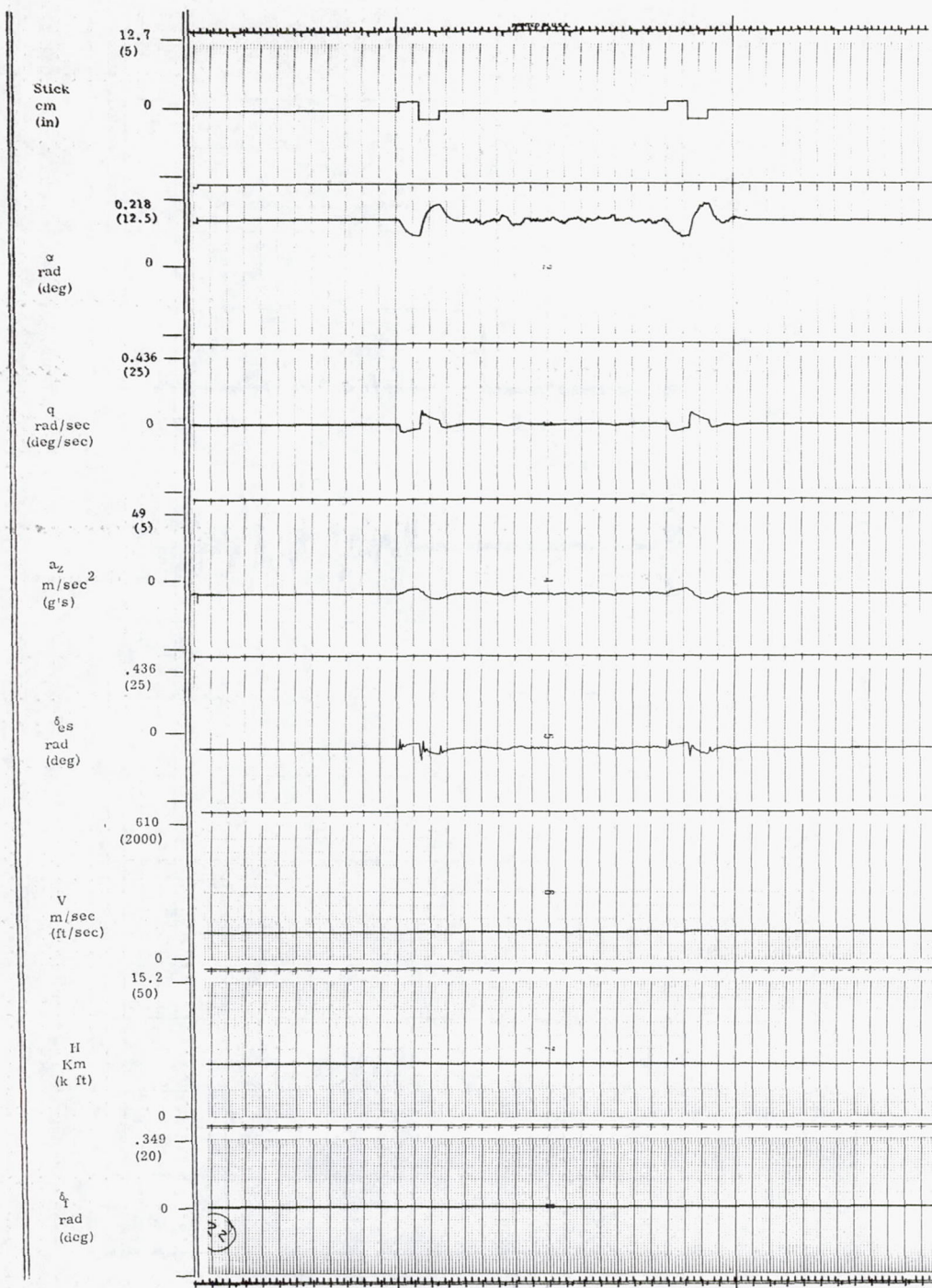


Figure 91. Limit-cycle design at FC5.

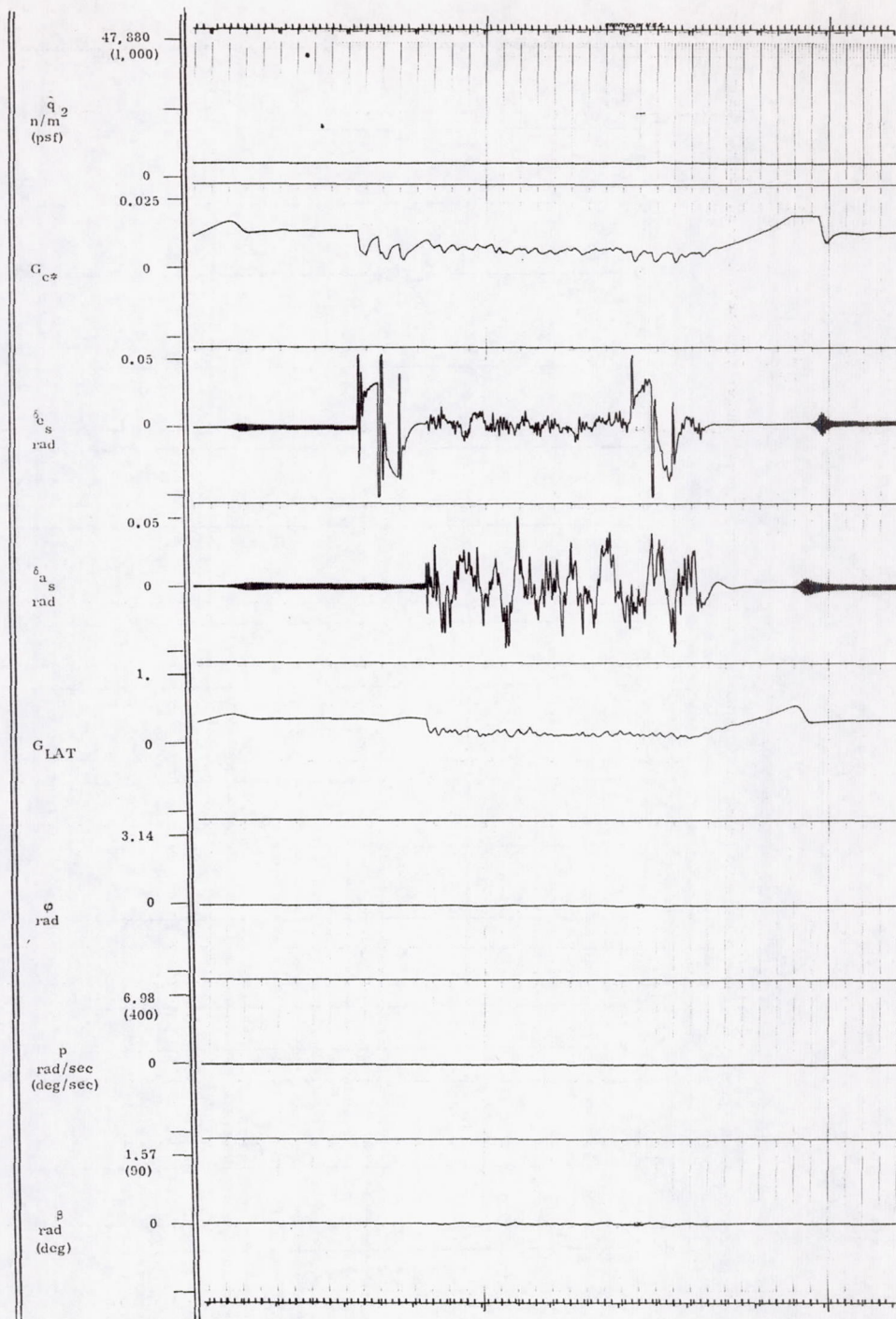


Figure 91. Concluded.

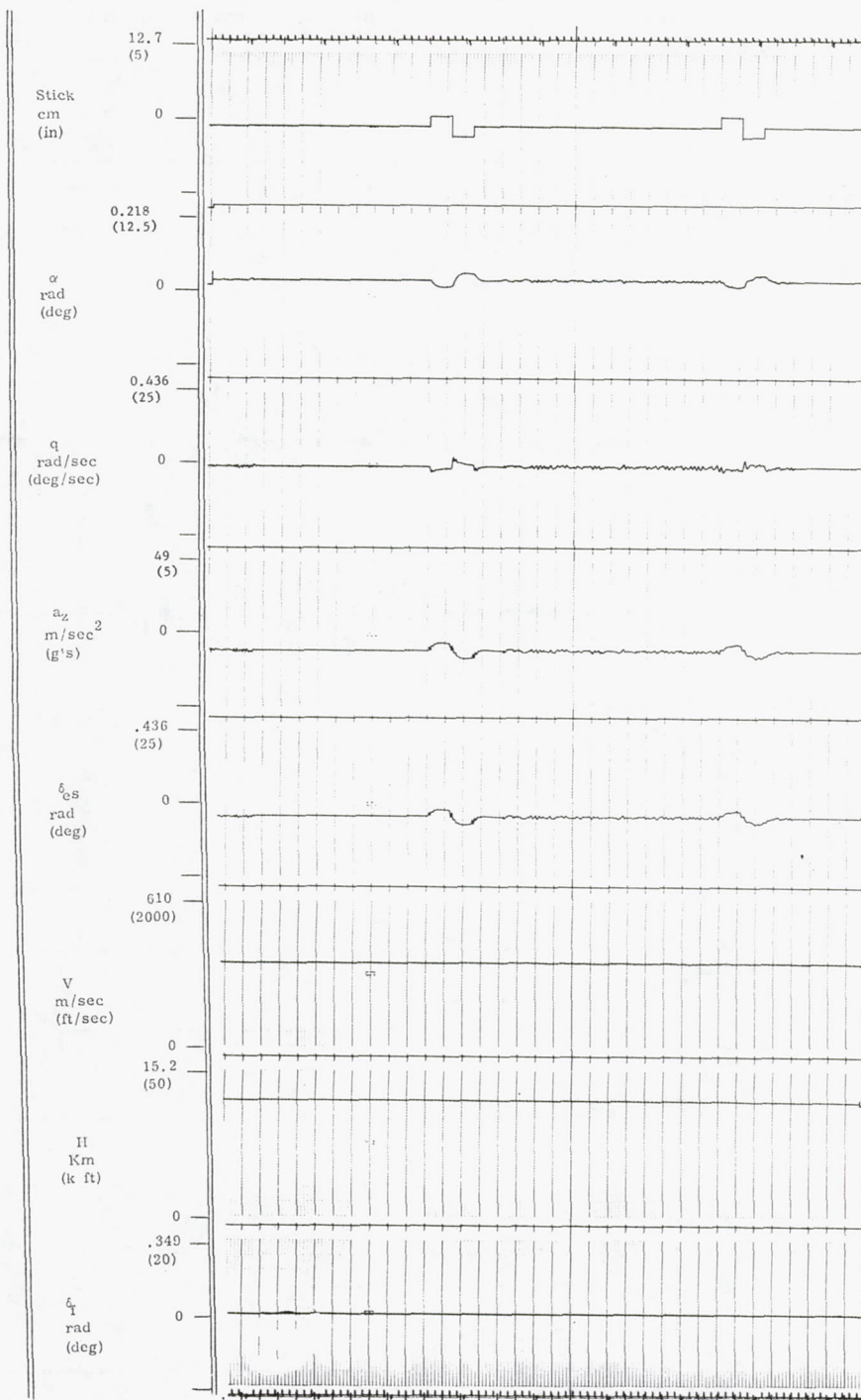


Figure 92. Limit-cycle design at FC8.

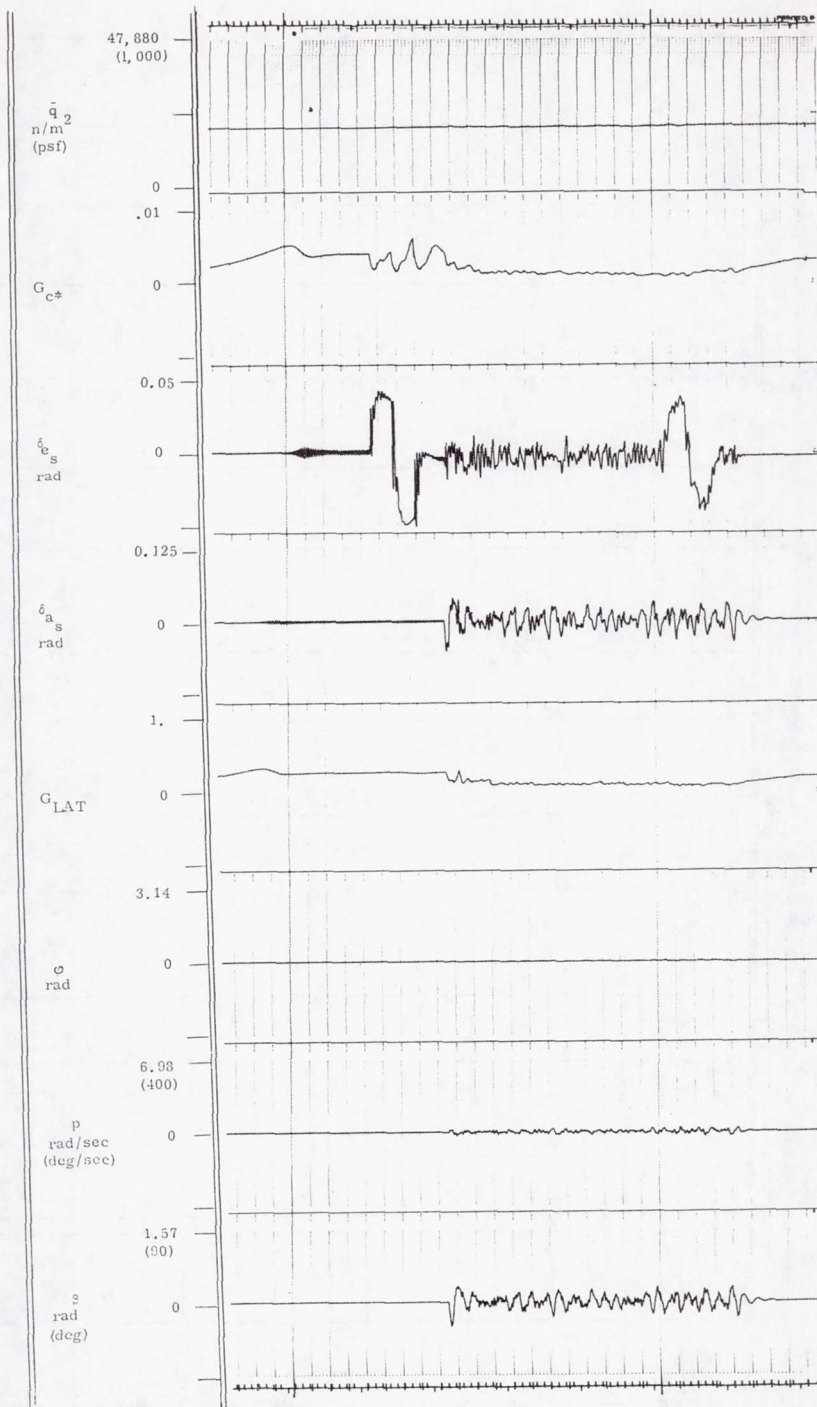


Figure 92. Concluded.

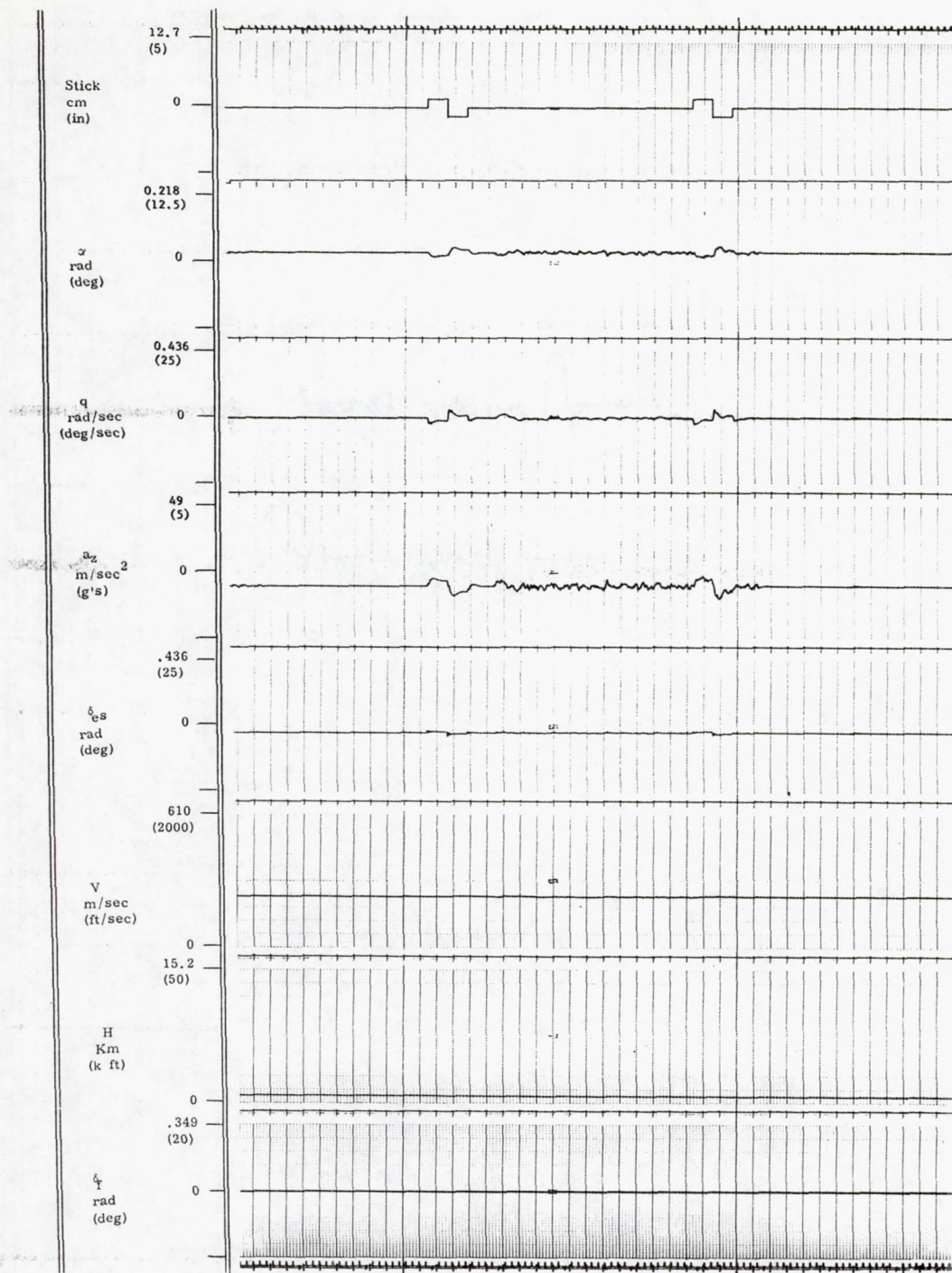


Figure 93. Limit-cycle design at FC10.

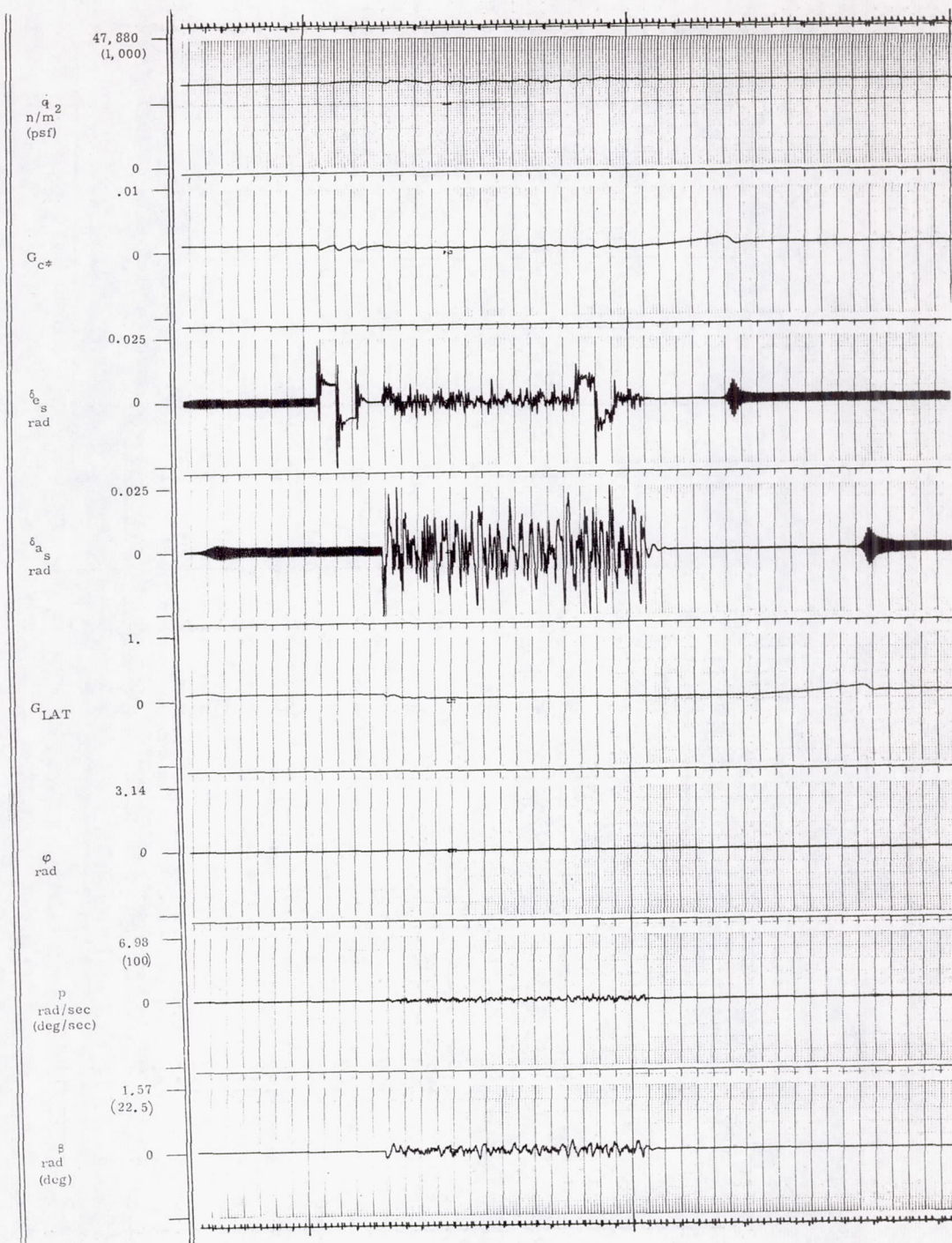


Figure 93. Concluded.

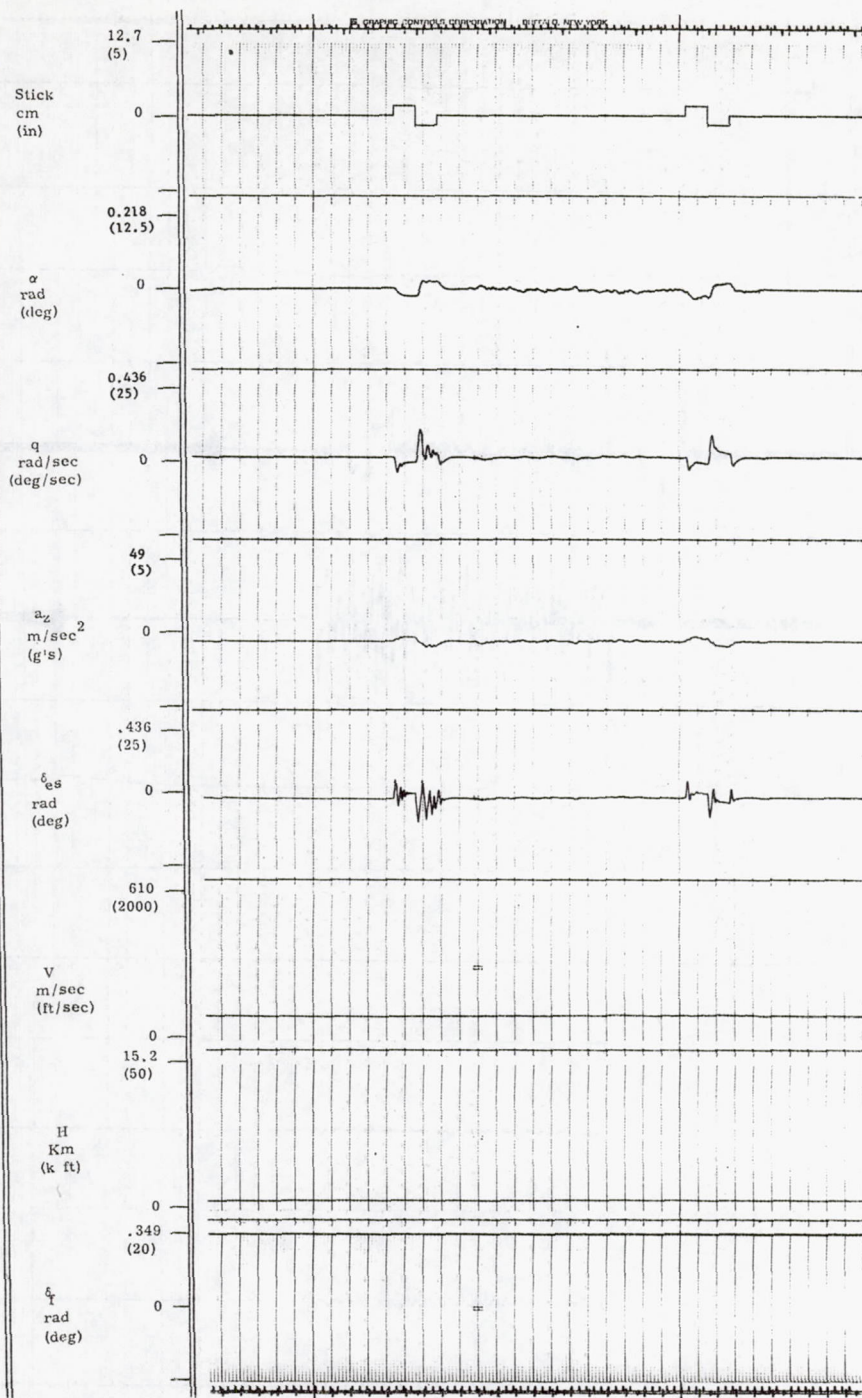


Figure 94. Limit-cycle design at FC17.

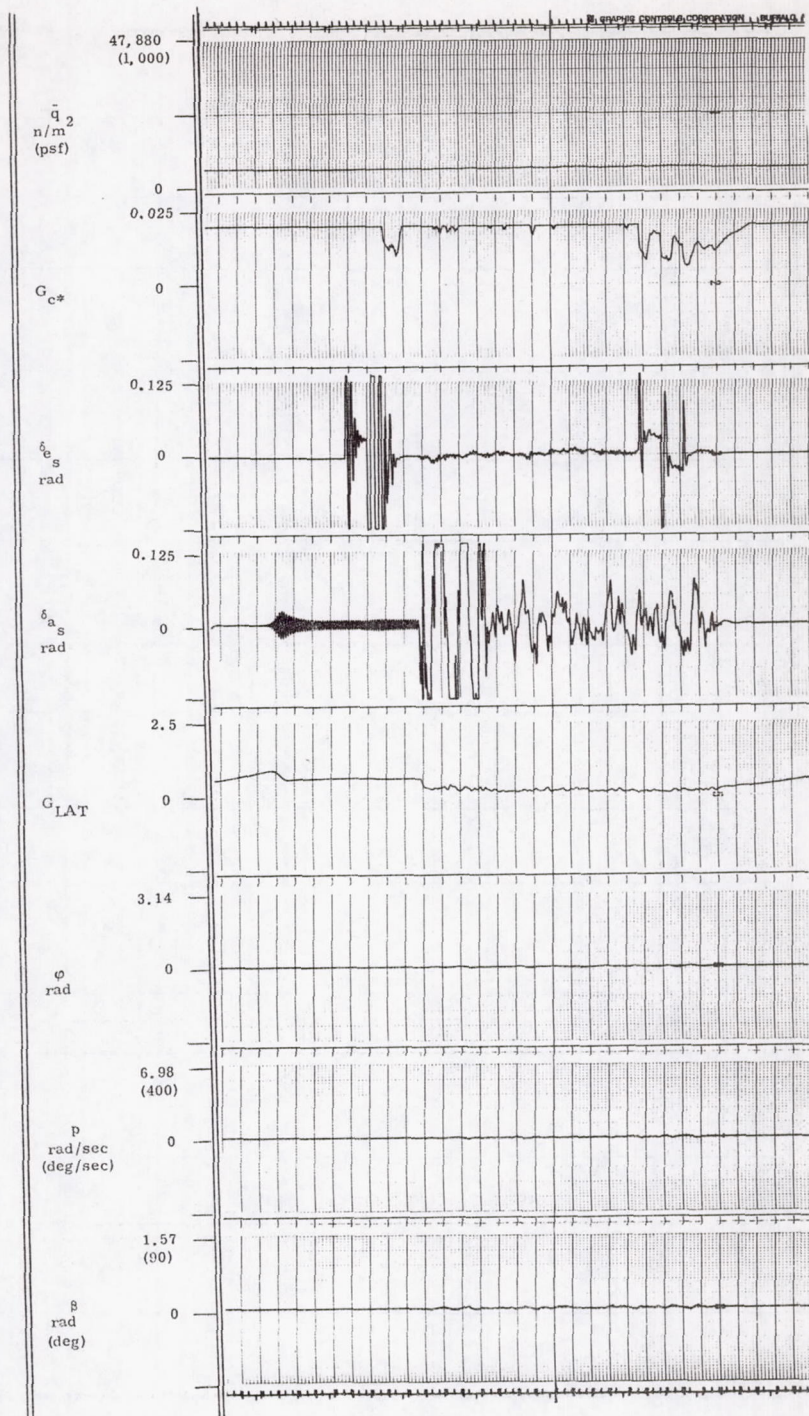


Figure 94. Concluded.

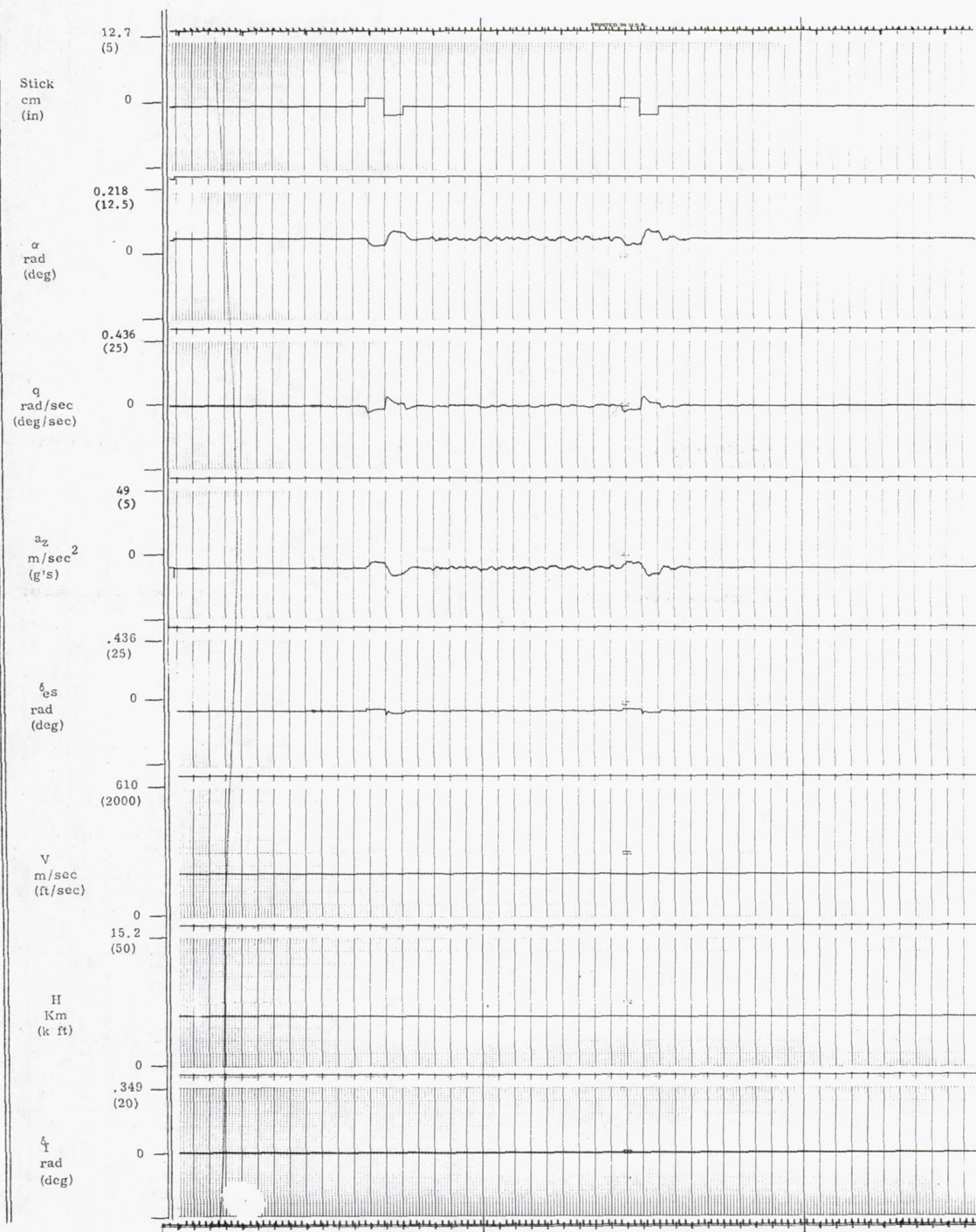


Figure 95. Limit-cycle design at FC1.
(sensor noise).

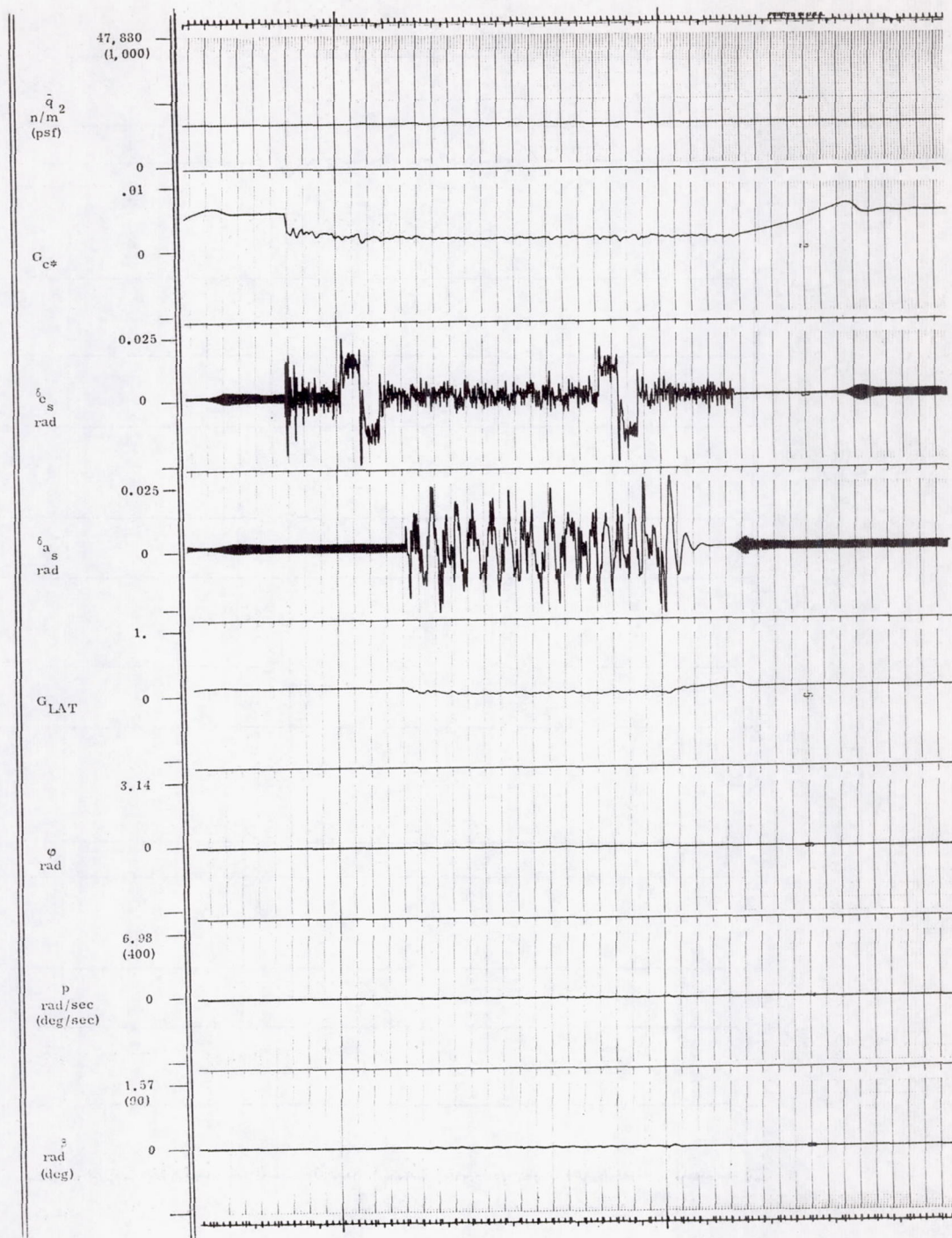


Figure 95. Concluded.

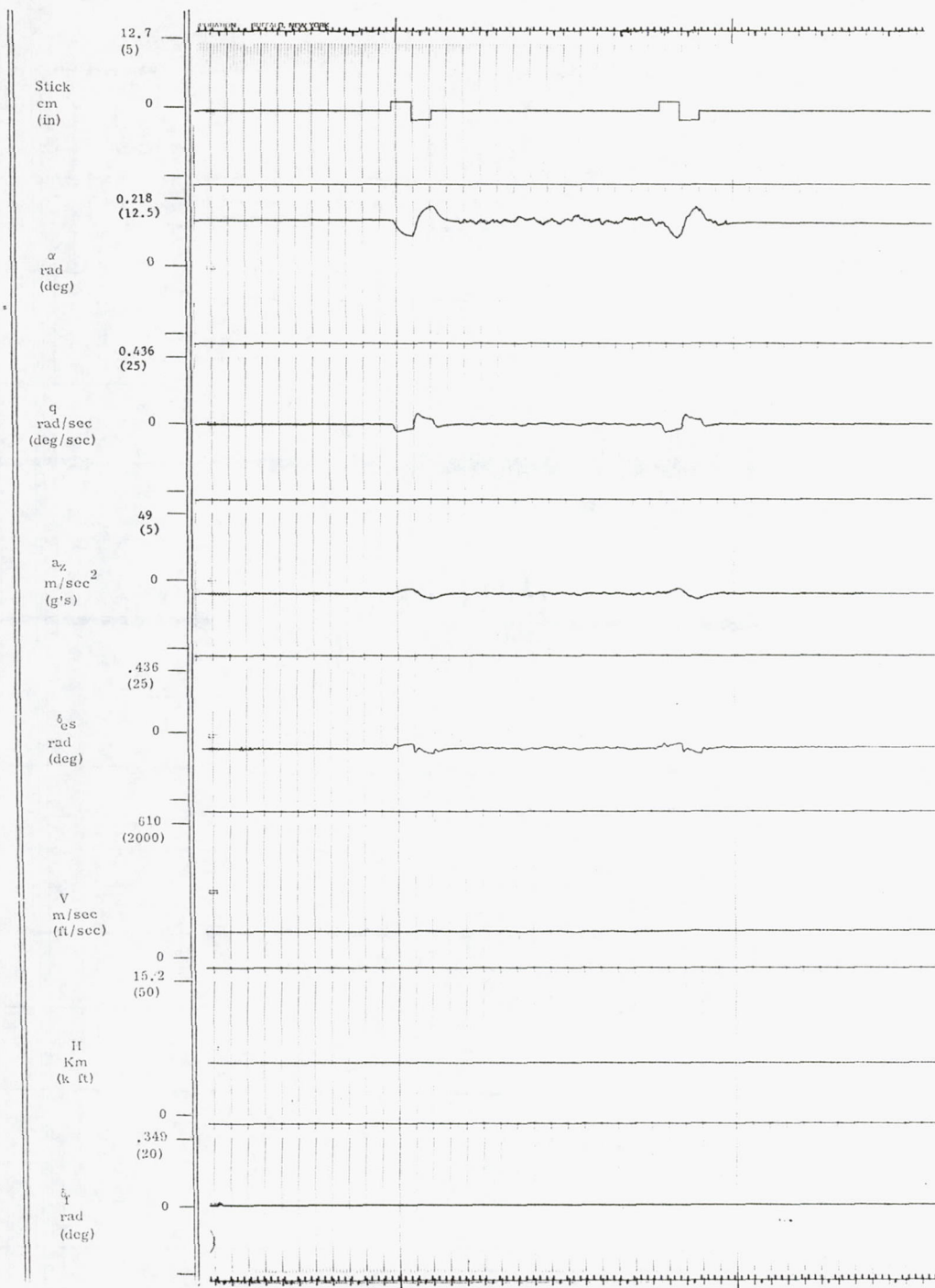


Figure 96. Limit cycle design at FC5 (sensor noise).

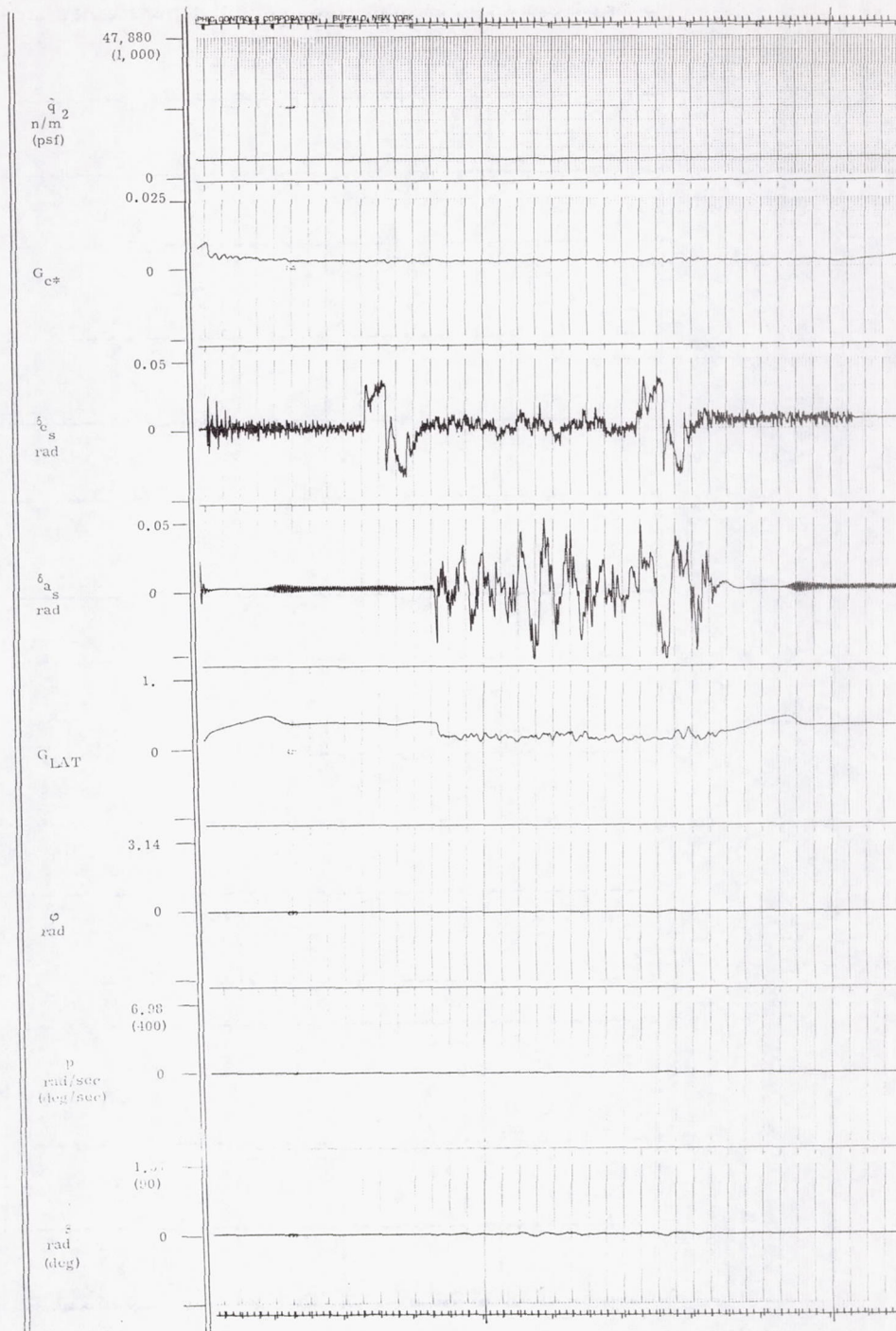


Figure 96. Concluded.

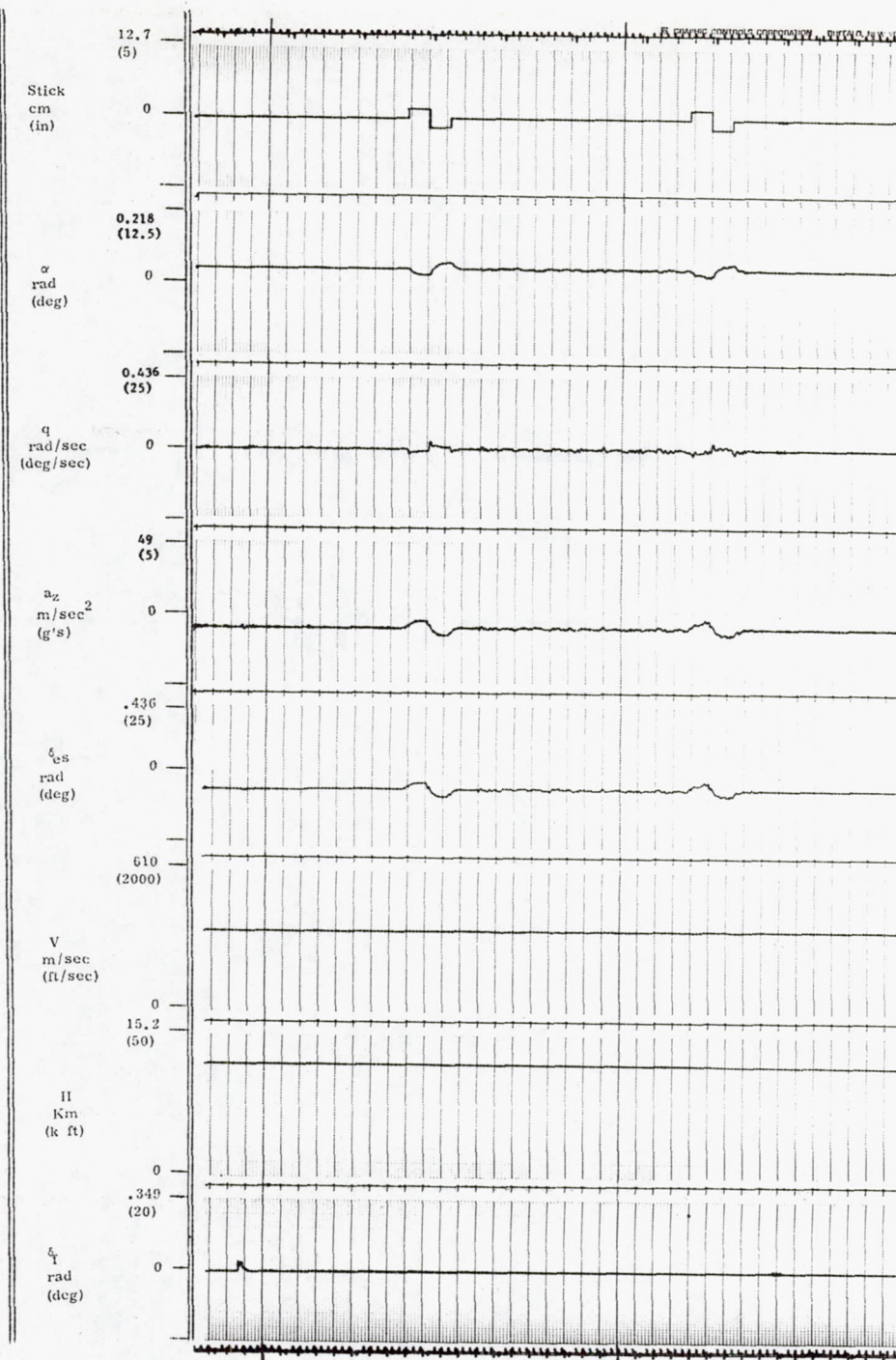


Figure 97. Limit-cycle design at FC8 (sensor noise).

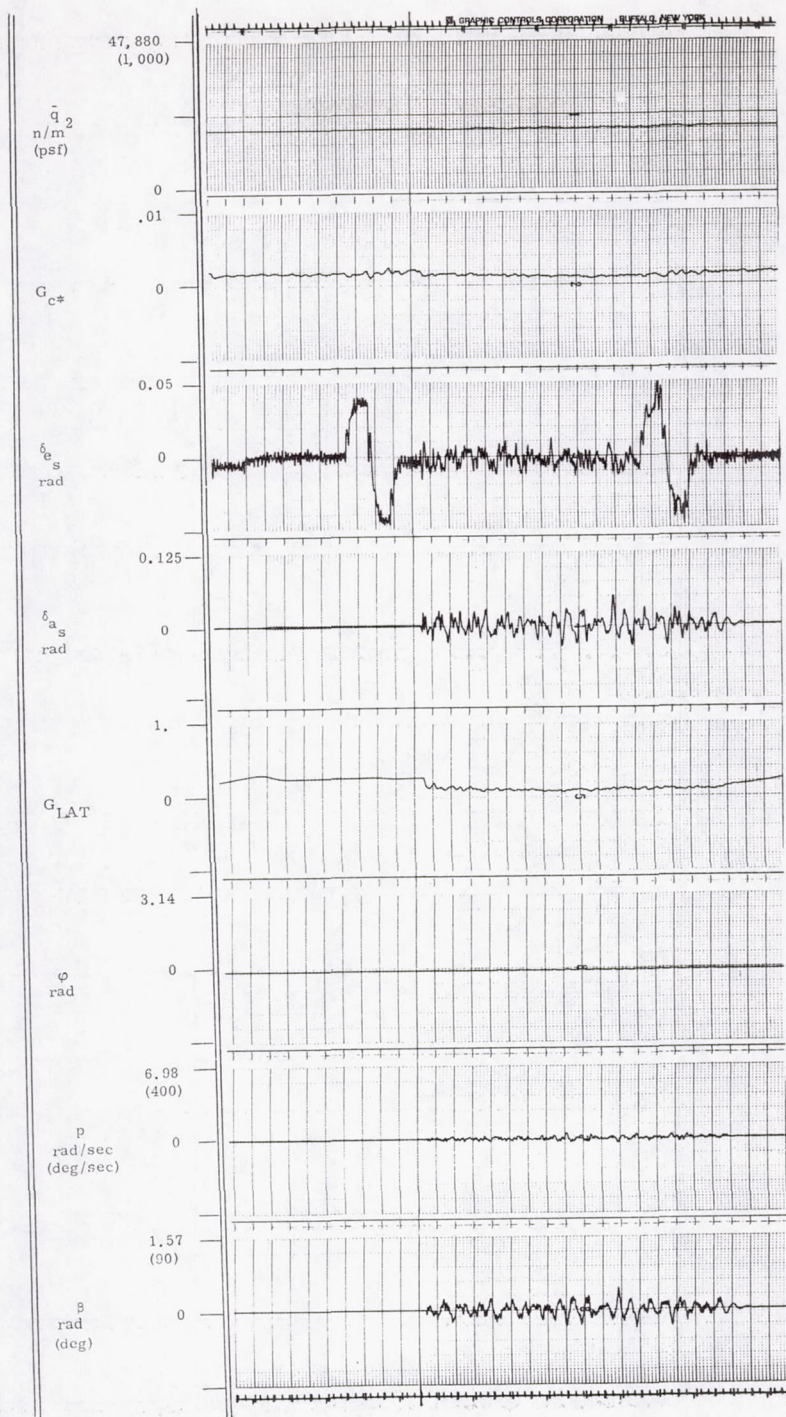


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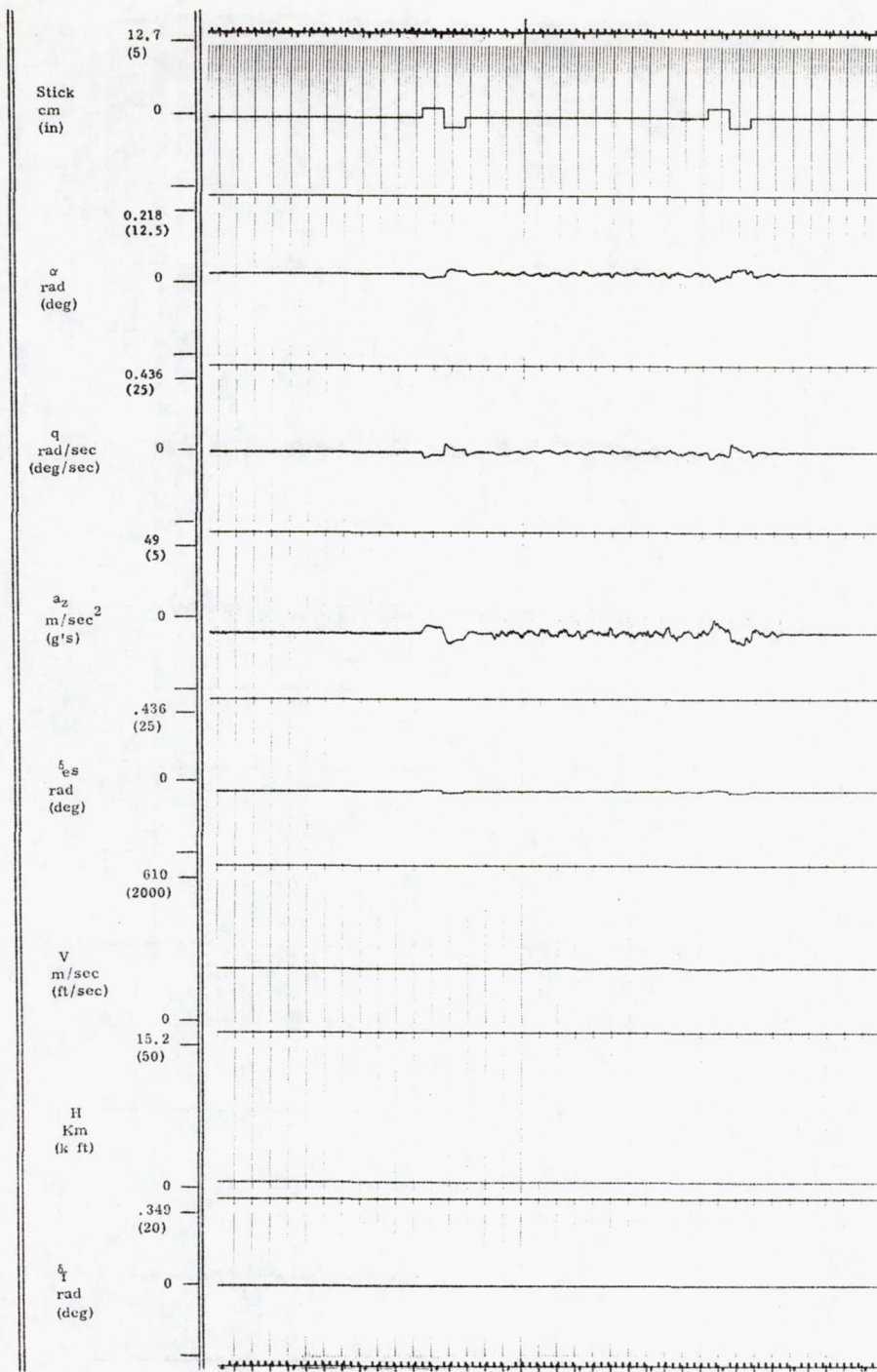


Figure 98. Limit-cycle design at FC10 (sensor noise).

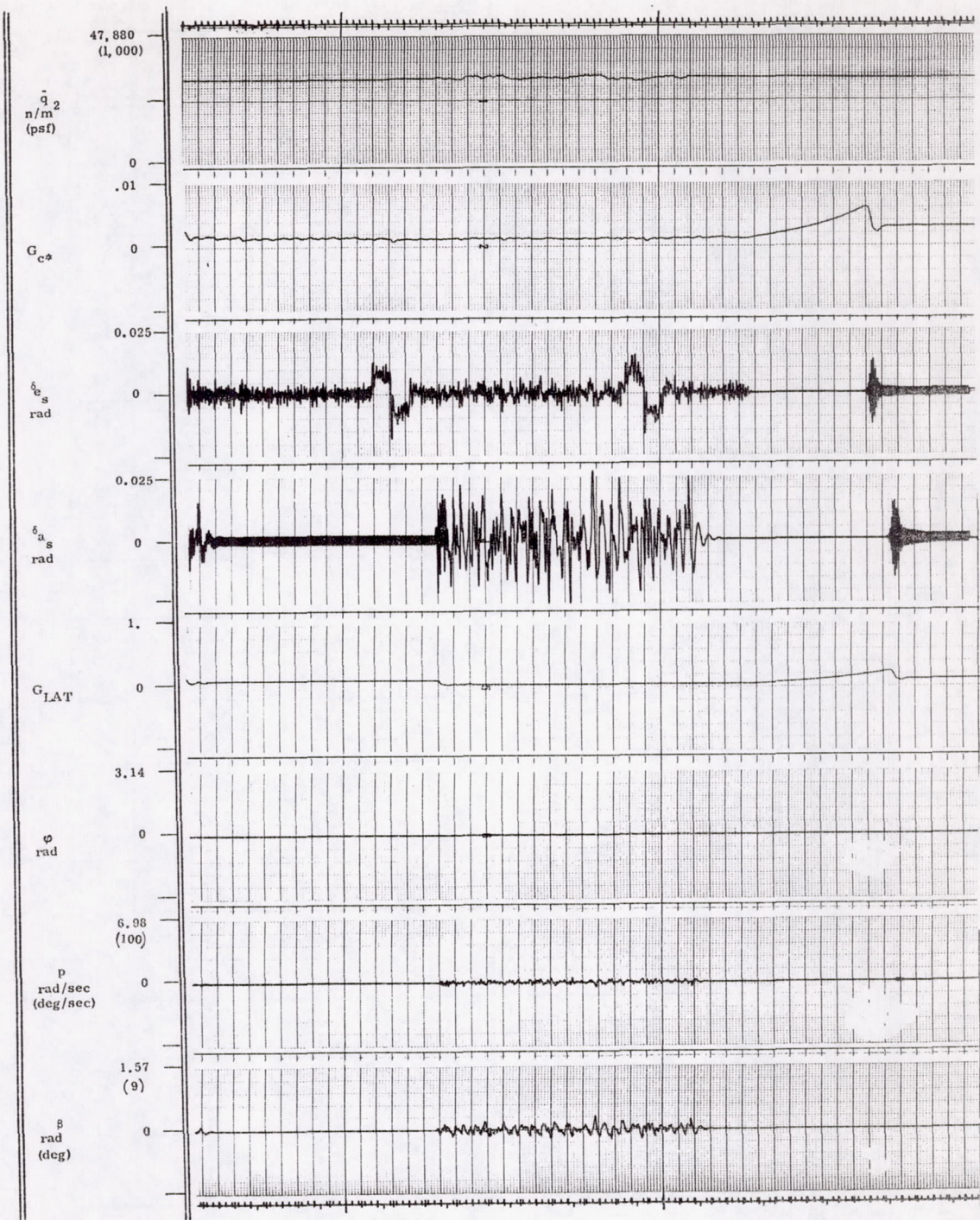


Figure 98. Concluded.

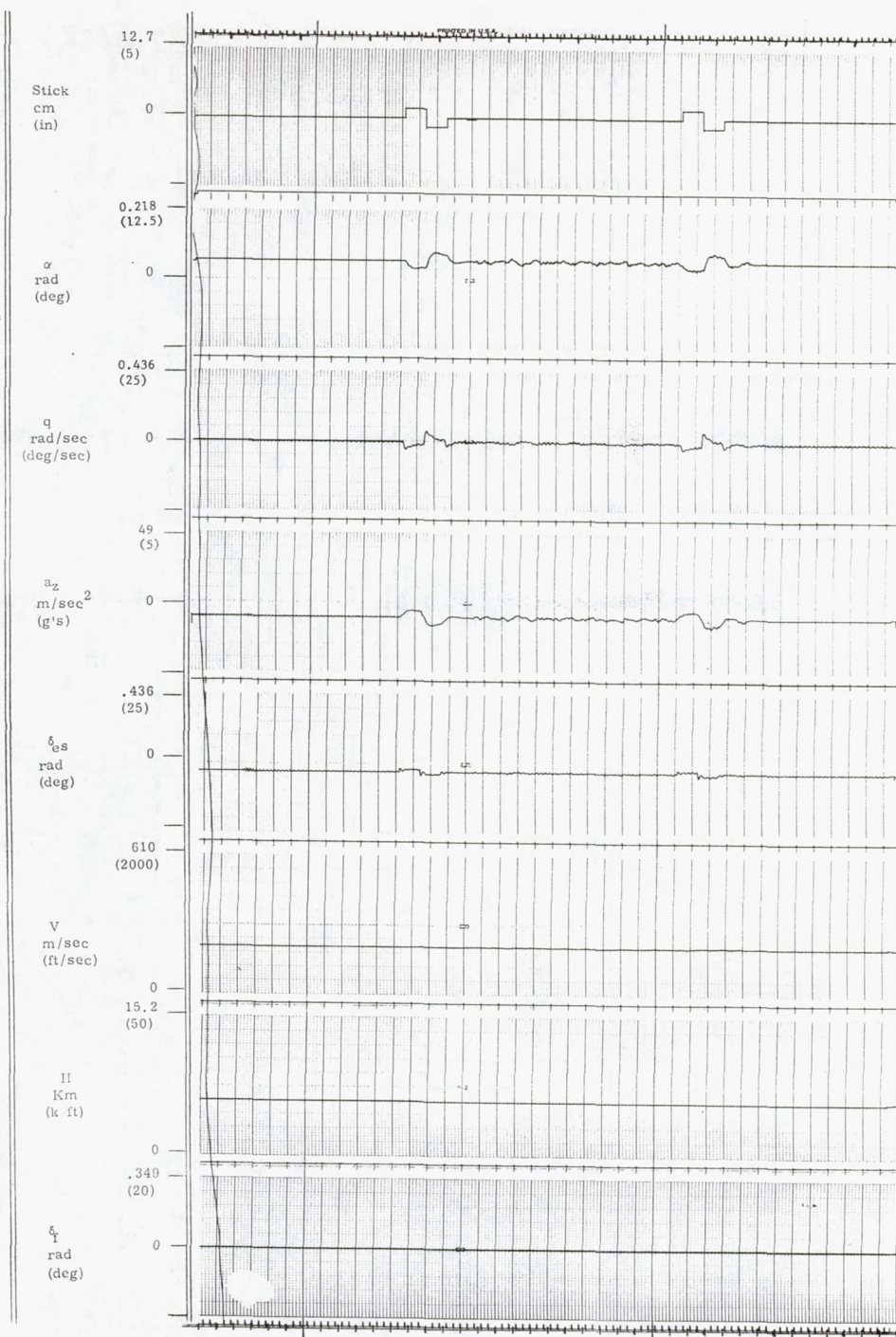


Figure 99. Limit-cycle design at FC1
(sensor noise plus hysteresis).

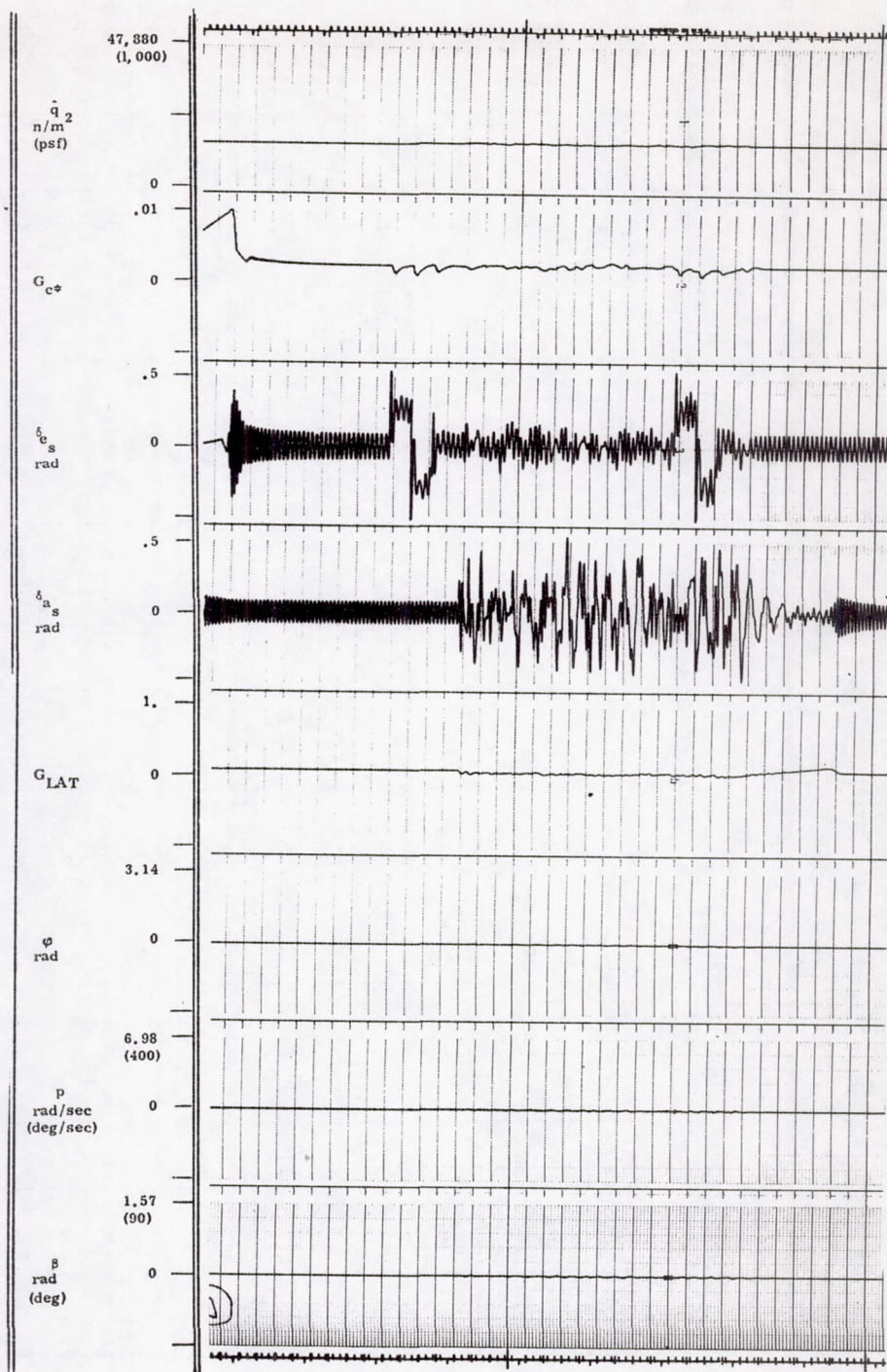


Figure 99. Concluded.

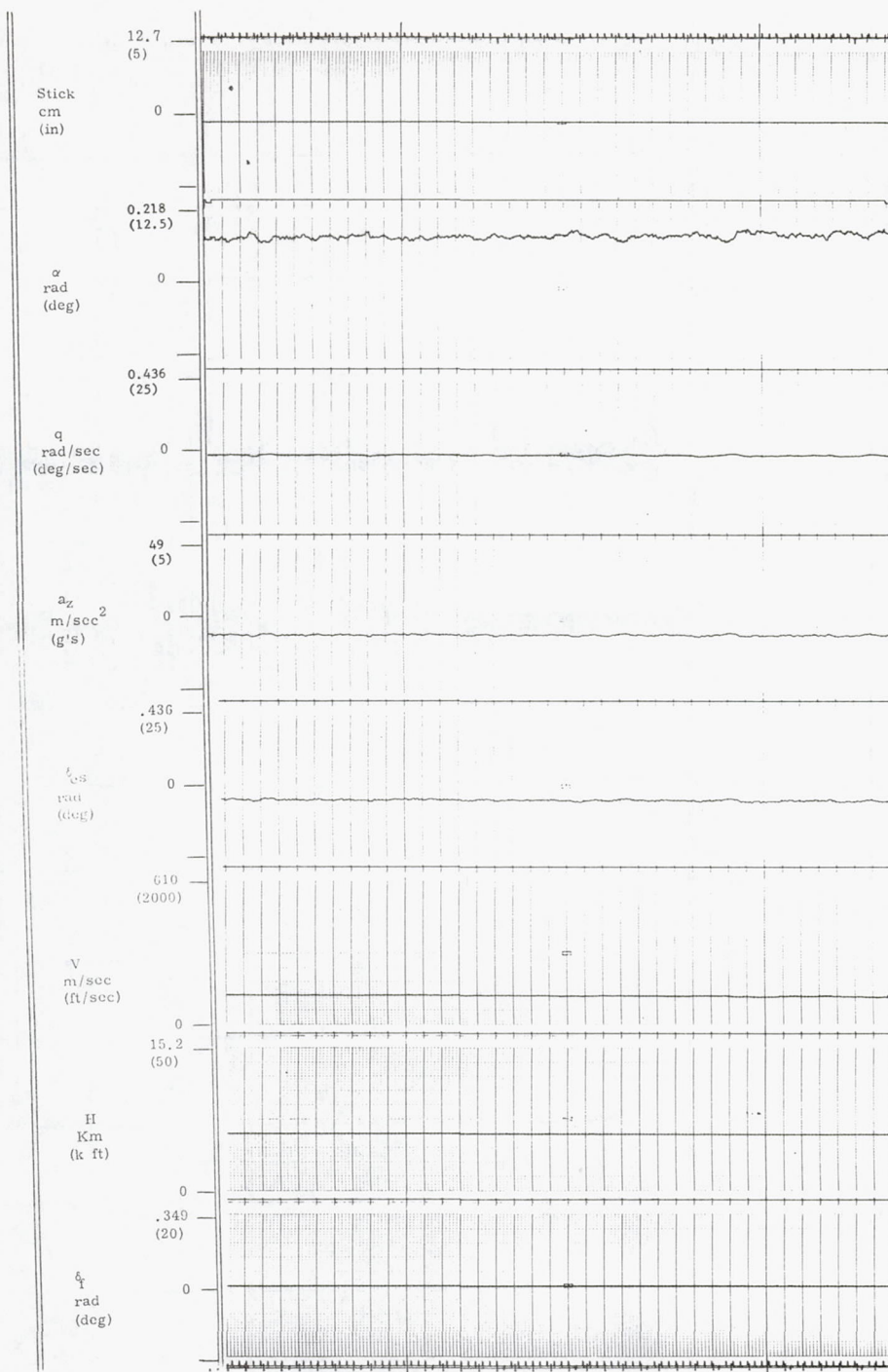


Figure 100. Limit-cycle design at FC5, IC FC10, gusts.

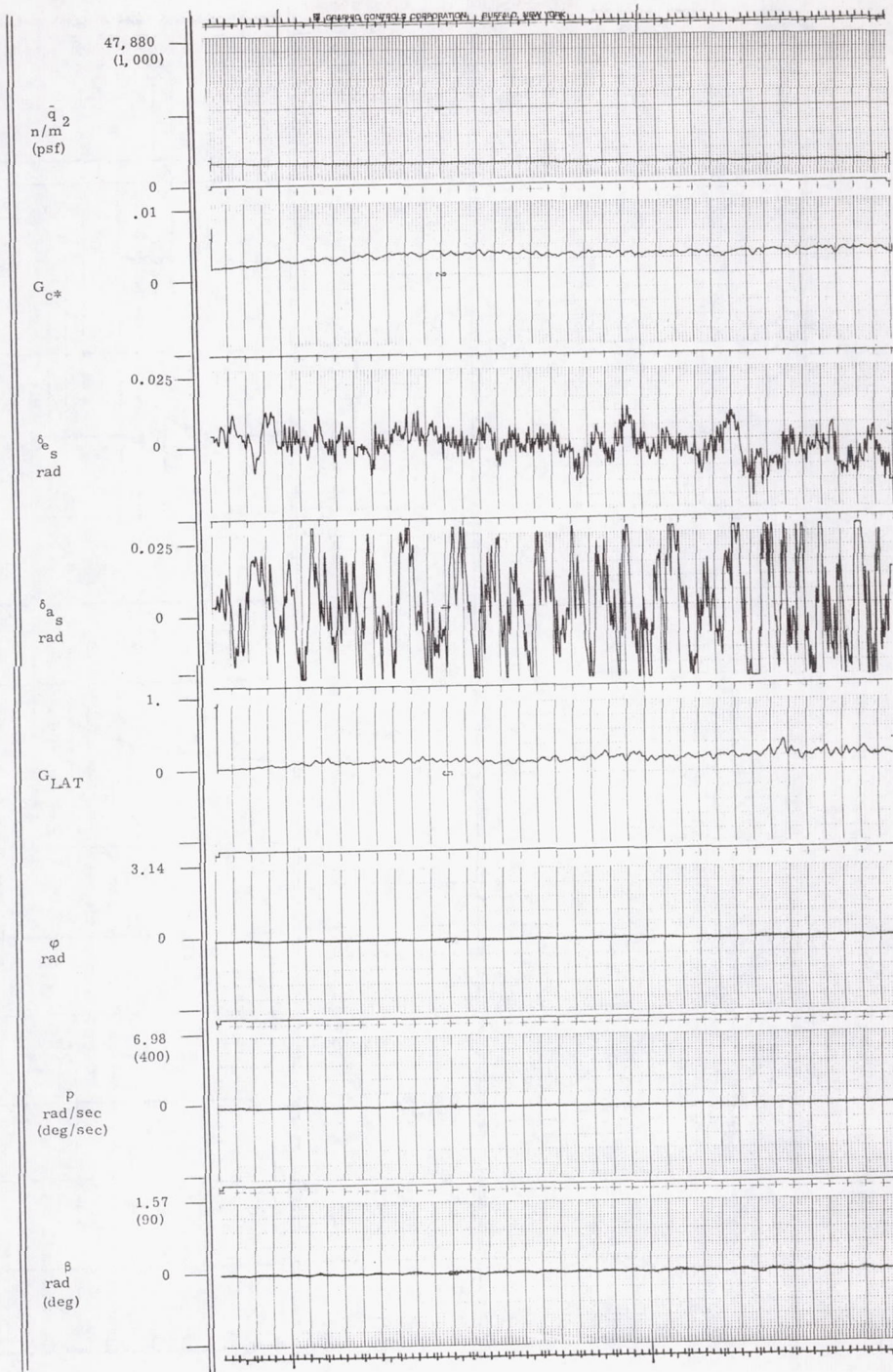


Figure 100. Concluded.

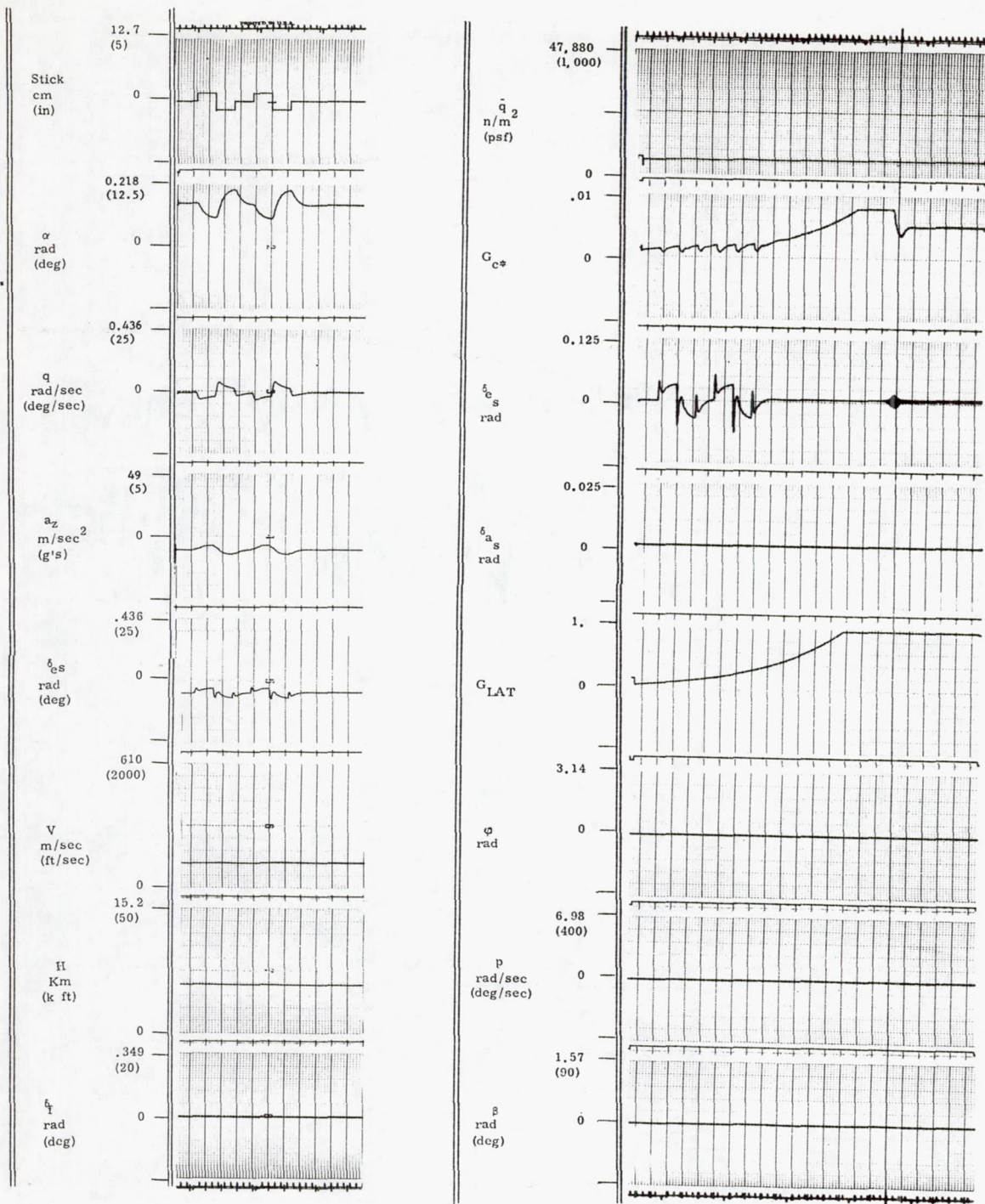


Figure 101. Limit-cycle at FC5, IC FC10, pilot commands.

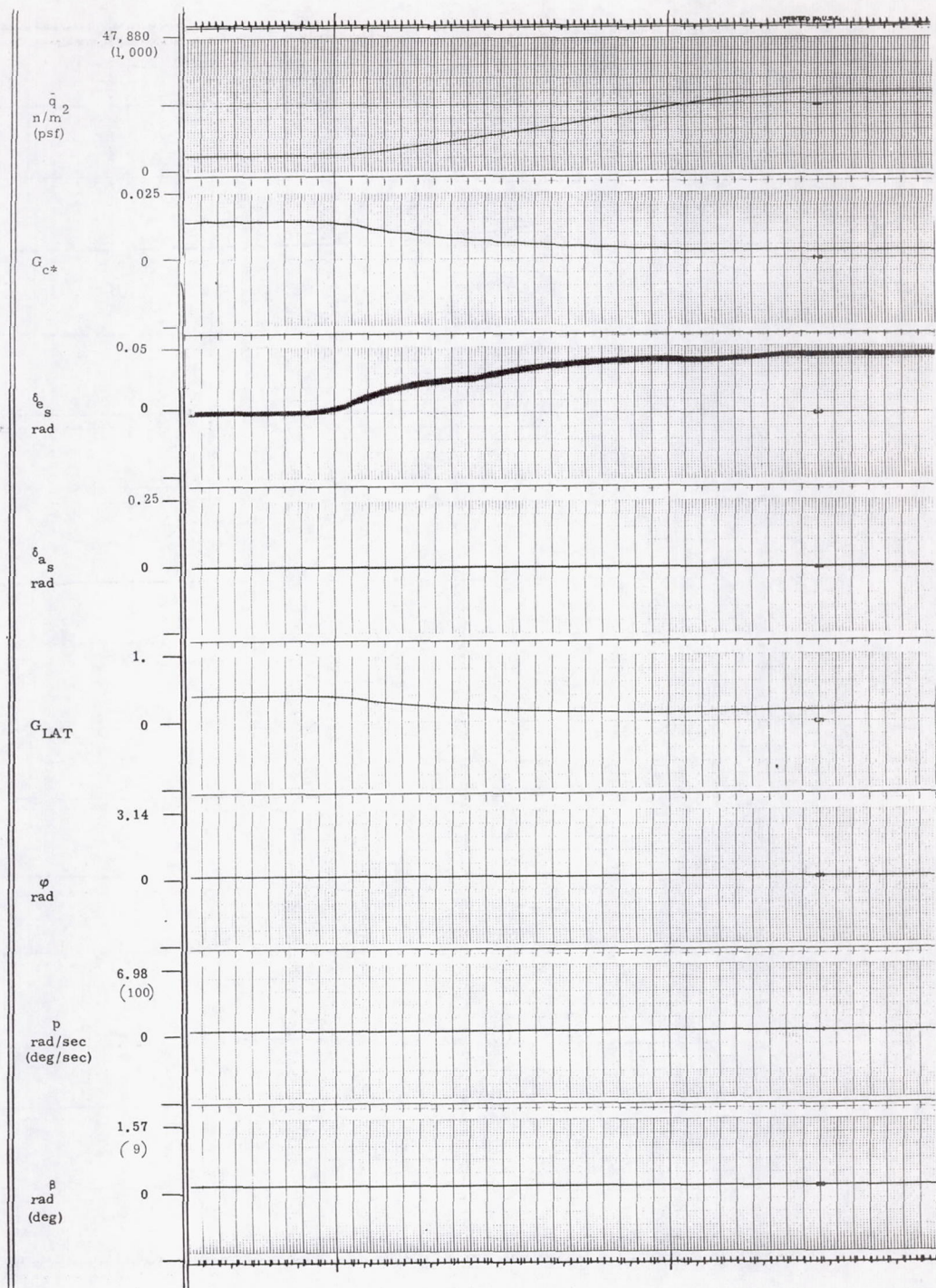


Figure 102. Limit-cycle design under acceleration.

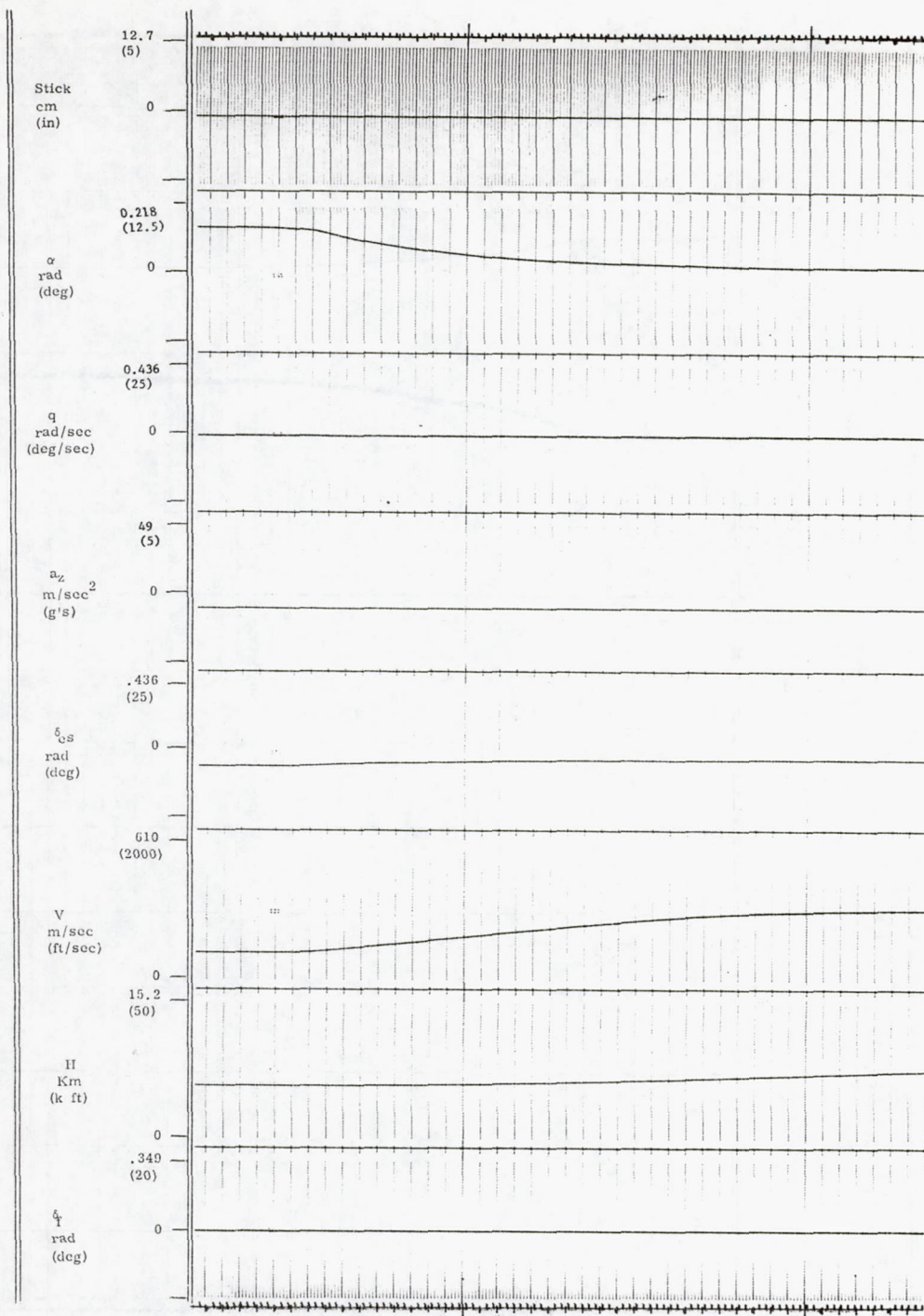


Figure 102. Concluded.

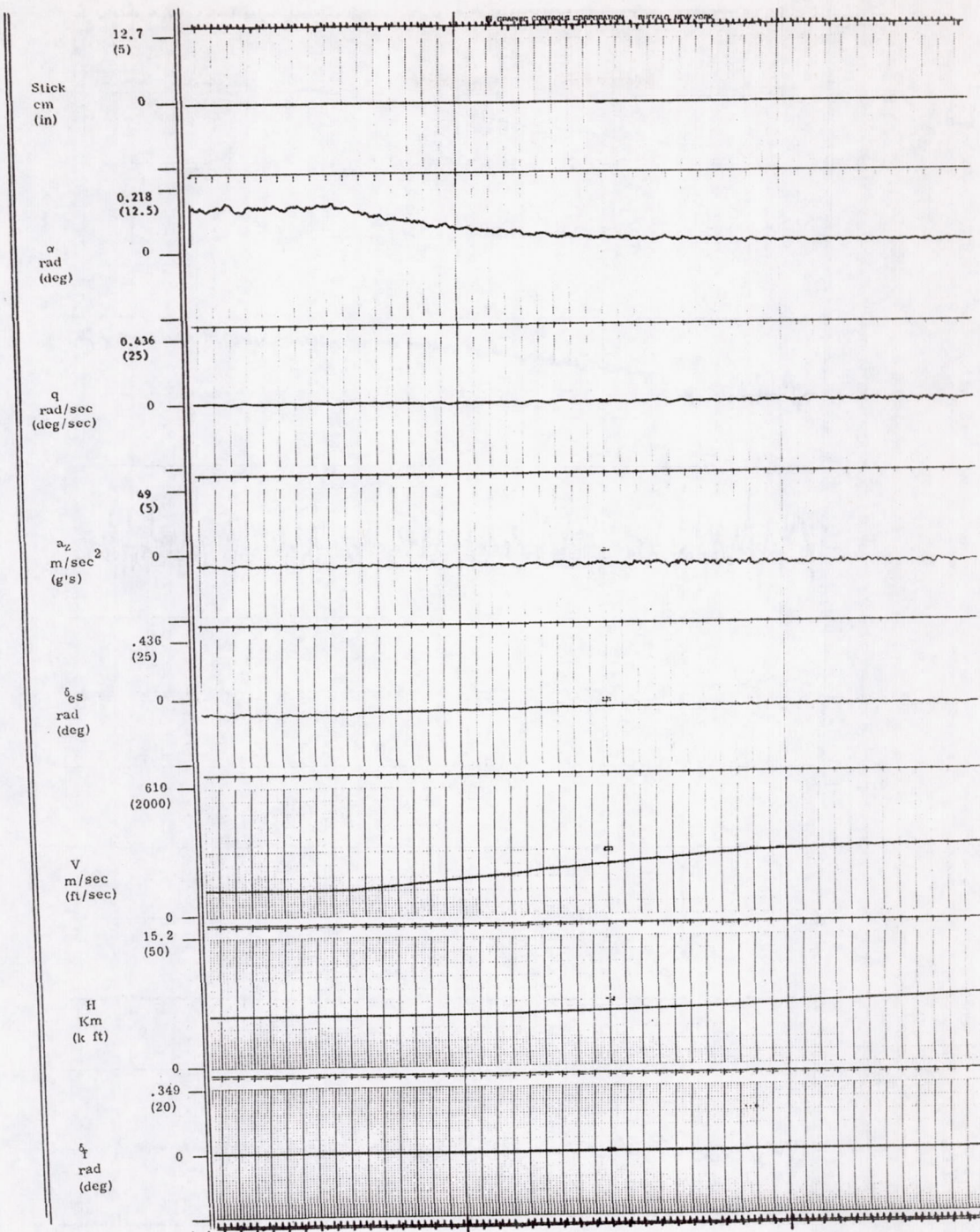


Figure 103. Limit-cycle design under acceleration (gusts).

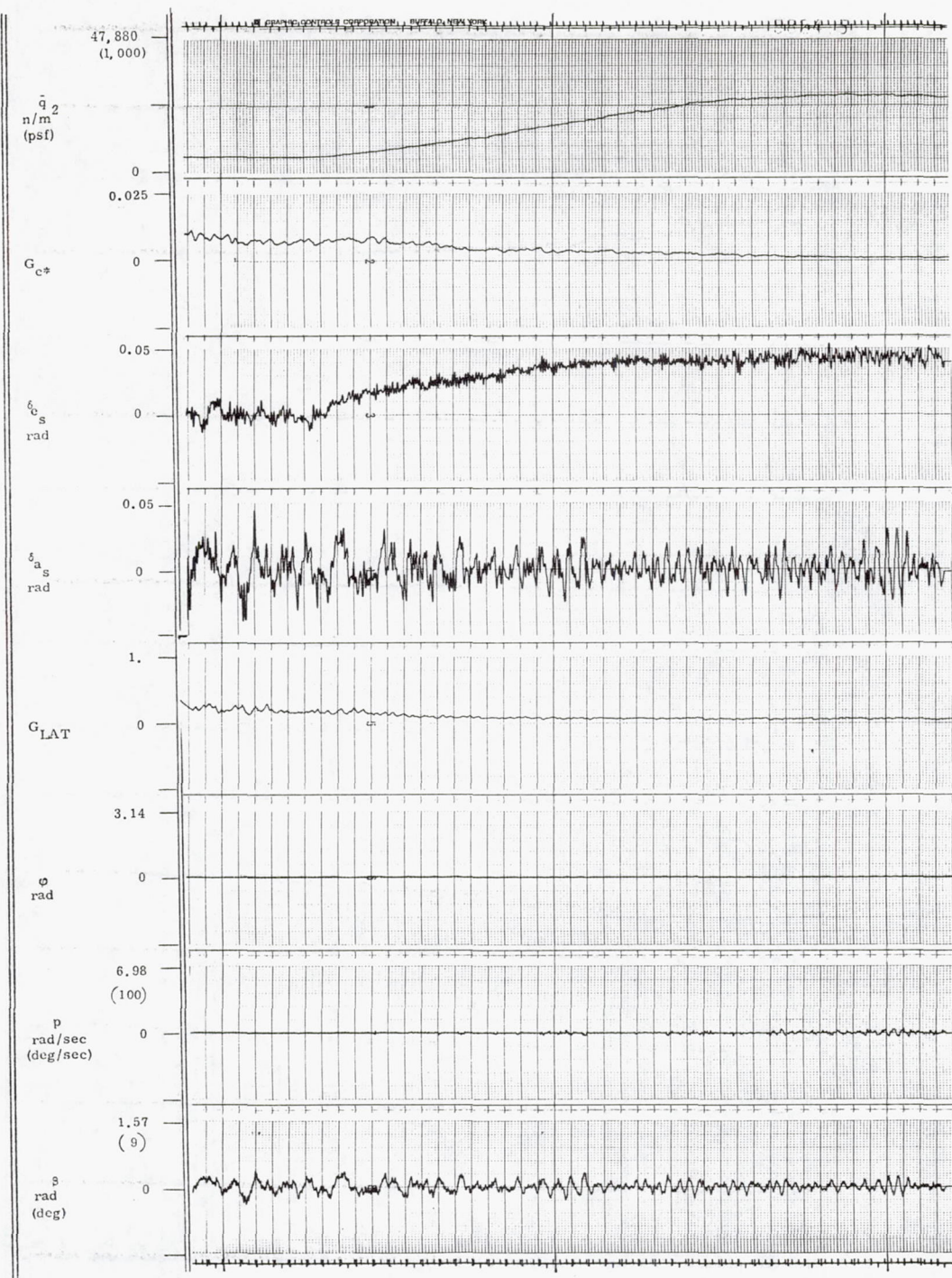


Figure 103. Concluded.

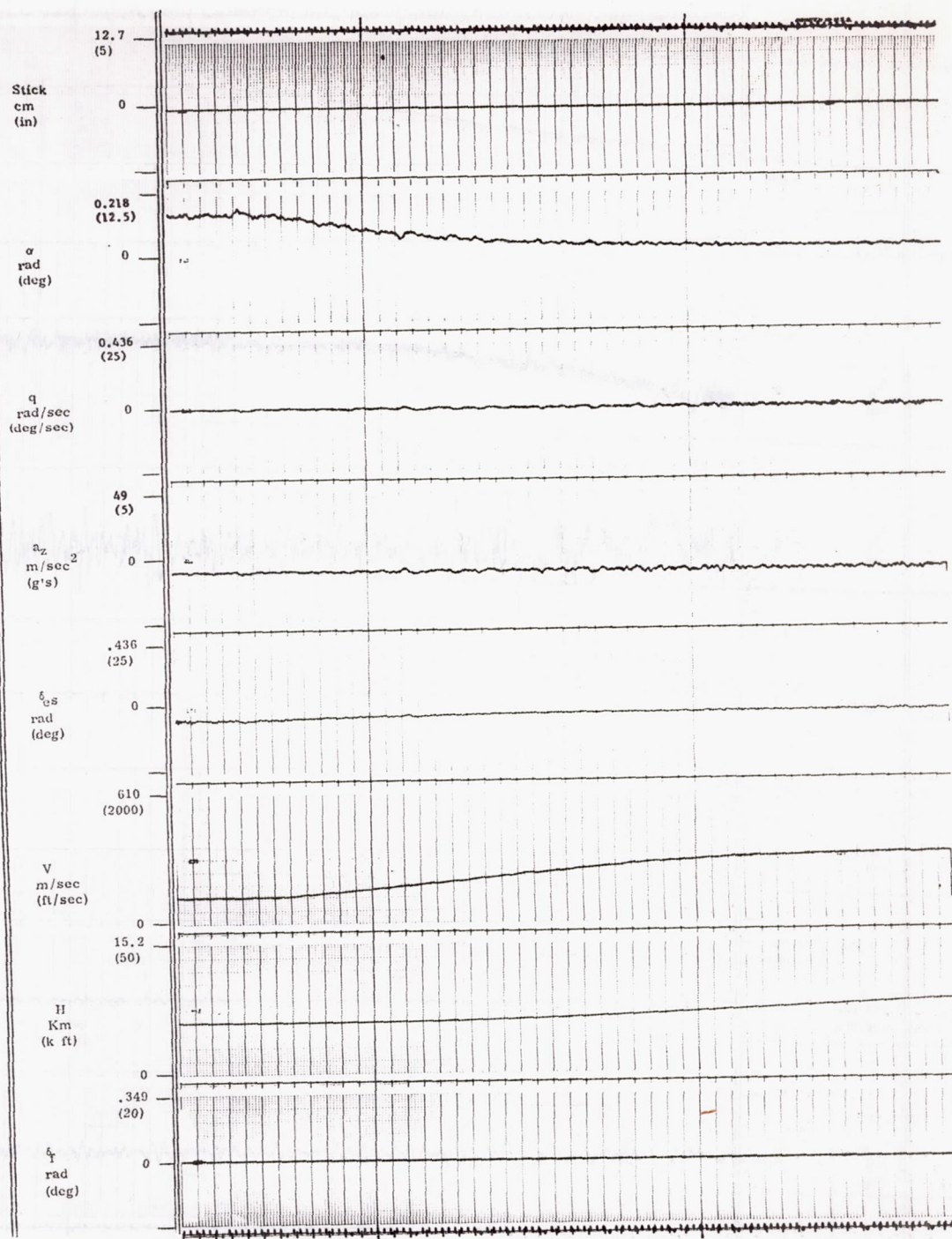


Figure 104. Limit-cycle design under acceleration (gusts plus sensor noise).

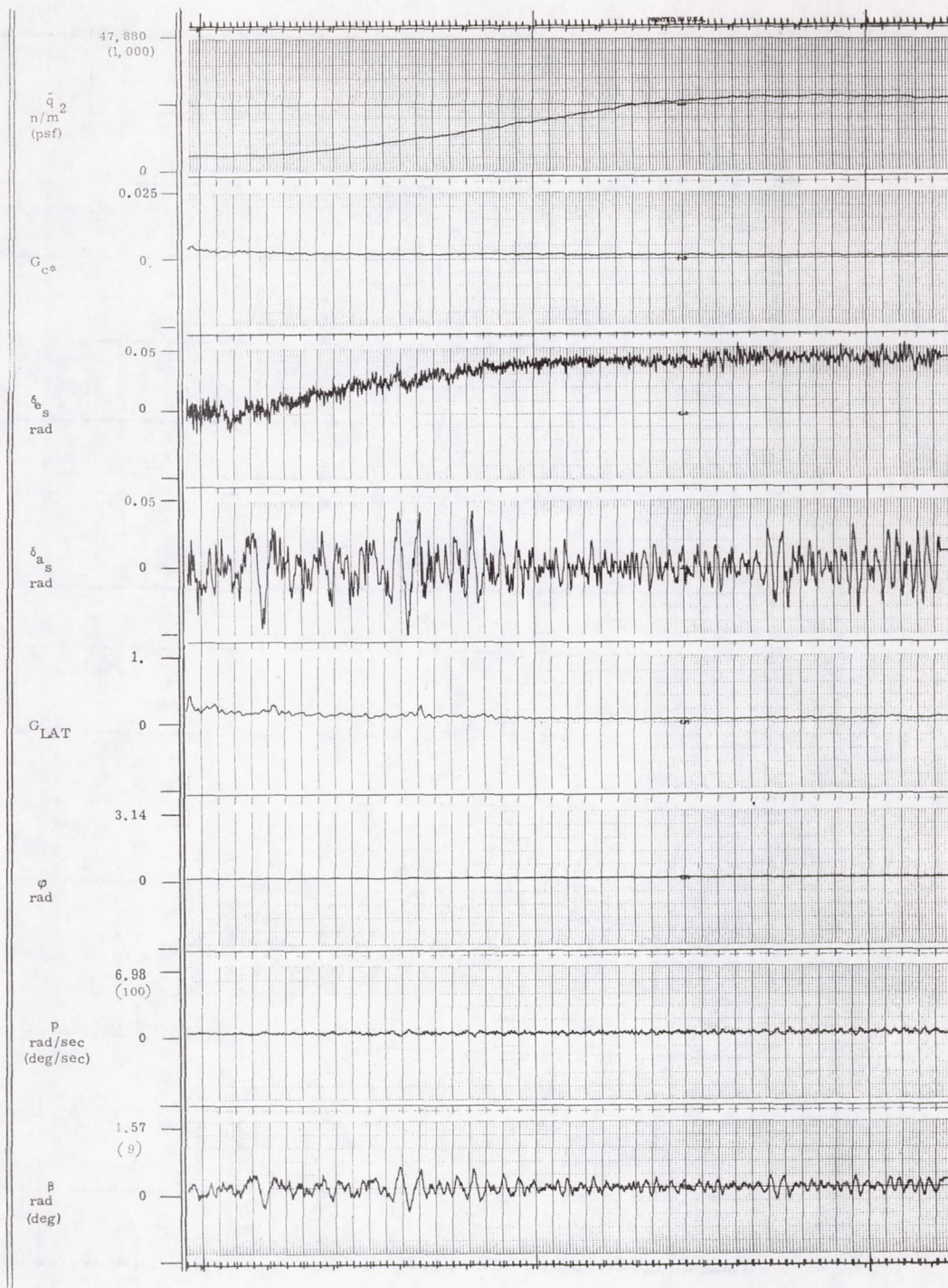


Figure 104. Concluded.

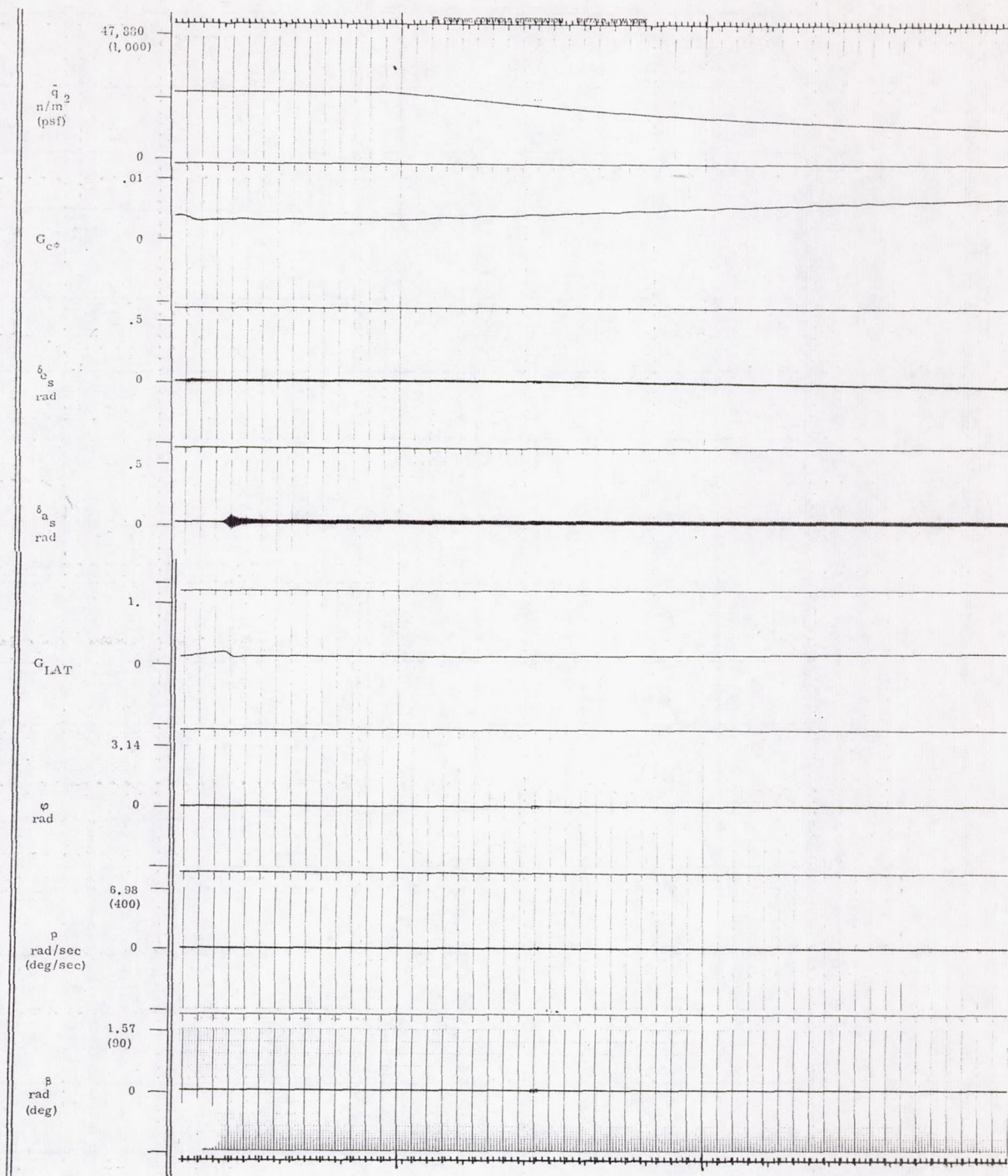


Figure 105. Limit-cycle design, deceleration from FC10.

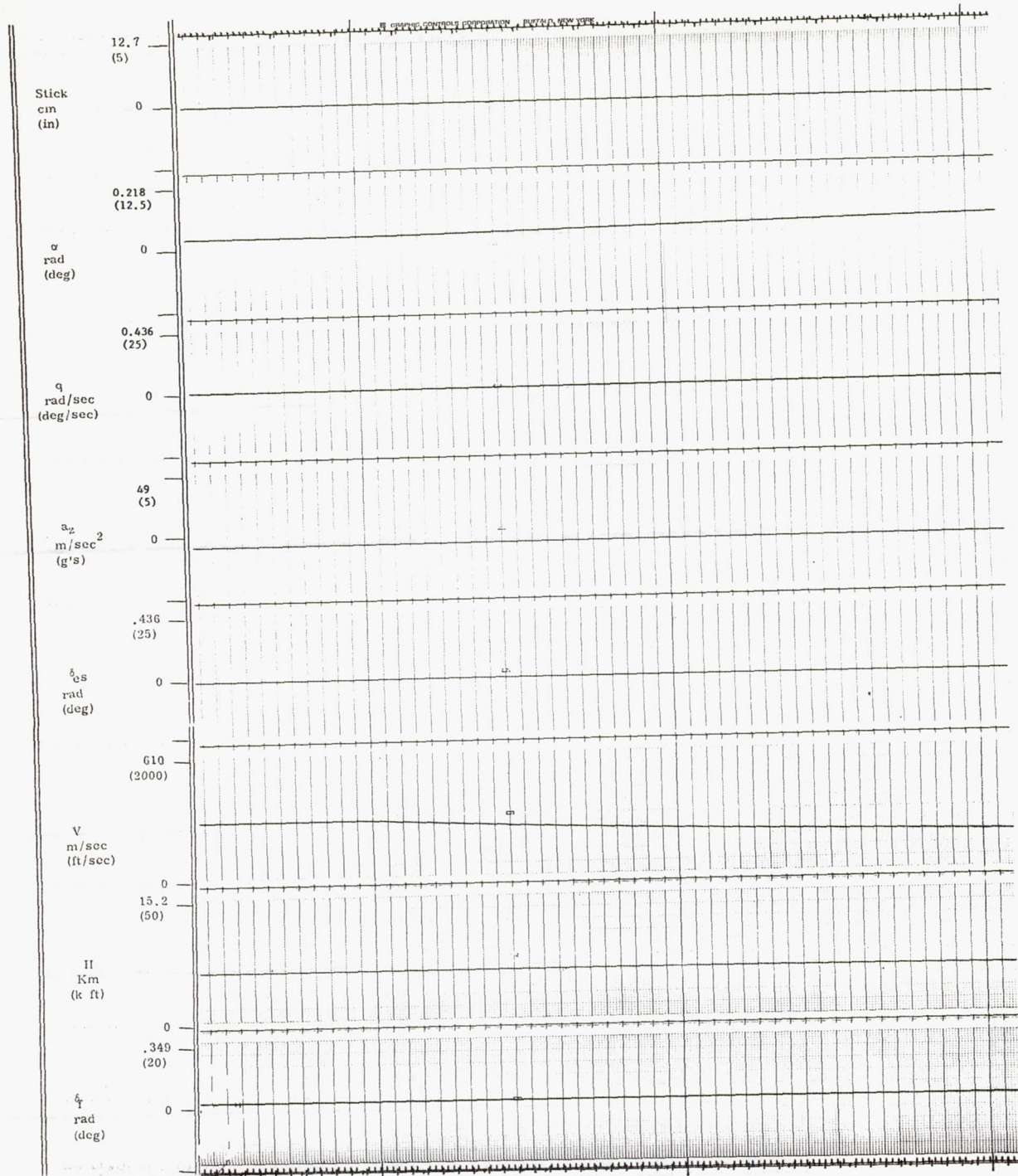


Figure 105. Concluded.

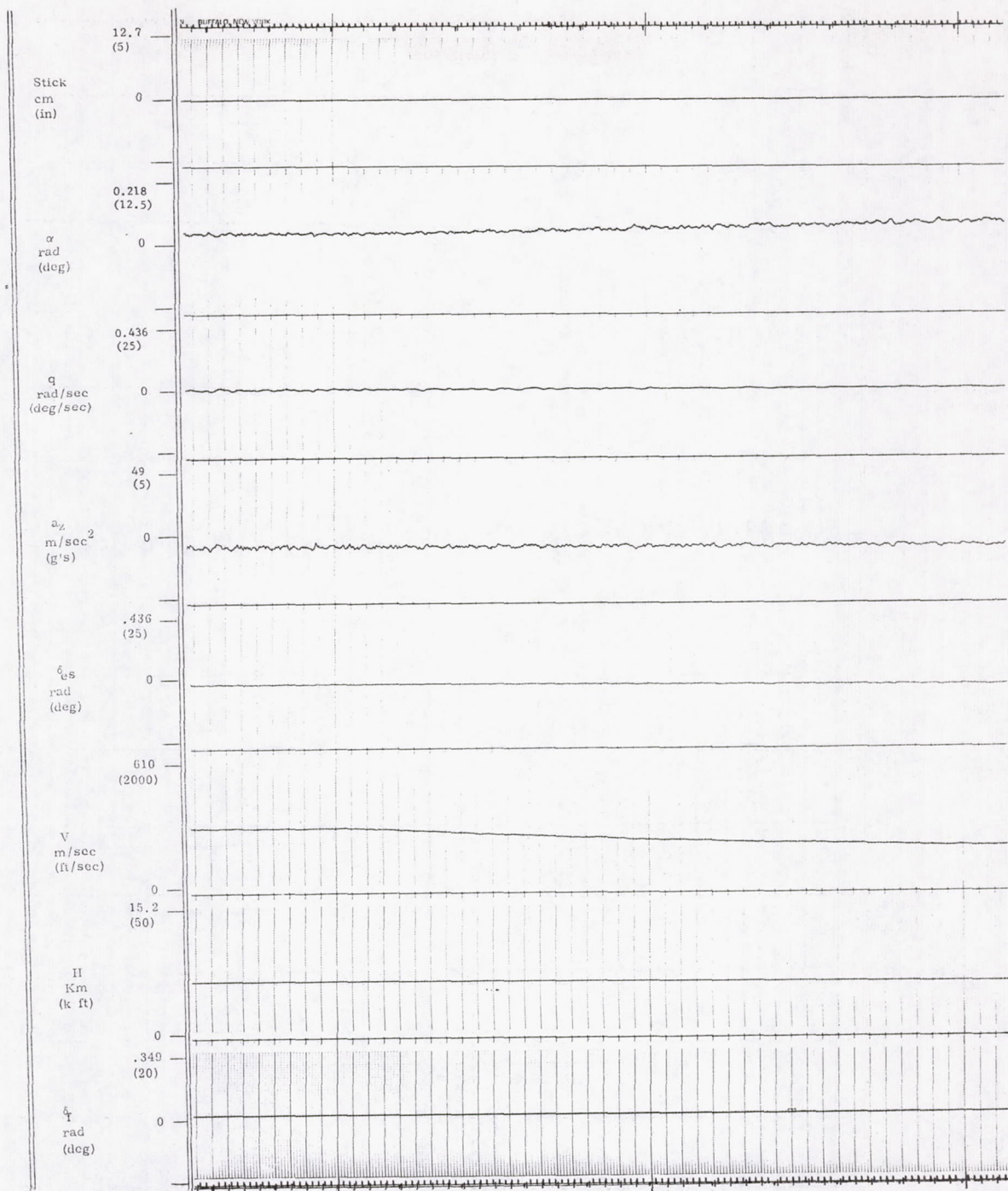


Figure 106. Limit-cycle design deceleration from FC10 (gusts).

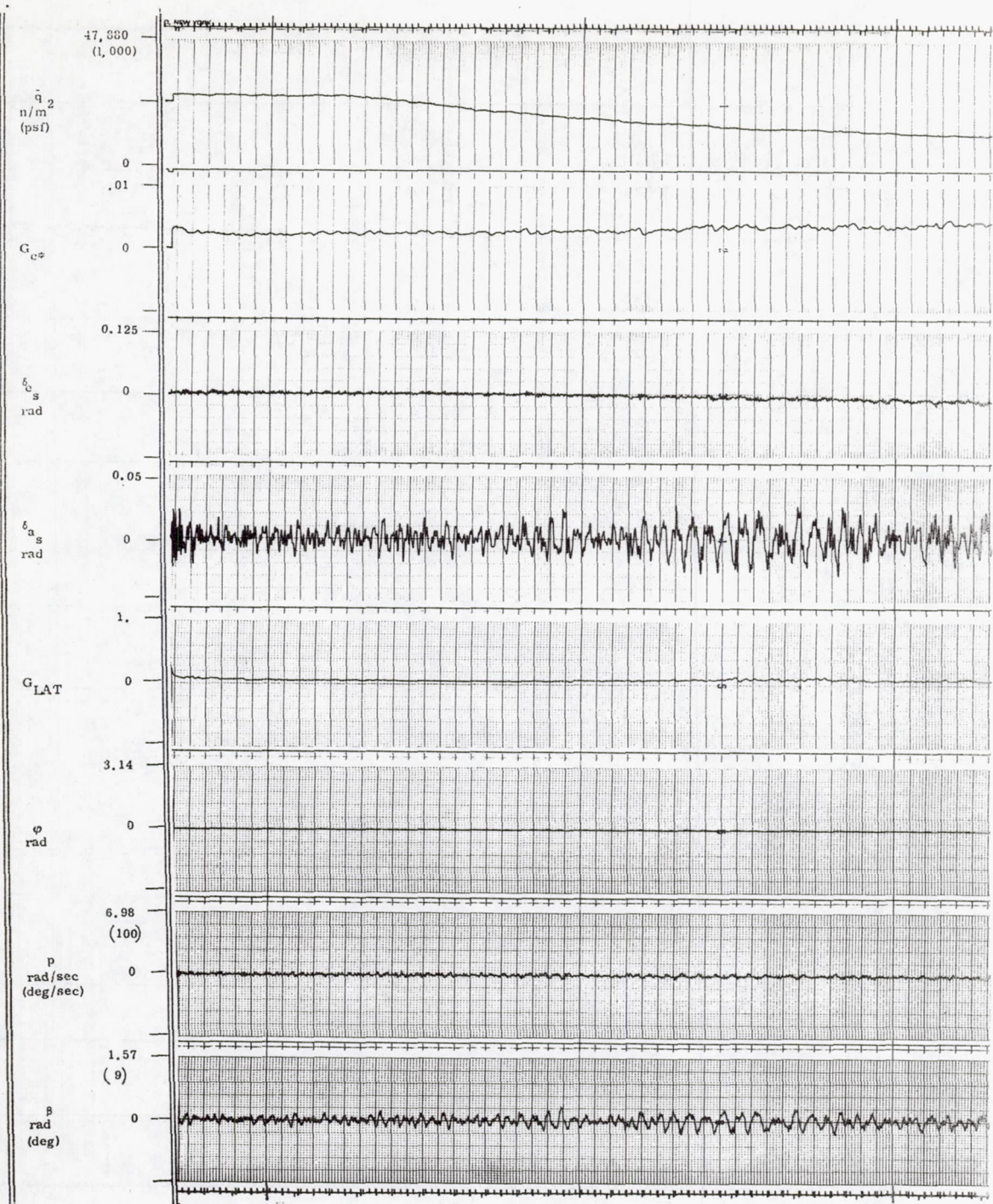


Figure 106. Concluded.

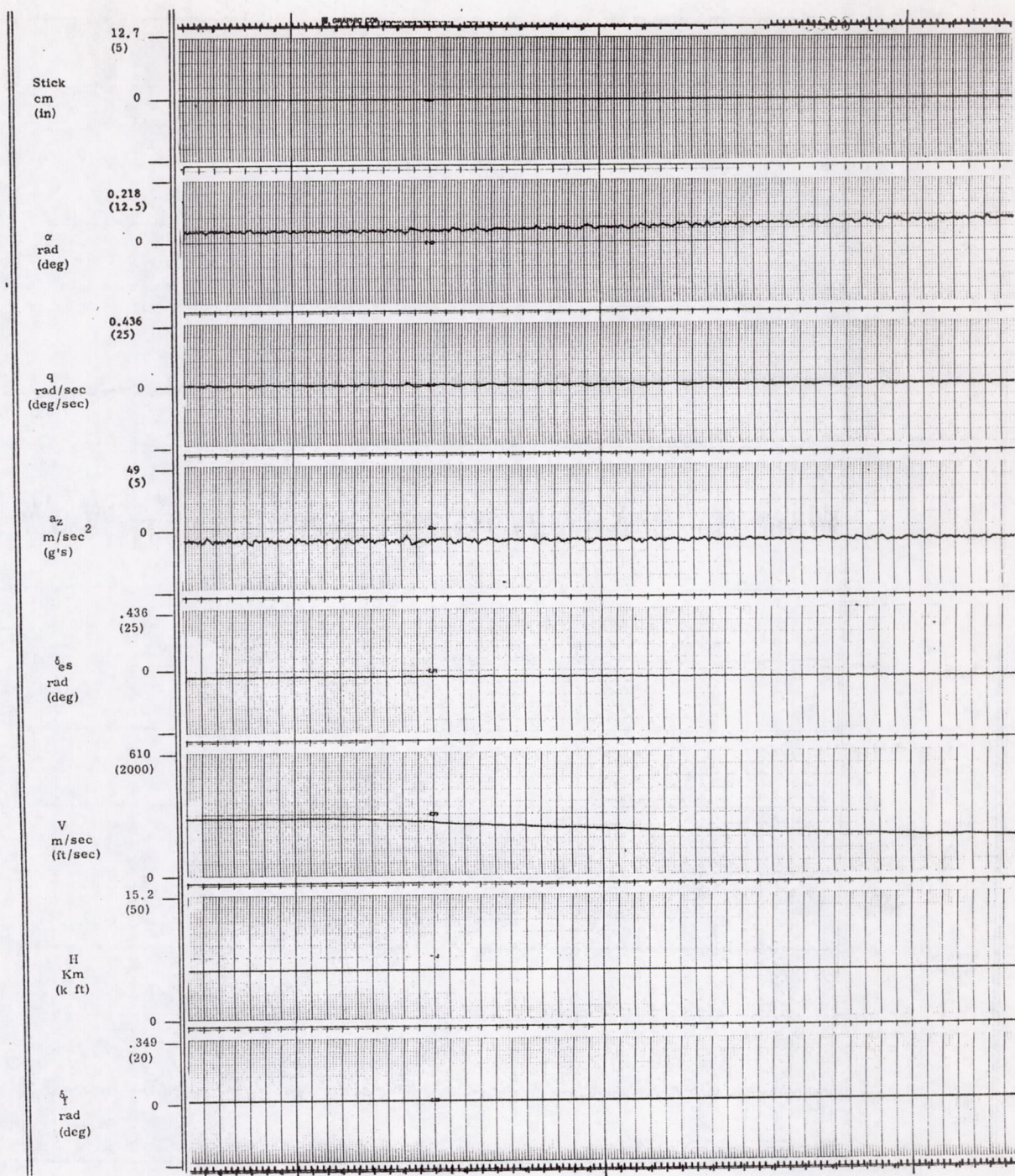


Figure 107. Limit-cycle design deceleration from FC10 (gusts plus sensor noise).

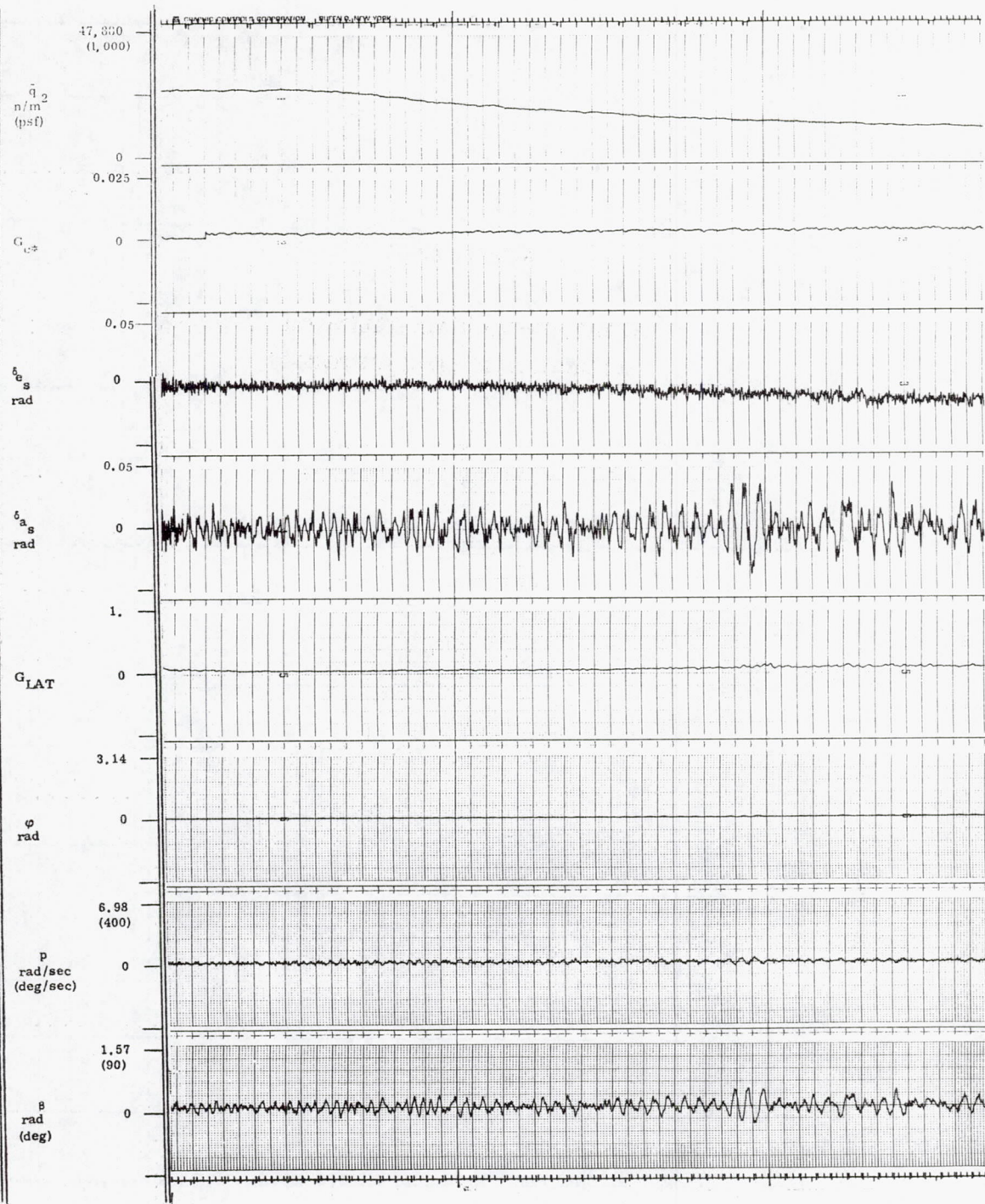


Figure 107. Concluded.

APPENDIX E

LIMIT-CYCLE GAIN-CHANGER LATERAL AXIS TIME HISTORIES

APPENDIX E

LIMIT-CYCLE GAIN-CHANGER LATERAL AXIS TIME HISTORIES

The series of analog traces (Figures 108 through 118) shows the response of the lateral-directional CAS to beta gusts and roll rate commands. The adaptive gains increase damping of the aircraft beta gust response from about 0.15 to about 0.25, with little or no change in frequency. In contrast, the nominal reduced CAS of Reference 1 tended to normalize the response frequency to about 0.4 Hz with similar damping. The tendency of the adaptive gains to reduce to the six-db level for command inputs is evident.

For further comparison, the gain was fixed at six db lower than critical, and the response is nearly identical to the adaptive response. The responses to lower gains are also shown since under turbulence and roll rate gyro noise, lower gain levels are realized with the gain changer. This data was also used to assess the applicability of using a gain scheduled on estimated surface effectiveness (or dynamic pressure) for use with the two other adaptive concepts. The performance degradation of operating the roll axis at a lower gain is minimal. The yaw axis gain does not need to be reduced in a scheduled system since it has sufficient gain margin.

Figure 119 shows roll rate commands and beta gust responses from the Langley F-8C simulation. Flight conditions 1, 5, 8, 10 and 17 are presented. They compare well to the analog responses for the corresponding flight condition.

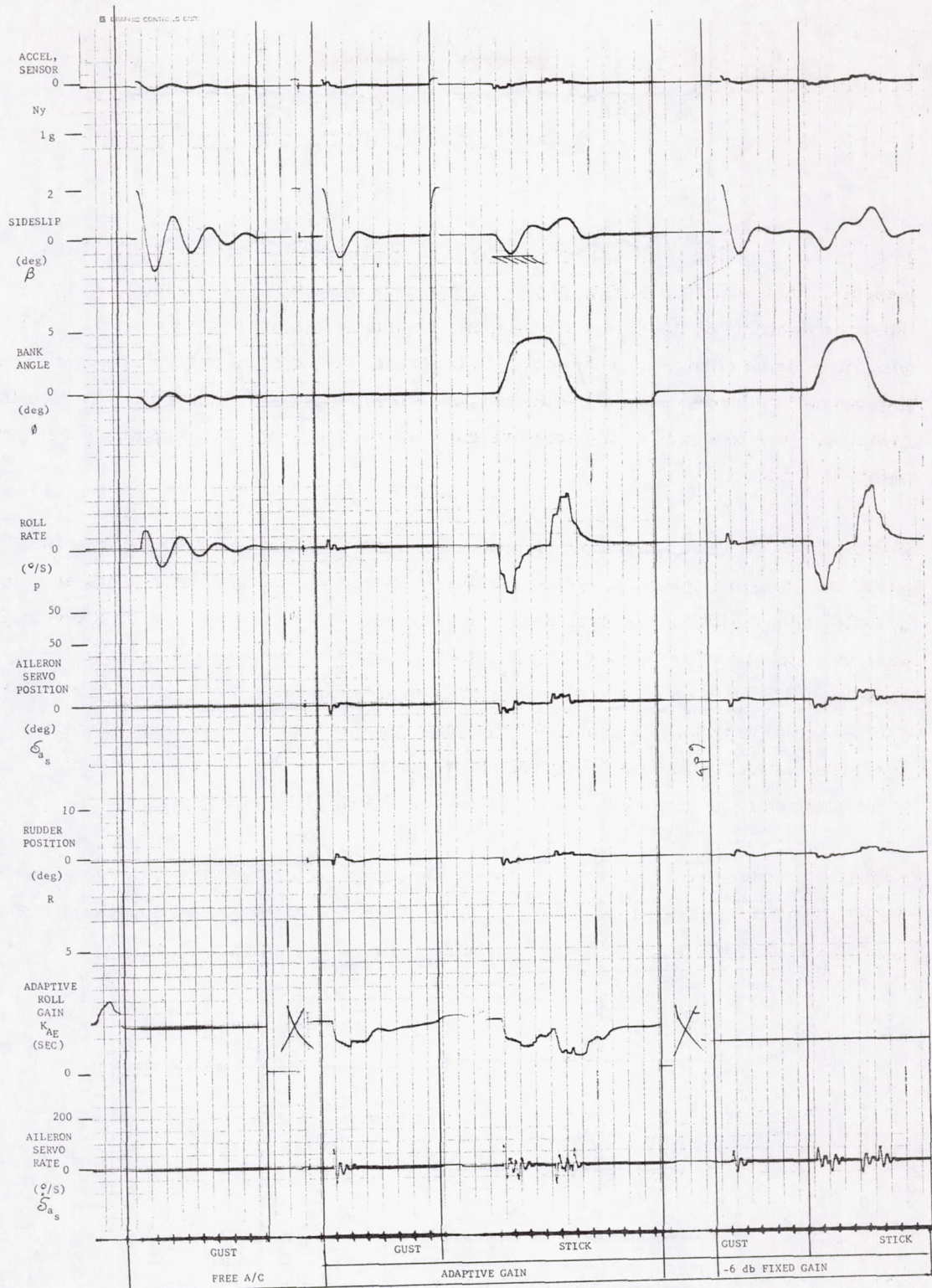


Figure 108. Flight Condition 1.

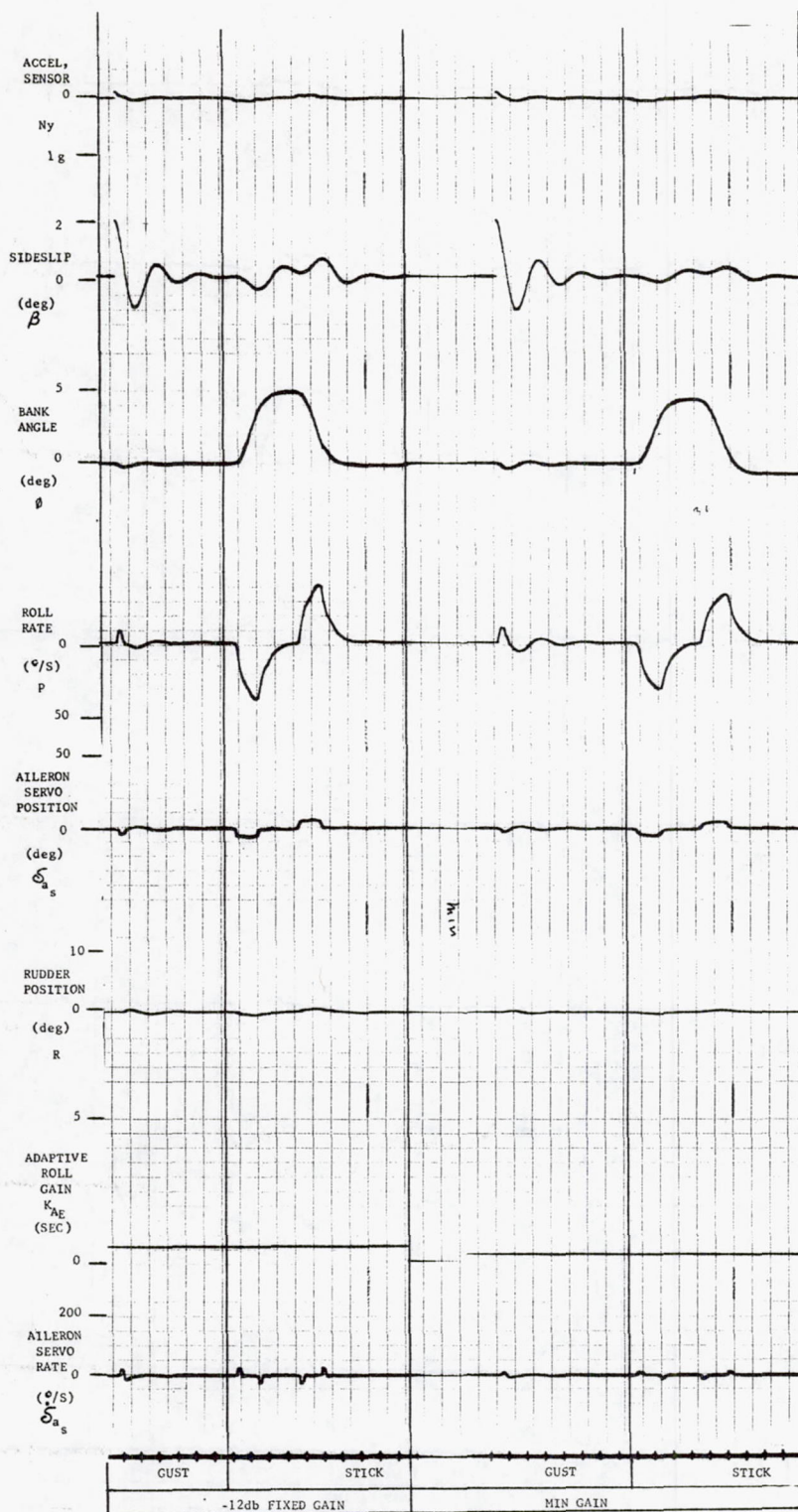


Figure 108. Concluded.

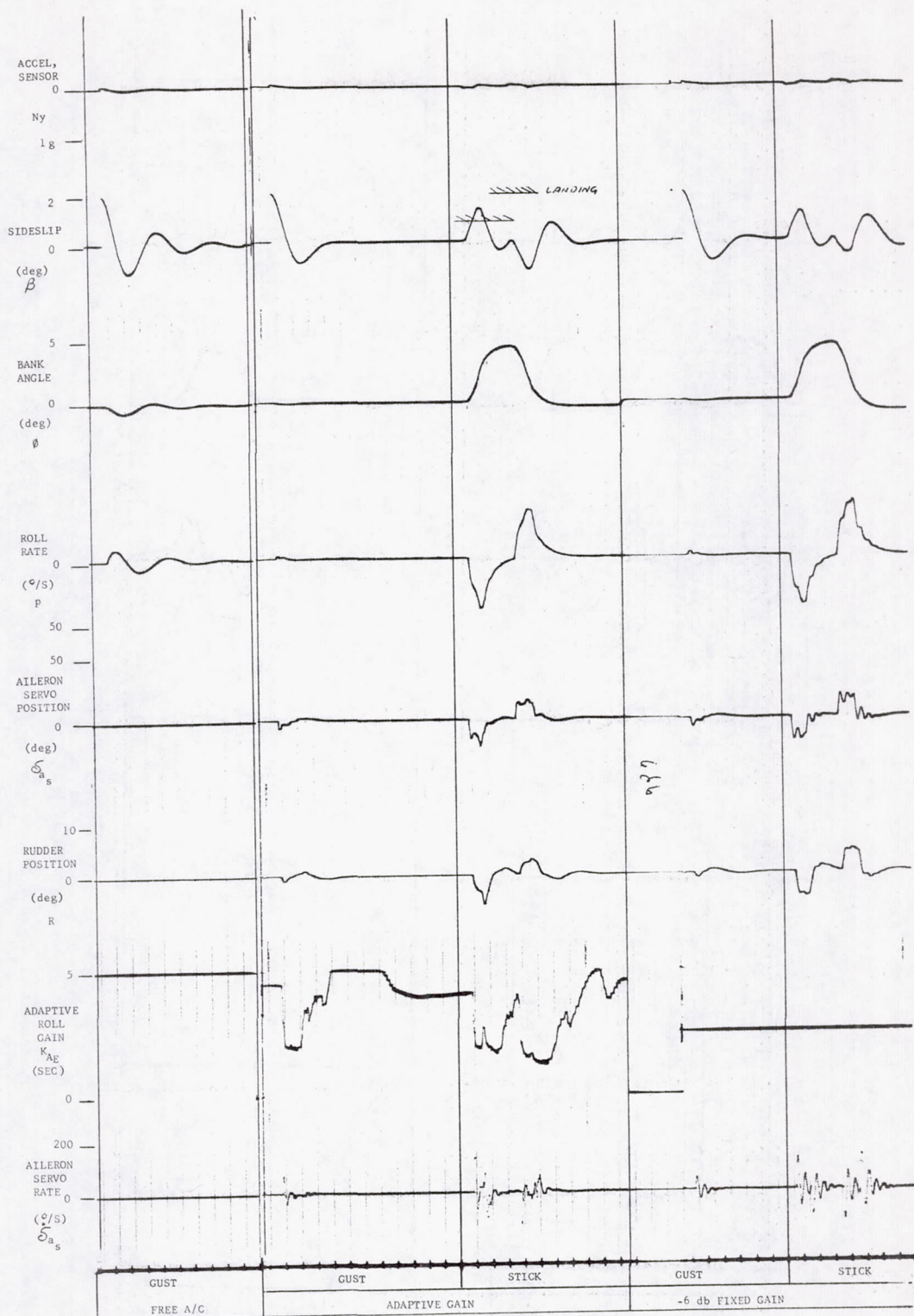


Figure 109. Flight Condition 5.

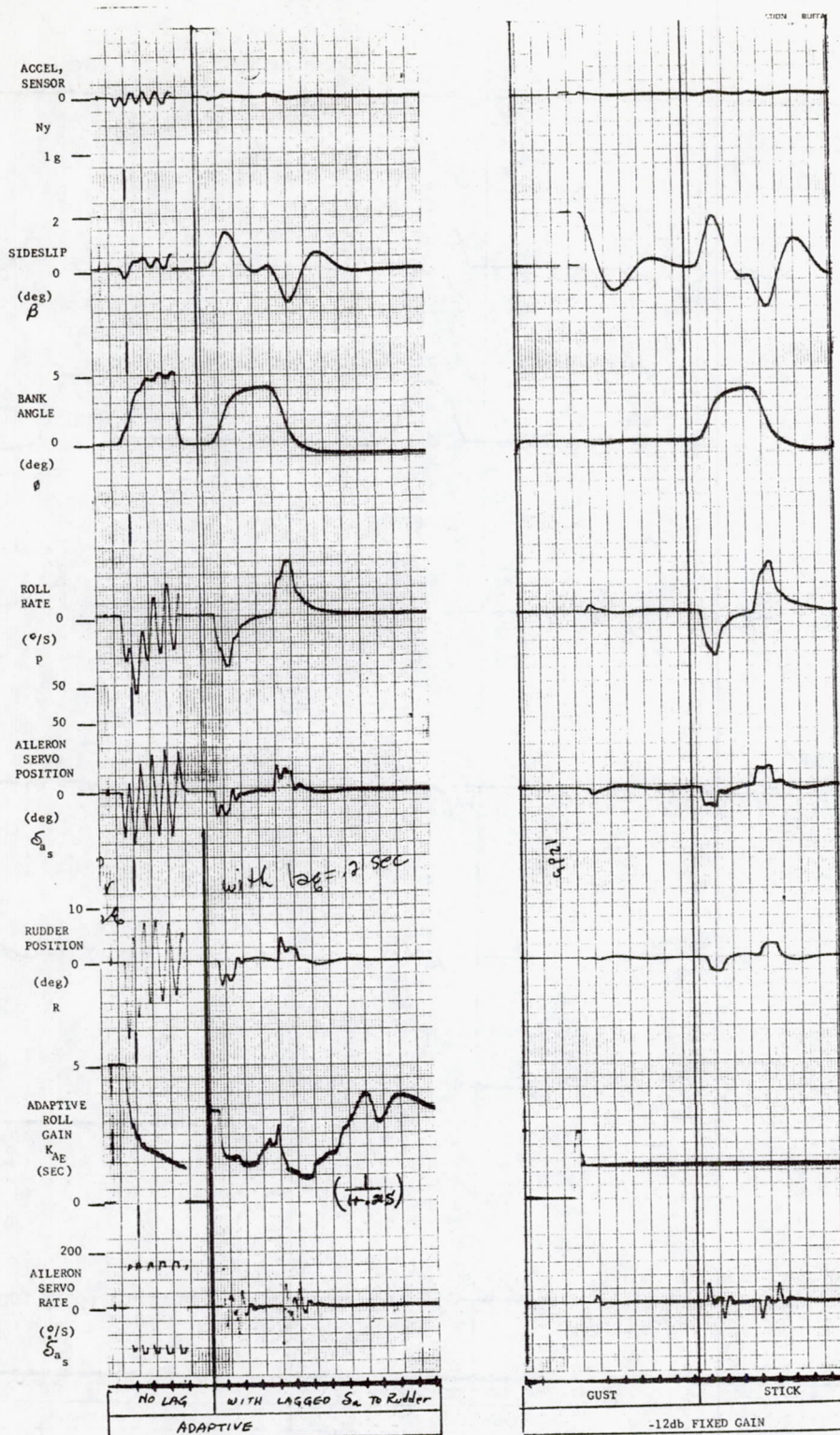


Figure 109. Concluded.

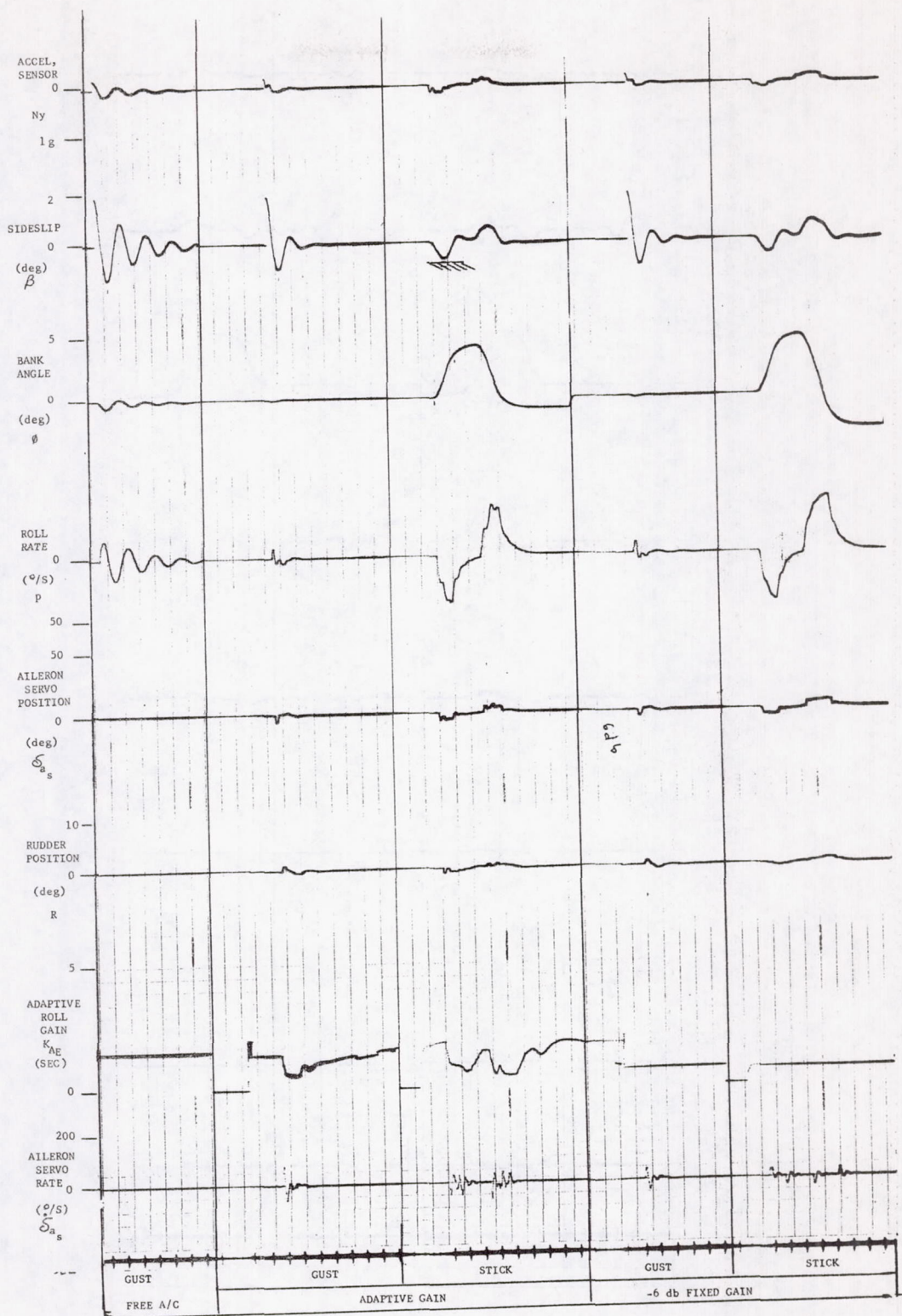


Figure 110. Flight Condition 6.

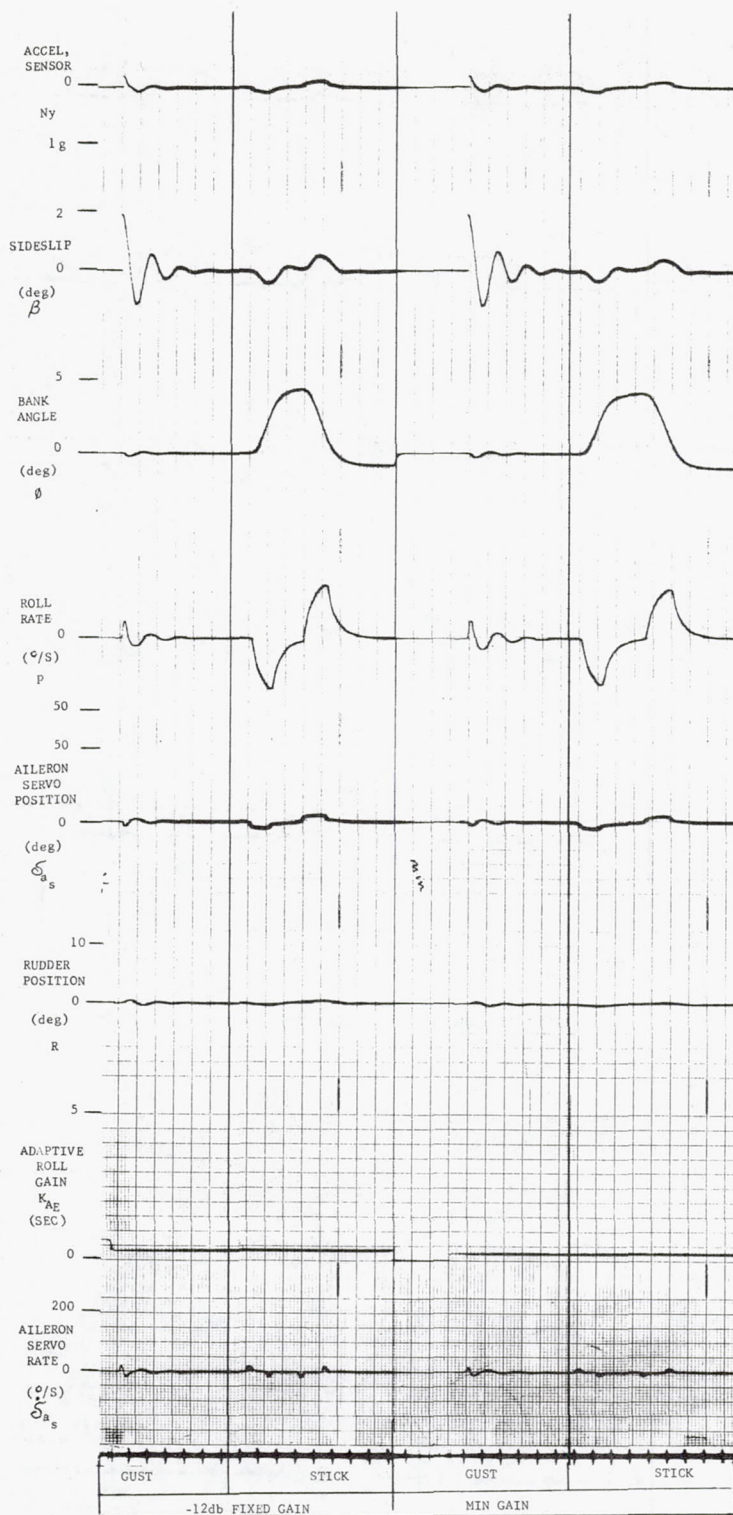


Figure 110. Concluded.

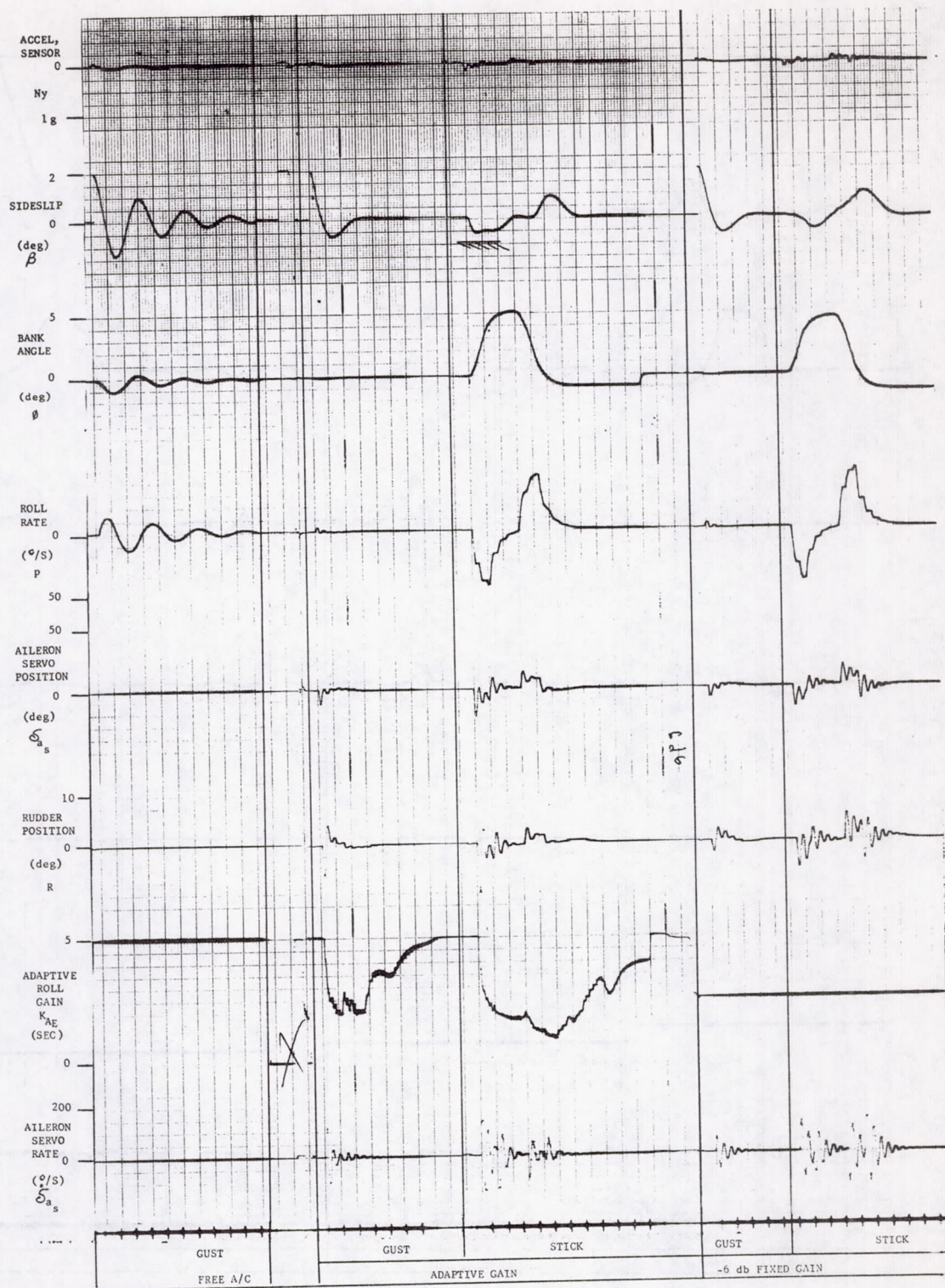


Figure 111. Flight Condition 7.

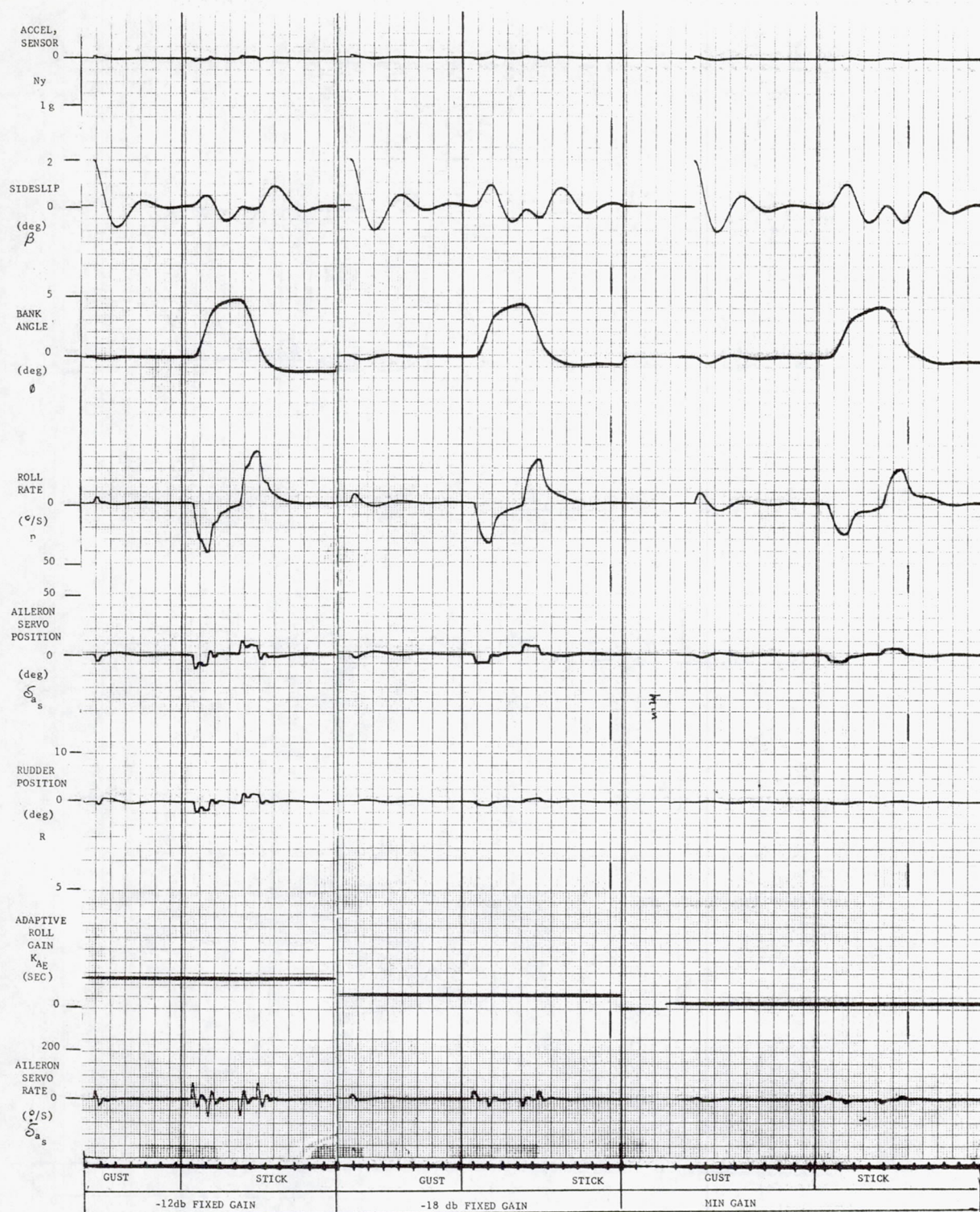


Figure 111. Concluded.

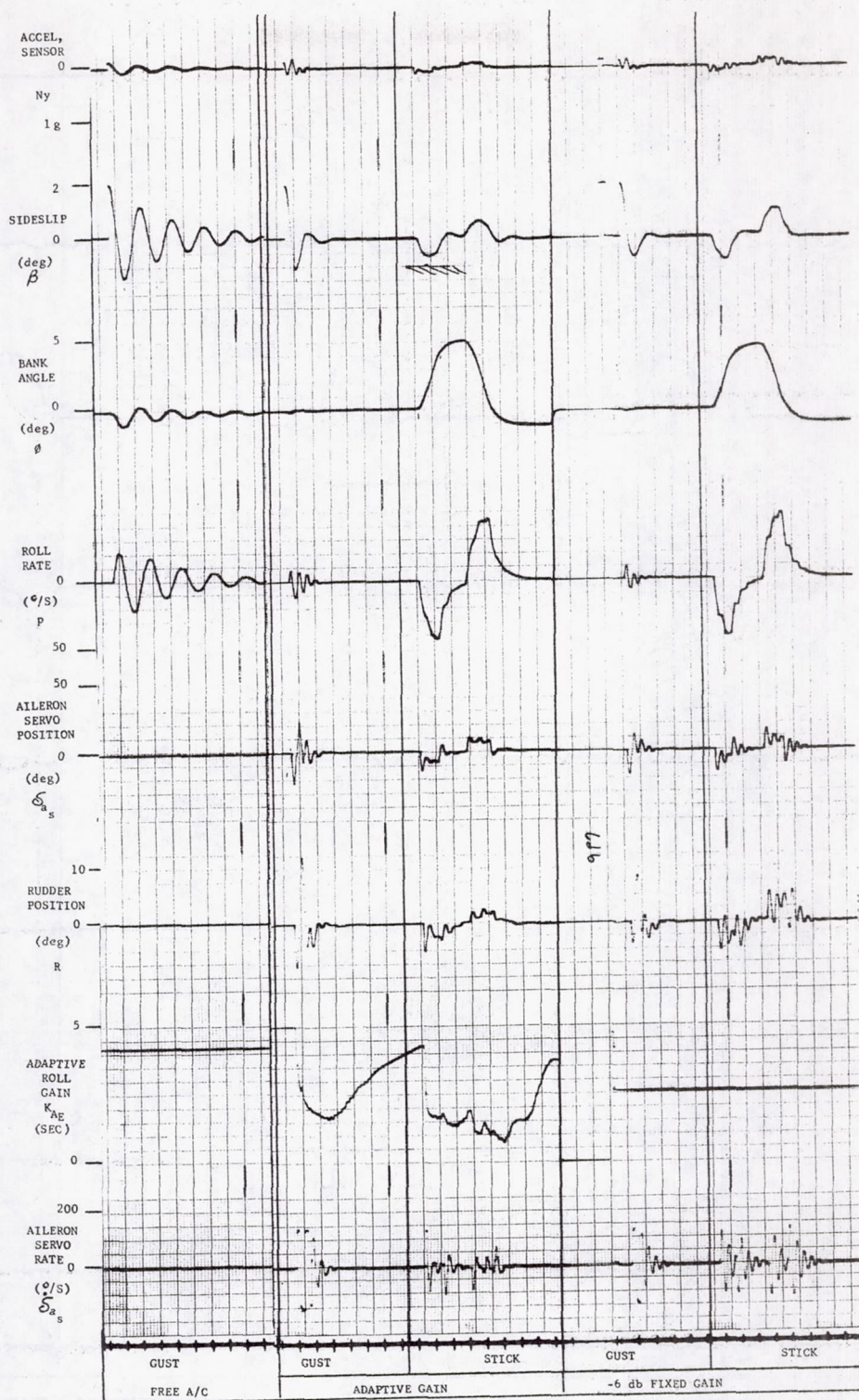


Figure 112. Flight Condition 8.

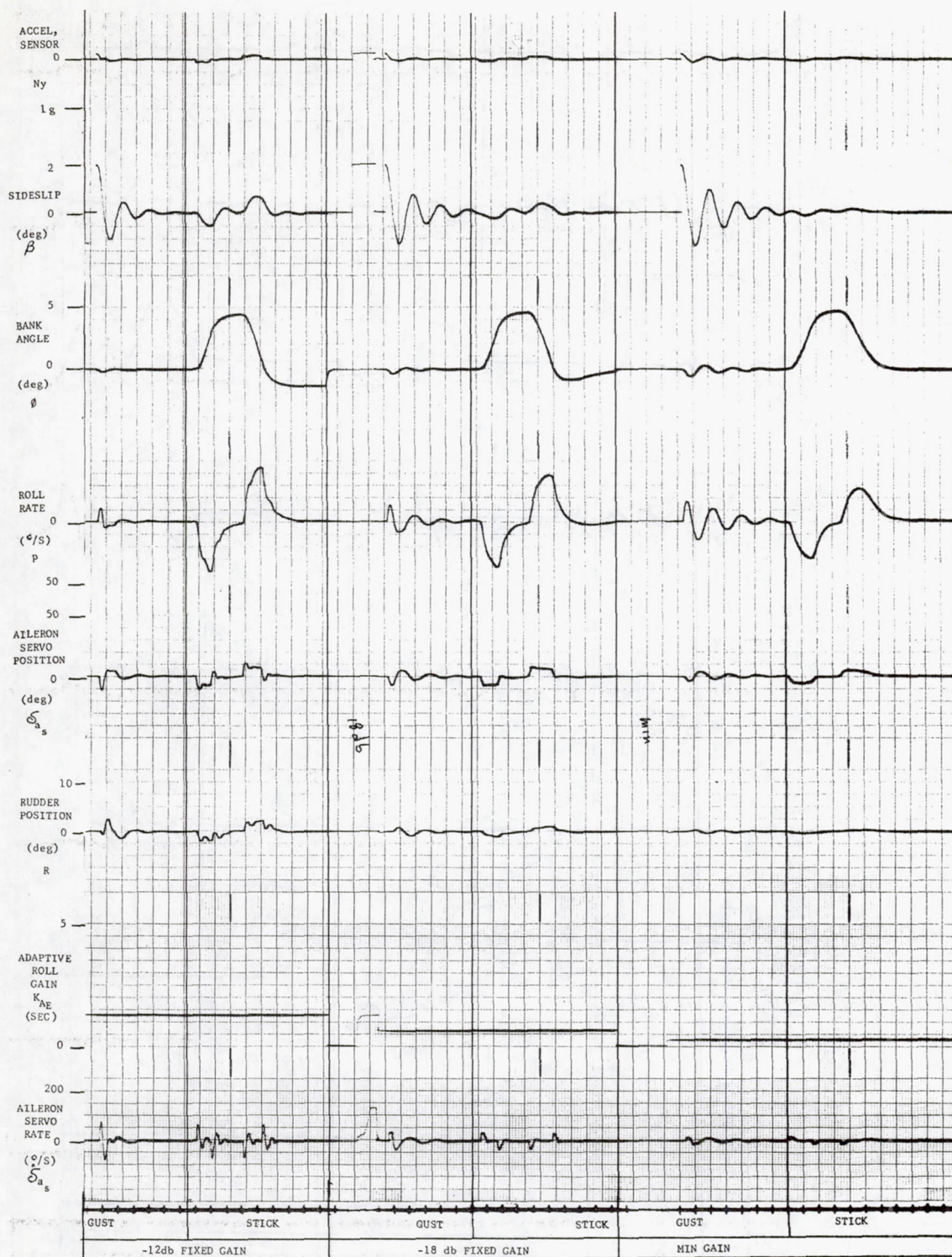


Figure 112. Concluded.

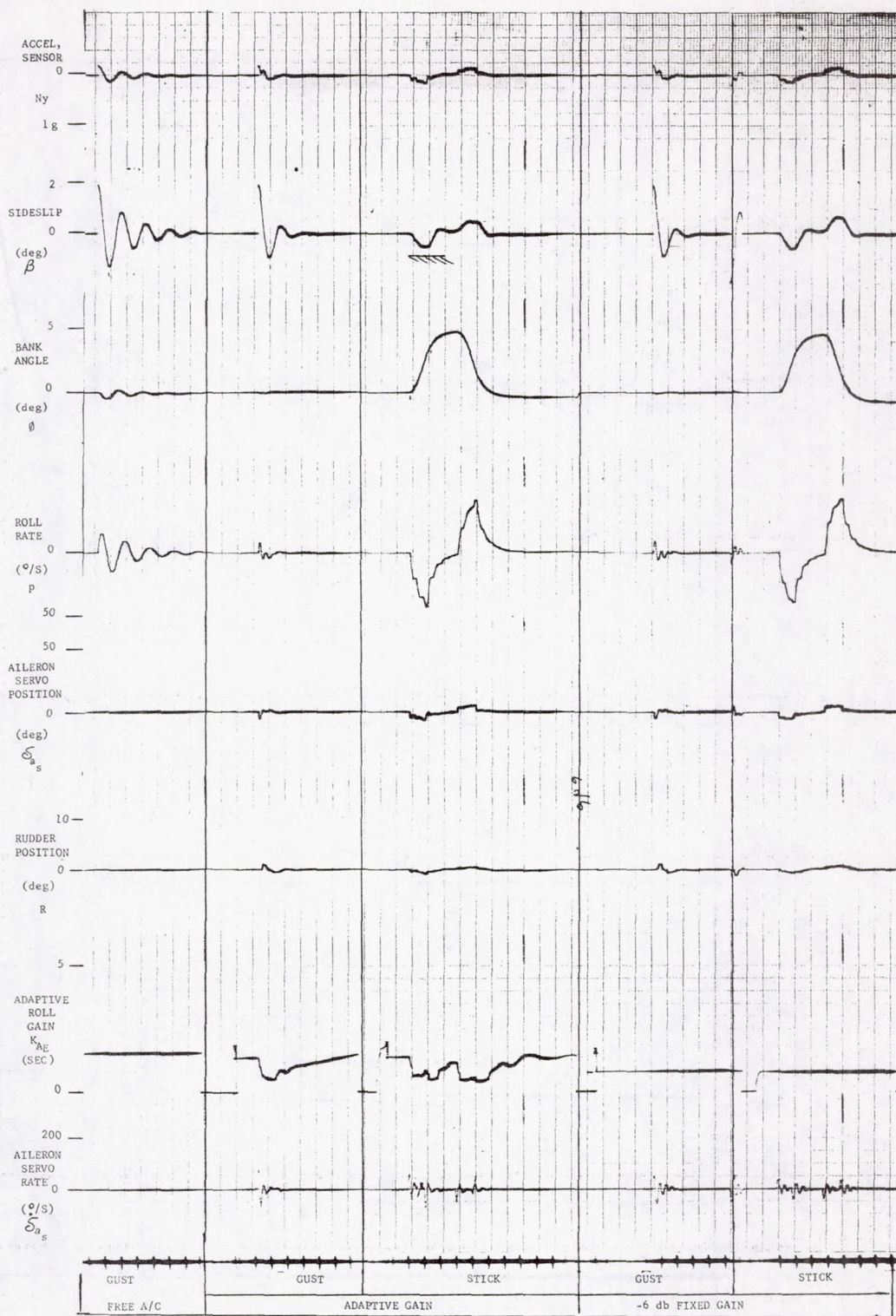


Figure 113. Flight Condition 9.

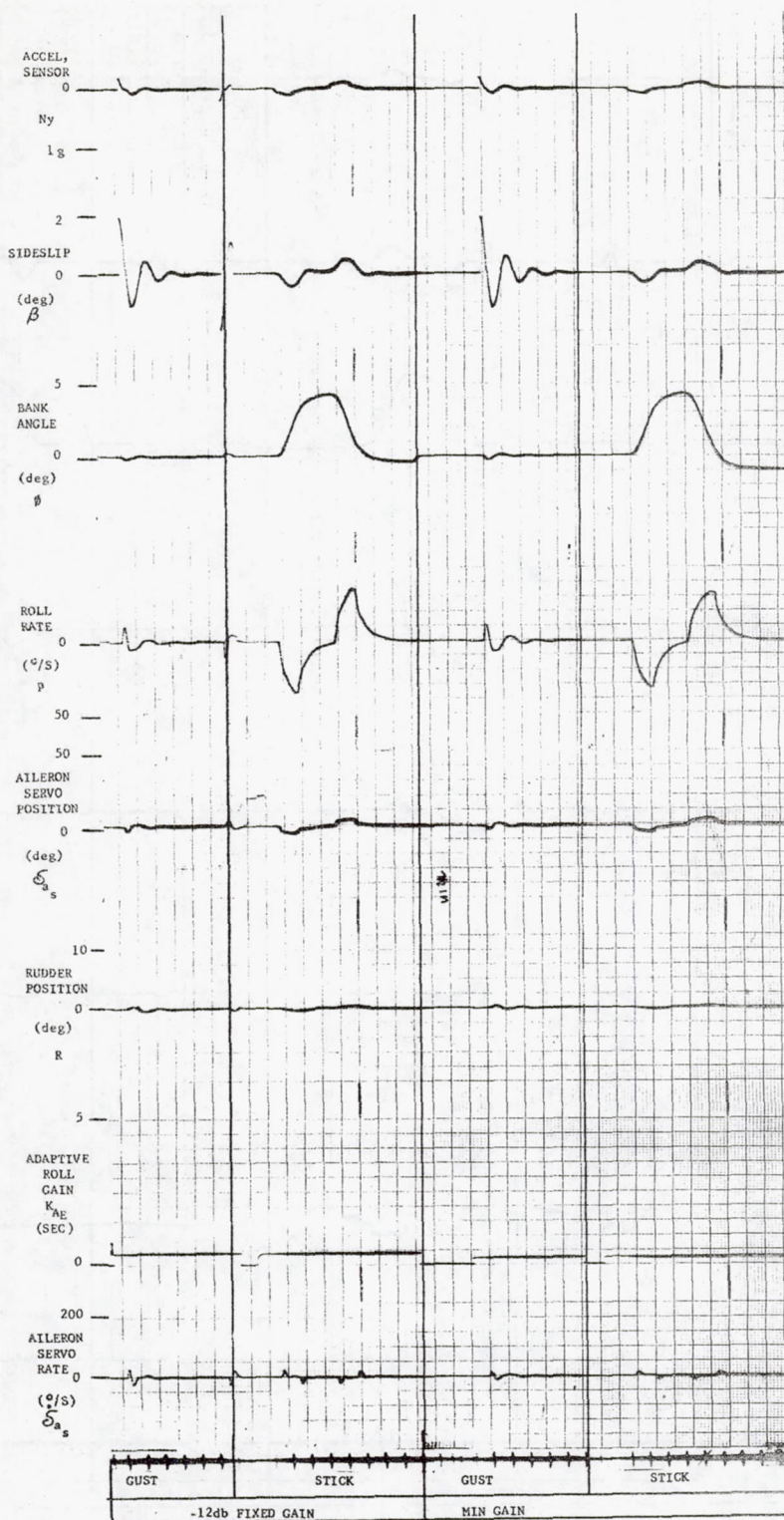


Figure 113. Concluded.

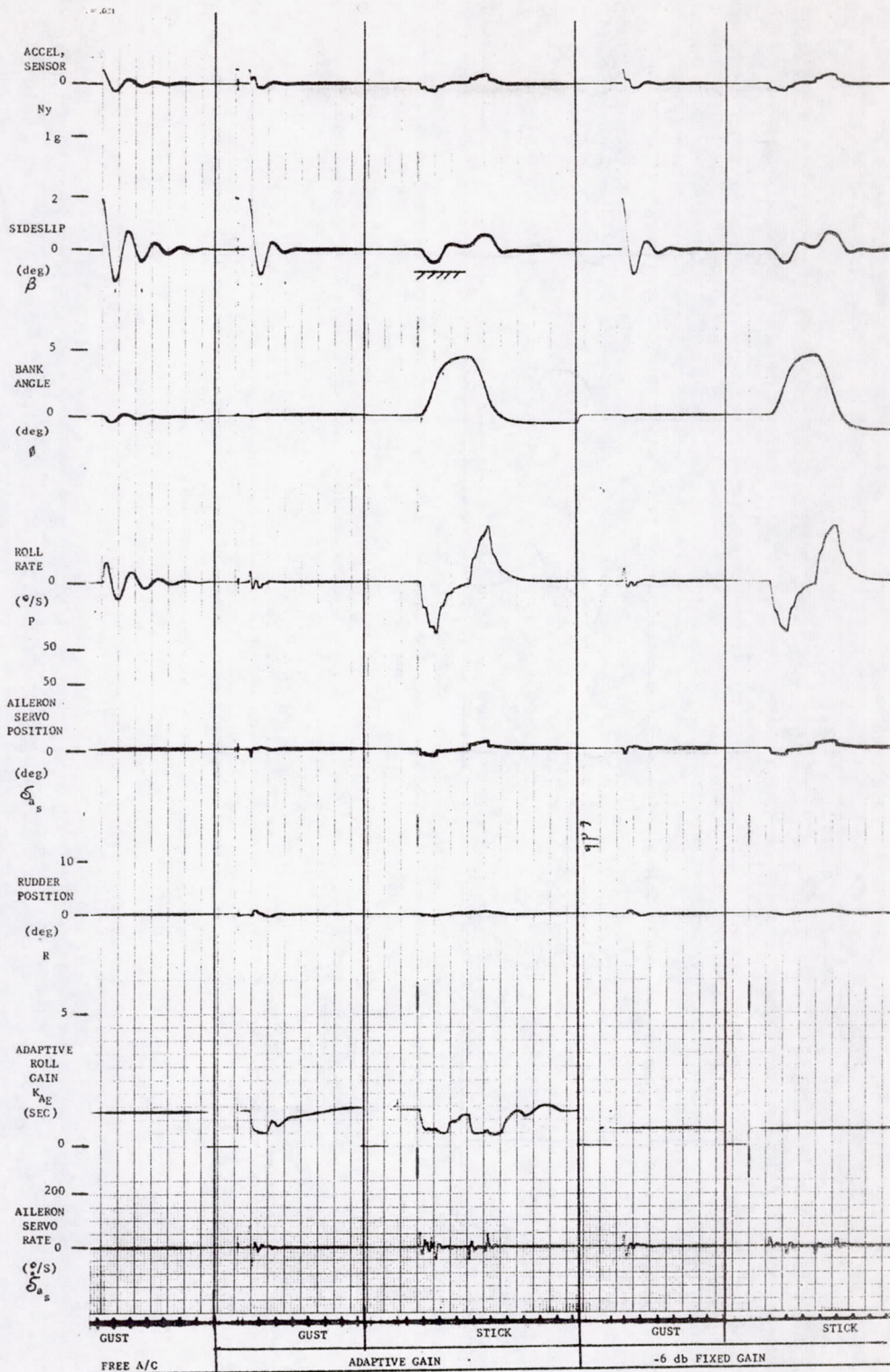


Figure 114. Flight Condition 10.

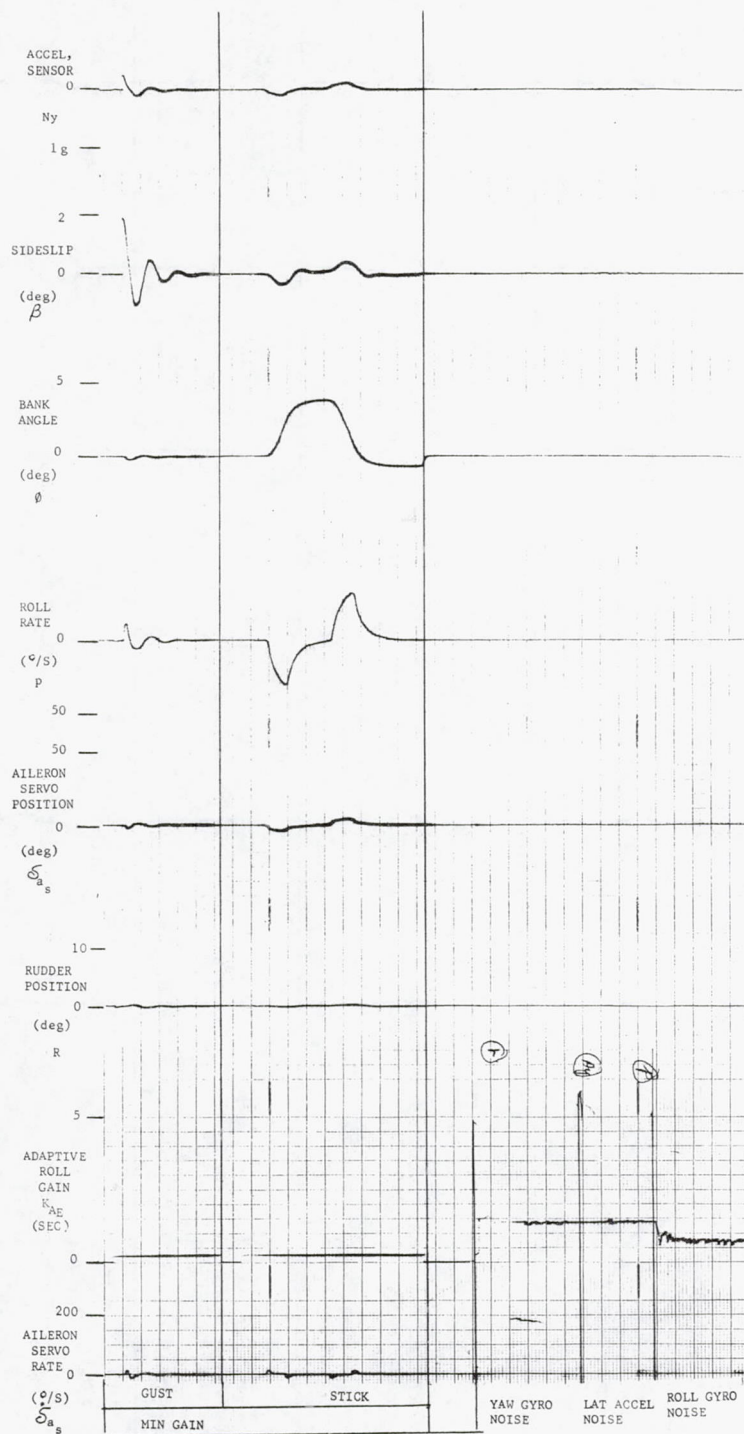


Figure 114. Concluded.

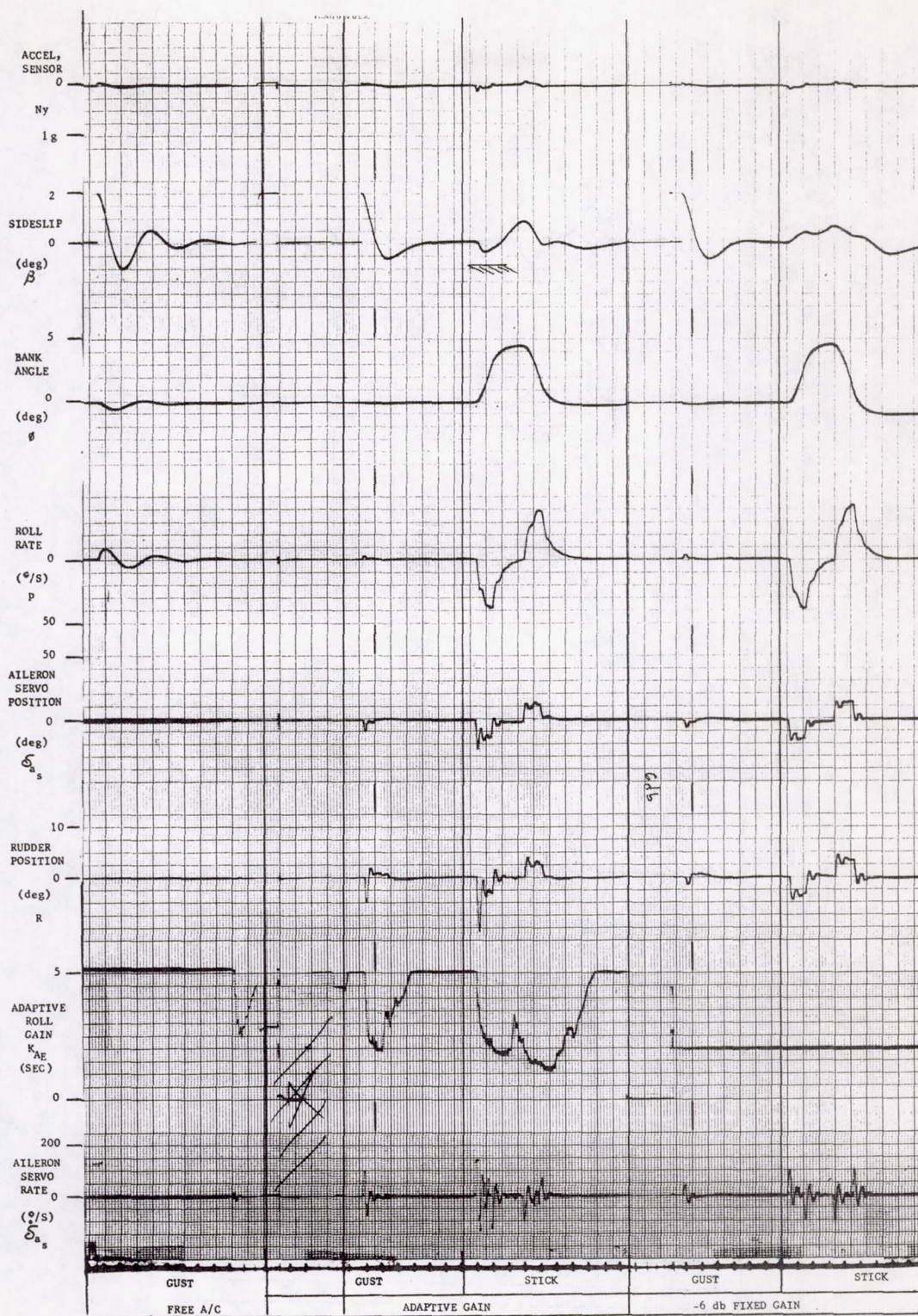


Figure 115. Flight Condition 11.

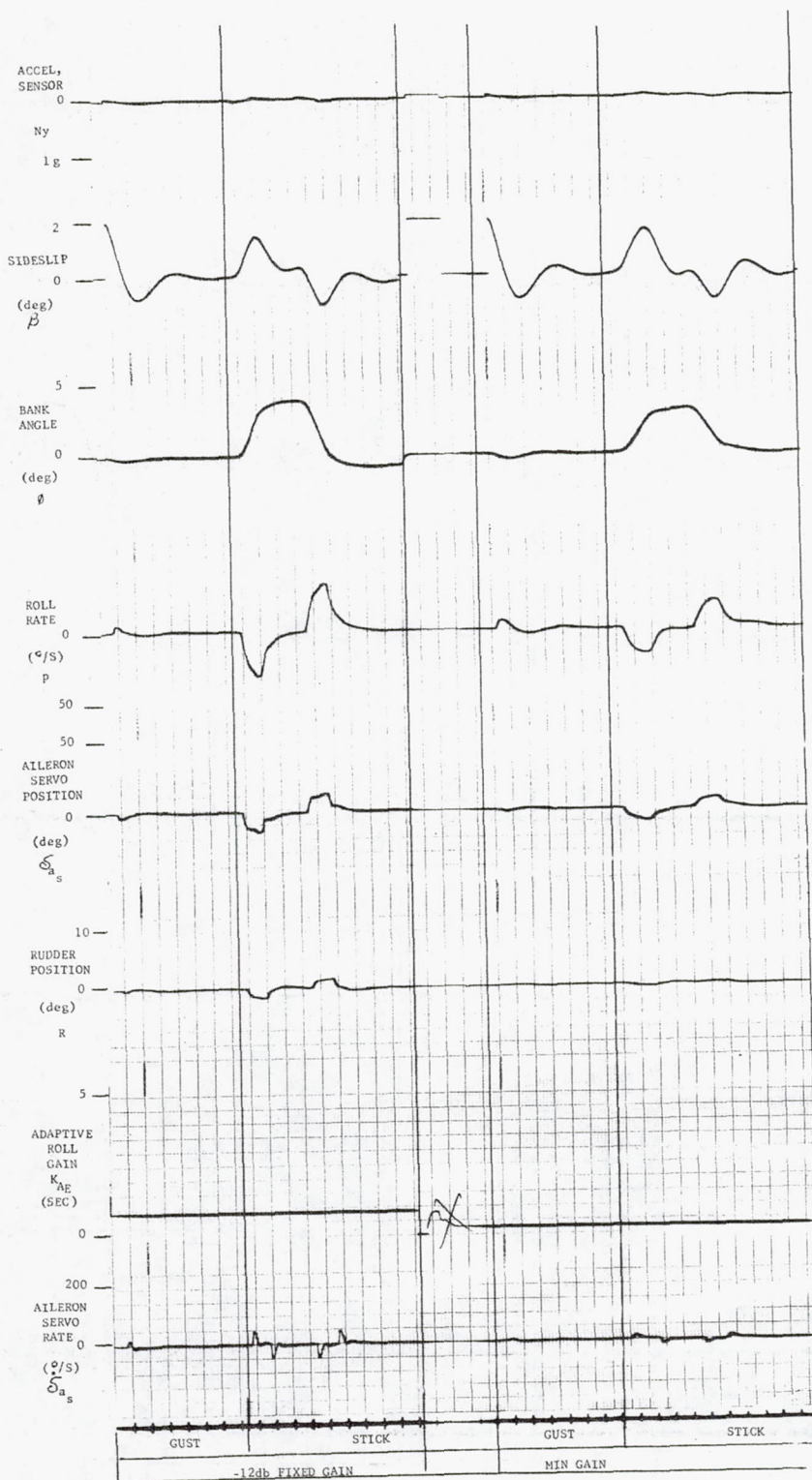


Figure 115. Concluded.

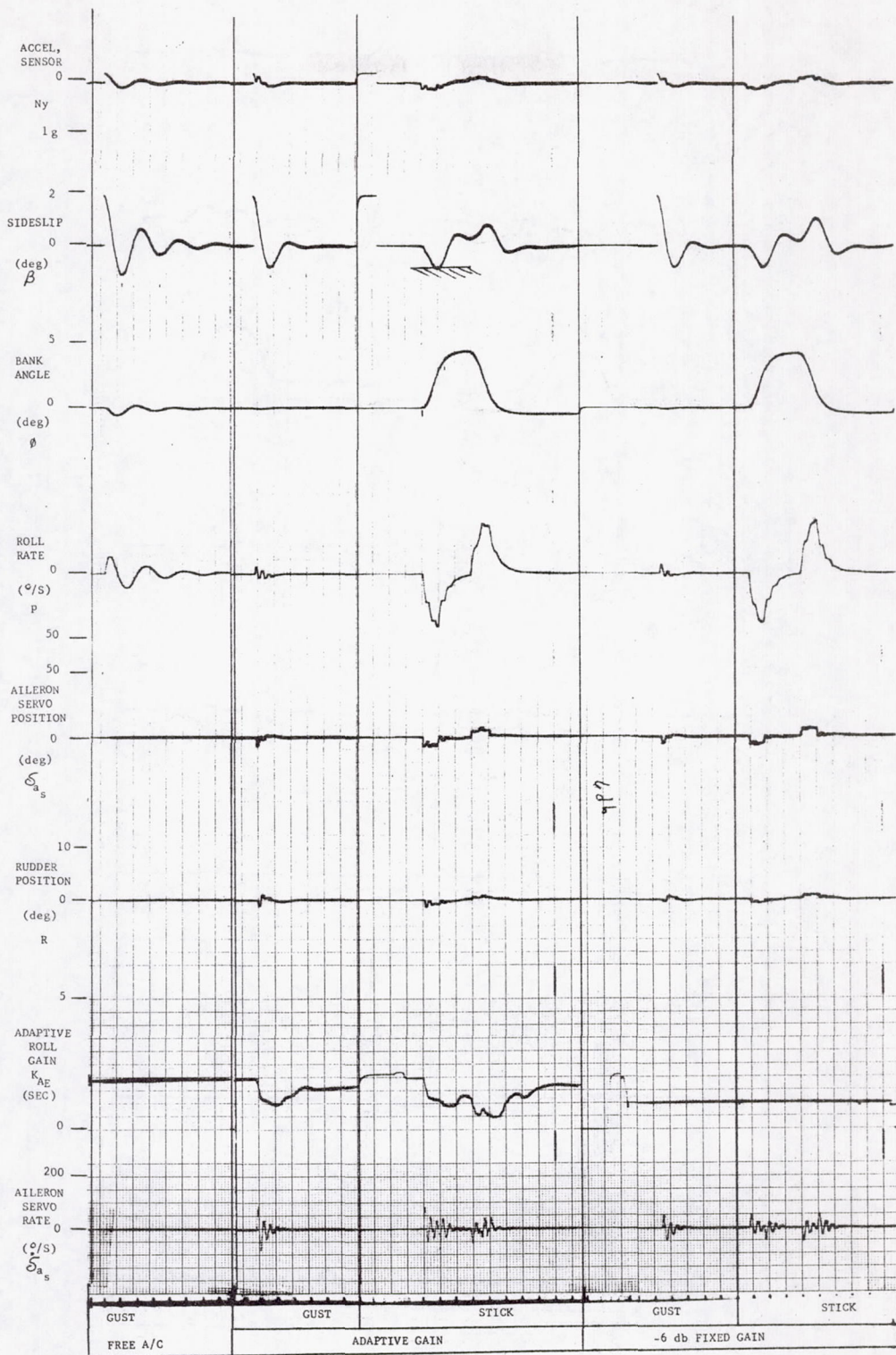


Figure 116. Flight Condition 12.

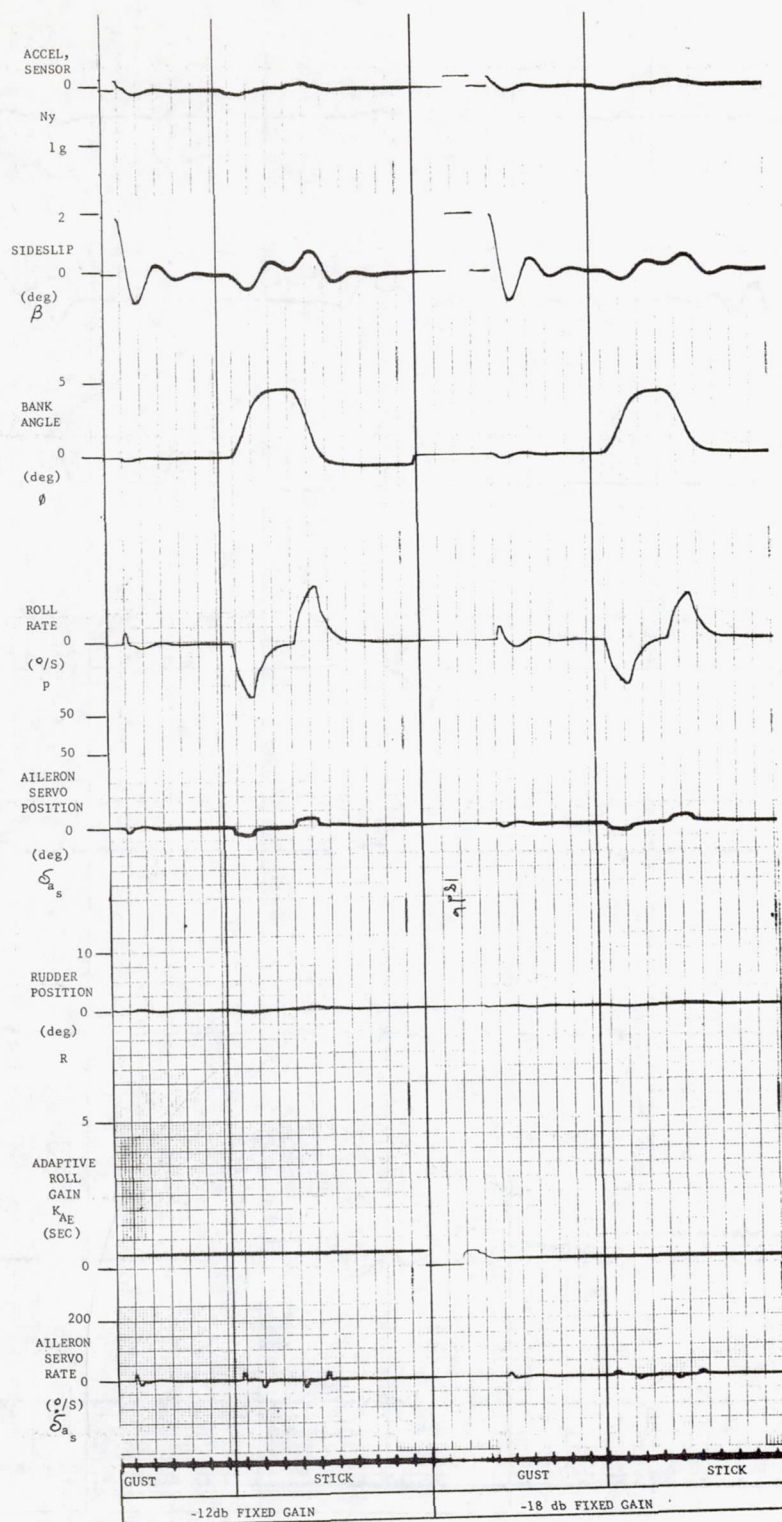


Figure 116. Concluded.

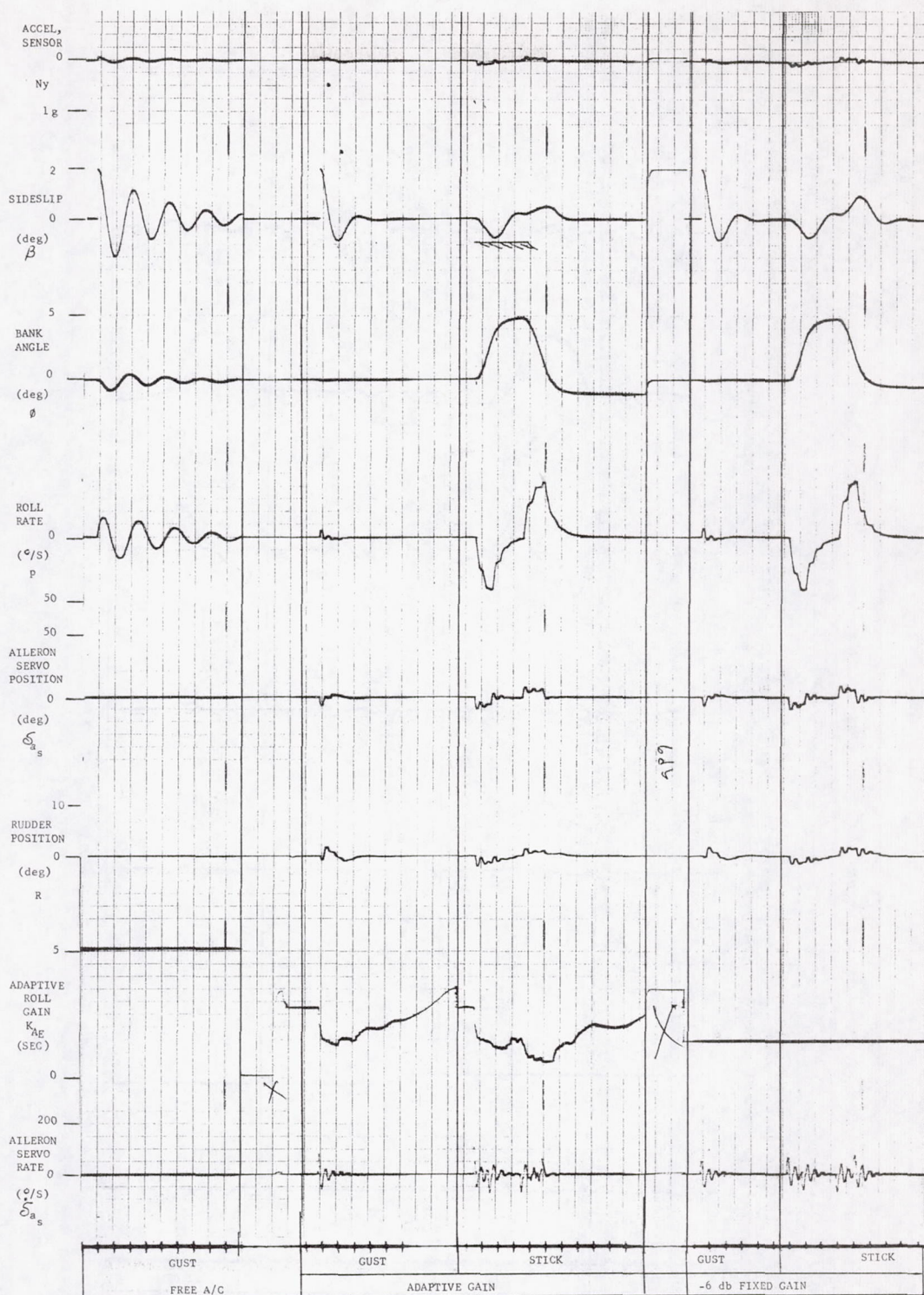


Figure 117. Flight Condition 16.

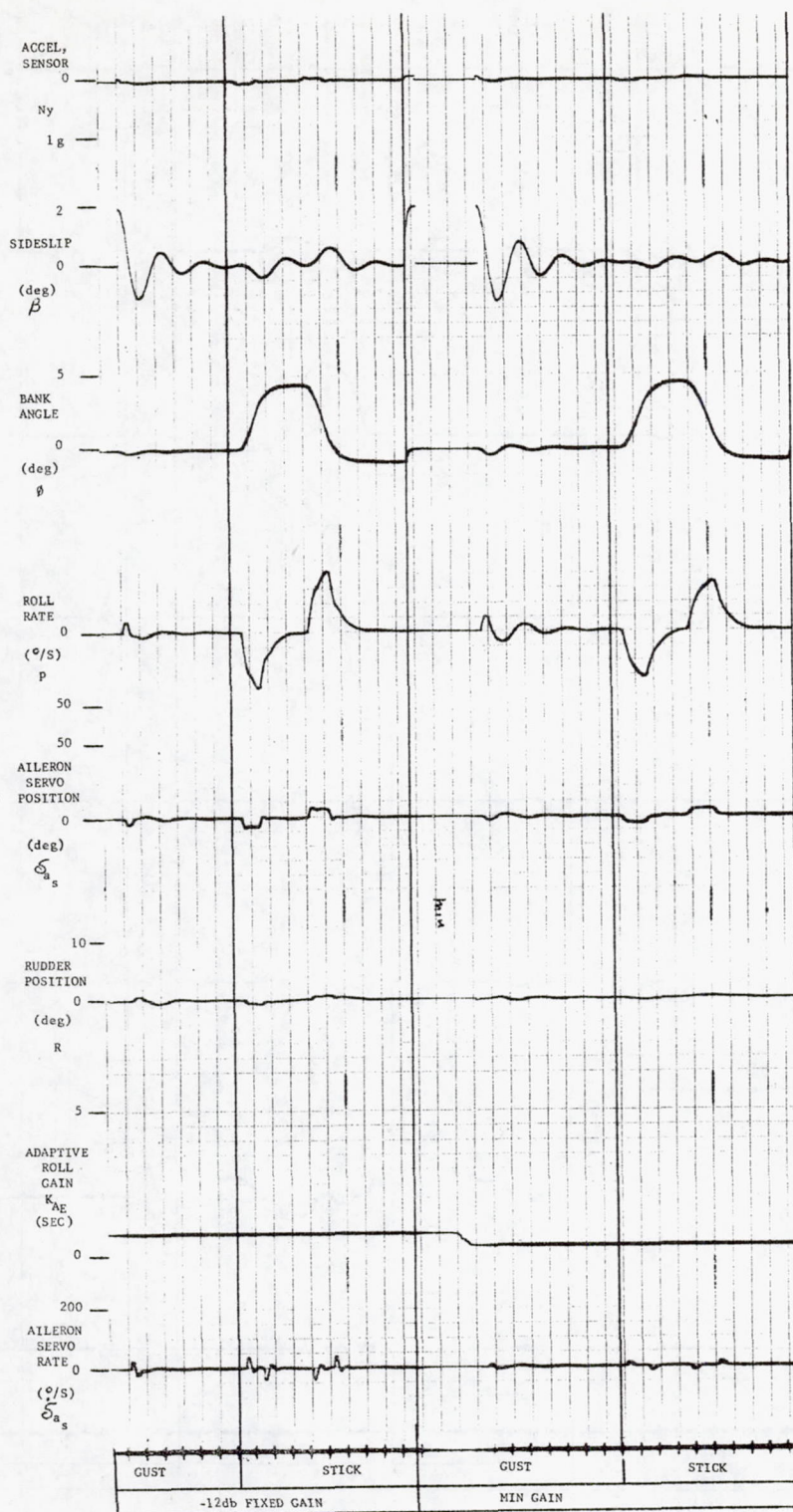


Figure 117. Concluded.

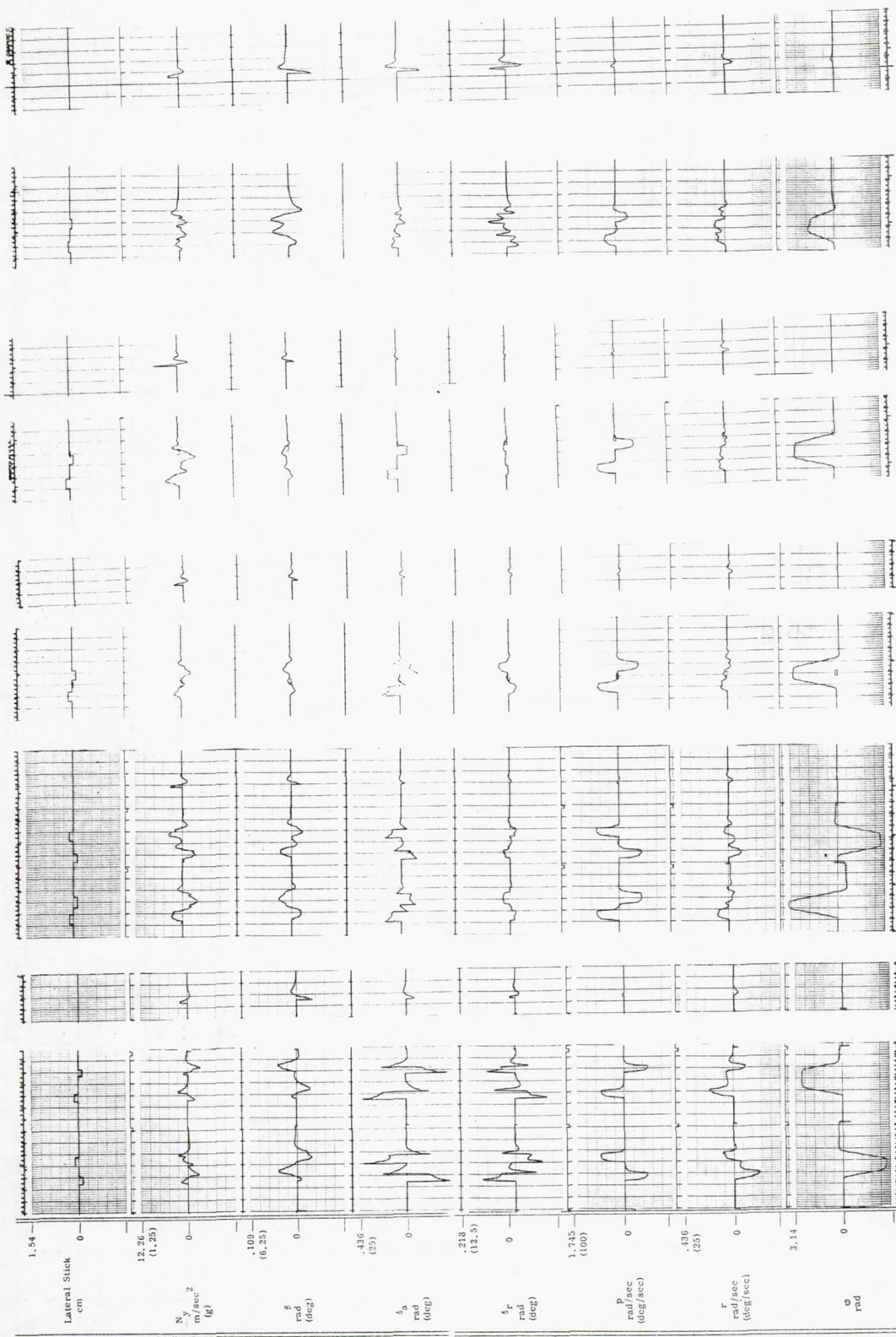


Figure 119. Lateral Responses from LRC F-8C Simulation



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—NATIONAL AERONAUTICS AND SPACE ACT OF 1958

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